

EXPERIMENTAL OBSERVATIONS ON RESONANCES

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I. ESTABLISHING THE EXISTENCE OF NEW PARTICLES

1. Remarks on lifetimes

Until about three years ago, all particles which were identified had the property of a relatively long life. By long I mean a lifetime order of magnitude longer than 10^{-24} sec, which is about the time it takes a light signal to transverse a nucleon. Most particles have an extremely long lifetime, in the range 10^{-7} - 10^{-10} sec. Our instruments are sensitive only to times in a limited range, the lowest limit reached is about 10^{-16} sec. We have been rather blind- and in a certain sense still are - to the region between 10^{-16} and 10^{-22} sec. Below about 10^{-22} we use the relation $\Gamma \cdot \tau = \hbar$. If we can measure widths $\Gamma \gtrsim 1$ MeV we can reach times $\tau \lesssim \hbar/\Gamma \simeq 6 \cdot 10^{-22}$ sec. Below $\tau = 10^{-24}$ sec the width would be > 600 MeV, i.e. of the order of the mass itself. Here it makes no sense to speak of particles. Figure 1 summarizes the experimental situation.

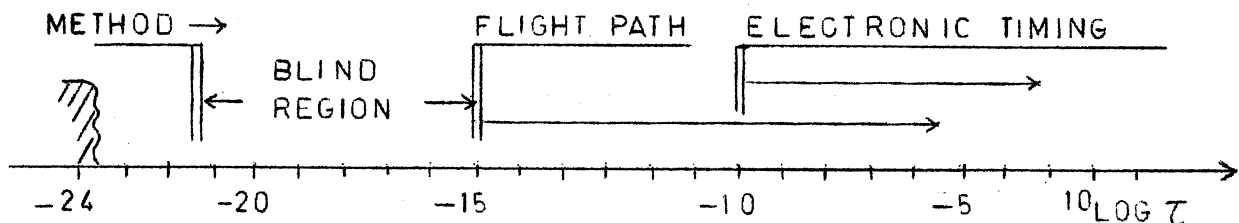


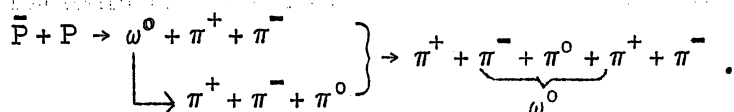
FIG. 1

The neutral pion is an example of a particle just about measurable by the flight path method. The Σ^0 is an example of a particle, probably lying in the blind region. Some of the new particles, called resonances [ρ , K^* , Y^* (1385)] are in the Γ region; some are probably at present in the blind region: η , ω , Y^* (1405). The blind region may be opened to some extent by cross-section measurements like the Primakoff method for π^0 .

The old particles (pions, K mesons, hyperons and muons) decay by weak or electromagnetic interactions. The new particles (resonances) decay by strong or electromagnetic interactions and are thus in general faster. Strangeness and parity are conserved in the last-mentioned interactions.

2. Peakology

Although the lifetimes of particles may be too short for direct measurements, their existence is revealed by a peak in the variation of a total cross-section with energy (e.g. some of the nucleon resonances or isobars) or by detecting the decay products (e.g. the pion resonances). In the latter case the final state of the interaction will contain particles other than the decay products of the resonance, e.g.



If in this case one always chose the three pions from the ω decay and plotted the distribution of their invariant mass (M^*):

$$M^{*2} = (\sum E_i)^2 - (\sum p_i)^2 ,$$

one would obtain a clustering around a central value for the M^* corresponding to the mass of the ω . In practice one does not know which charged pions to include in the triplet decay and therefore one has to plot the M^* distribution for all combinations of π^+ , π^- , π^0 in the five particle final state. If there were no resonance involved in the final state of this reaction one would expect the M^* distribution to be smoothly

distributed between a lowest limit (the sum of the masses of three pions) and the upper limit set by the total energy available. In the example taken, one gets such a distribution for pion triplets that do not come from the ω meson. The presence of the ω is seen as a peak superimposed on this background. The individual values of the energy in the peak due to the resonance are spread around the central value for two reasons; one is due to the errors of measurement, the other to the natural line width of the parent particle mass. One thinks here in terms of a Breit-Wigner distribution:

$$P(E) dE = \frac{\text{const}}{(E - E_0)^2 + \frac{\Gamma^2}{4}} dE,$$

where Γ is the full width at half maximum, E_0 and E the mass, and our energy (M^*) variable respectively. The time dependence of a quantum mechanical decaying state may be described:

$$\psi_t = \psi_0 e^{-\gamma \frac{t}{2}} e^{-iE_0 t/\hbar}$$

where E_0 is the energy. Here γ is the decay constant, because the probability of the state at time t is

$$|\psi_t|^2 = |\psi_0|^2 e^{-\gamma t} \quad (\text{exponential decay}).$$

The mean lifetime τ is related to γ in the usual way, $\gamma \cdot \tau = 1$. The state can be written as a superposition of stationary states of various energies E , i.e.

$$\psi_t = \int_{-\infty}^{\infty} A(E) e^{-iEt/\hbar} dE.$$

The amplitude $A(E)$ is obtained by Fourier's integral theorem and its square gives the probability of the energy E

$$P(E) = |A(E)|^2 = \frac{\text{const}}{(E - E_0)^2 + \frac{\Gamma^2}{4}} \quad (\text{Breit-Wigner formula}),$$

where Γ has been introduced instead of γ ; the relation being $\Gamma = \hbar \cdot \gamma = \hbar/\tau$. This shows that the relation $\Gamma \cdot \tau = \hbar = 6.58 \cdot 10^{-22}$ MeV sec is exact and not an inequality of the type derivable from the uncertainty principle.

In general, in a reaction producing a resonance in the final state one sees the decay products of the resonance and the particles directly produced as in the above example. This results in a smooth invariant mass distribution with a peak superimposed. In the energy histograms everybody now looks for peaks, signalling the existence of a parent or resonance particle. I would like to mention a danger at this stage, namely the risk of seeing too many peaks or peaks too often and also the risk of missing a peak that might be there. In some papers one reads unclear statements saying that the observed peak is statistically significant. The problem is in its simplest form of the following type. We have a histogram (ideograms should be avoided) as in Fig. 2.

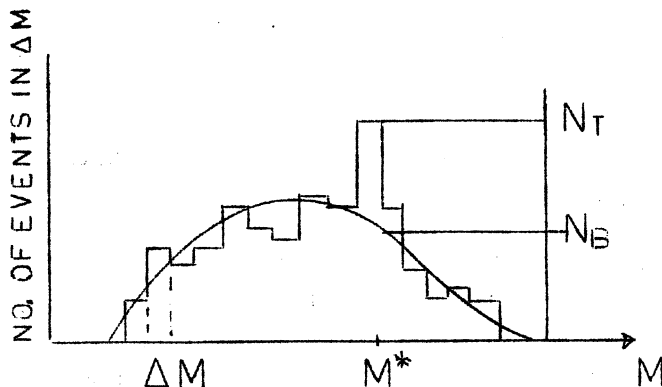


FIG 2

We wish to test if there is a significant peak above the background at M^* or if it is merely a fluctuation of the background. First of all, significance is a question of judgement. From Cramér's statistical textbooks the following useful rules are suggested:

- i) if the probability for $N \geq N_T$ is $\leq 0.1\%$, based on the hypothesis that the expected value is N_B , the peak is said to be highly significant;
- ii) if $0.1\% < P(N \geq N_T) \leq 1\%$, the peak is significant;
- iii) if $1\% < P(N \geq N_T) \leq 5\%$, the peak is almost significant;
- iv) if $P(N \geq N_T) > 5\%$, the peak is not significant.

Note: Cramér actually deals with fluctuations both ways. In conformity with that we could require even stricter rules, e.g. only for $P(N \geq N_T) \leq 0.05\%$ is the peak highly significant.

One important thing is that the experimenter should decide for himself before the experiment on these levels of significance, or others if he so prefers. The concept of statistical risk is also important. If one decides to publish discoveries of peaks based on the significance level of 10% , then one may find that too many "discoveries" disappear in the long run. It is safer, less risky, to stick to a level of 1% or less. Nowadays, when we have hundreds of histograms published every year, we are not to be surprised if a few objects which were significant one year turn out not to be so the next year. I have been told that the $T = 2$ meson at about 600 MeV ($\pi^+ \pi^+$ and $\pi^- \pi^-$) recently disappeared when the experiment in which it was found was continued.

One more thing is important. Even in one single histogram there are many bins; 10 is a low number, but let us use it for convenience. If we do not know a priori where the peak is expected, we have 10 chances to find a fluctuation. To get a level of significance of 1% for the whole histogram, we must set our standard as high as 0.1% for each bin. If we know the expected position, this is not necessary. The latter case applies, for instance, to a situation where we look for η^0 production (or any other already known state) in a new reaction. We then know where to look for a peak.

I now come to the question of how to test the significance. Some people take the $\sqrt{N_T}$ and test whether $N_T - N_B > 3 \sqrt{N_T}$ or generally $> \alpha \sqrt{N_T}$. Others use $\sqrt{N_B}$ and test whether $N_T - N_B > 3 \sqrt{N_B}$. Still others compute the

strength of the line $S = N_T - N_B$ and its compounded error $\Delta S = \sqrt{N_T + N_B}$ and test whether S is significantly different from zero, i.e. $S \geq 3 \sqrt{N_T + N_B}$. All three cannot be correct as they might lead to different results. Let me exemplify. I decide first on which probability level, I will regard a peak as significant. Let me choose the rules just stated. If the background is $N_B = 16$ events and on top of that 10 events, one reaches 26 events ($= N_T$). The observed difference of 10 is equal to $2.5 \sqrt{N_B}$; the probability of a background fluctuation ≥ 10 is 0.6%. The peak is thus significant but not highly significant. This is the correct treatment, because it is based on the hypothesis that there is no peak on top of the background. If we test on the basis of $\sqrt{N_T} = \sqrt{26}$ we get $P = 3\%$ and on the basis of $\sqrt{N_T + N_B} = \sqrt{42}$, we get $P = 6\%$, which is not significant, but not correct treatment either. Many difficulties arise when determining the background level in the region of the peak when it is wide. The sure thing is to get peaks well above the background as in Fig. 3 (ω production; cf. Fig. 4, the basis for ω discovery).

II. DETERMINATION OF PROPERTIES OF RESONANCE PARTICLES

1. Mass

The most used method already introduced in the last section is very general and consists of a calculation of the invariant mass (M^*) of the decay products in their rest system. Note:

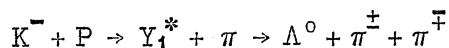
$$M^{*2} = (\sum E_i)^2 - (\sum \vec{p}_i)^2.$$

If the final state contains only one other particle (numbered 3) apart from the resonance particle under study, we have a two-body reaction which can be treated as follows:

$$\begin{aligned} M^{*2} &= (\sum E_i)^2 - (\sum \vec{p}_i)^2 \\ &= (E_0 - E_3)^2 - (\vec{p}_0 - \vec{p}_3)^2, \end{aligned}$$

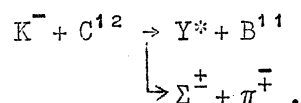
where E_0 and p_0 are the energy and momentum of the initial state.

If we work in the rest system of the initial state (or discuss a reaction at rest in the lab) then $\vec{p}_0 = 0$ and $M^{*2} = E_0^2 + m_3^2 - 2E_0E_3$ so that M^* varies with E_3 , the energy of the third particle (all other things being constant). This means that a sharp value of M^* is reflected into a constant value of E_3 and of course also its momentum p_3 . This was how the first excited hyperon Y_1^* (1385) was discovered in the reaction



at 1.15 GeV/c (K beam) by the Berkeley group.

Similarly Frisk (Stockholm) has observed a sharp line of $|\vec{p}_1 + \vec{p}_2| (= p_3)$ in a reaction of K^- with emulsion nuclei, interpreted as



I will return later to this case, since if the results are borne out when higher statistics are available, it appears to be the resonance best suited for a detailed study by the nuclear emulsion technique. For neutral states and also more general situations, the missing mass computation is of great value.

2. The lifetime τ (or width Γ)

This quantity is known only for a few of the new resonance particles. In bubble chamber experiments, which dominate this field of research, the widths obtained have varied rather widely. What this means is difficult to say; one can think of measurement errors or, if these are not sufficient, final state effects. In two cases, the η^0 (548 MeV) and the ω^0 (782 MeV), the width seems to be too small to measure with available techniques. In a third case, Y^* (1405), the situation is unclear. With bubble chambers, a width of about 50 MeV (resolution claimed 5 MeV) was obtained by several groups. Using nuclear emulsions, the sharp peak observed at Stockholm mentioned in Section II.1, for the momentum of the Y^* from K^- absorption on C^{12} , would imply a width of 2 MeV for the Y^* (1405). The emulsion work requires confirmation based on much higher

statistics before discussing the possibility of, e.g. two resonant states, etc. For the π -nucleon resonances, the behaviour with energy of the π nucleon total cross-section gives the width.

3. Quantum numbers

The quantum numbers are listed below:

B = baryon number (baryon B = + 1, antibaryon B = - 1, meson and lepton B = 0);

S = strangeness (restricted at present to S = 0, ± 1 , - 2);

T = isobaric spin (T = 0 charge singlet, T = $\frac{1}{2}$ doublet, T = 1 triplet);

J = spin;

P = parity;

G = G parity.

The first three are obtained most easily. The production or decay channels give B and S immediately from the fact that these numbers are separately conserved in the strong interaction processes. In several cases the T (isobaric spin) has been determined by search for states with the same mass but different charge. Thus the ω mass of 782 MeV is obtained only in the neutral combination $\pi^+ \pi^- \pi^0$ not in any charged combination; thus ω is a T = 0 particle. Also the use of Clebsch-Gordan coefficients in comparing the observed intensities in various decay channels has been used. This is exemplified for the ρ meson. The reaction studied by Erwin et al.¹⁾ was as follows.

Table 1

	T = 0	T = 1	T = 2	<u>Exper.</u>
$\pi^- + P \rightarrow P \pi^- \pi^0$	0	1	1	1
\downarrow $\rightarrow n \pi^+ \pi^-$	2	2	2/9	1.7 ± 0.3
\downarrow $\rightarrow n \pi^0 \pi^0$	1	0	4/9	$\leq 0.25 \pm 0.25$

Table 1 shows the expected branching ratios for $T = 0, 1$ and 2 . The experimental result points to $T = 1$ for the ρ meson. It should be mentioned that the data refer to peripheral π - p reactions (in essence π - π scattering).

The assignment of spin and parity is of great importance for the classification of the particles and for a deeper understanding of their structure. These quantum numbers are still not known for some resonances, especially not for the hyperon states (like Y_0^* , etc.). However, we recall that only recently has the Σ - Λ parity been established (by a CERN group) with greater confidence as positive and that the spin of the Σ is only very recently fixed with such confidence to be $1/2$.

The situation is much better for the mesonic states (Table 2) where K^* (888 MeV) is 1^- (vector), K is 0^- (pseudoscalar).

Table 2

<u>J^{PG} for non-strange mesons</u>	0^{--} $\pi \quad T = 1$	0^{-+} $\eta \quad T = 0$	} pseudoscalar mesons
	1^{--} $\omega \quad T = 0$	1^{-+} $\rho \quad T = 1$	} vector mesons

Here the π has been included to make the symmetry evident. In fixing the values of spin and parity for ω and η , the Dalitz diagram, once invented for the K meson (τ decay), is useful. The procedure is roughly as follows.

- a) A Dalitz diagram is a two-dimensional plot of events for three-body reactions, e.g. $\omega \rightarrow \pi^+ + \pi^- + \pi^0$. In the c.m.s. of the parent particle the energy sum is $T_1 + T_2 + T_3 = Q = \text{constant}$. So each event can be represented by a dot in a two-dimensional diagram (only two energies are independent). Figure 5 shows three ways of making such a diagram.

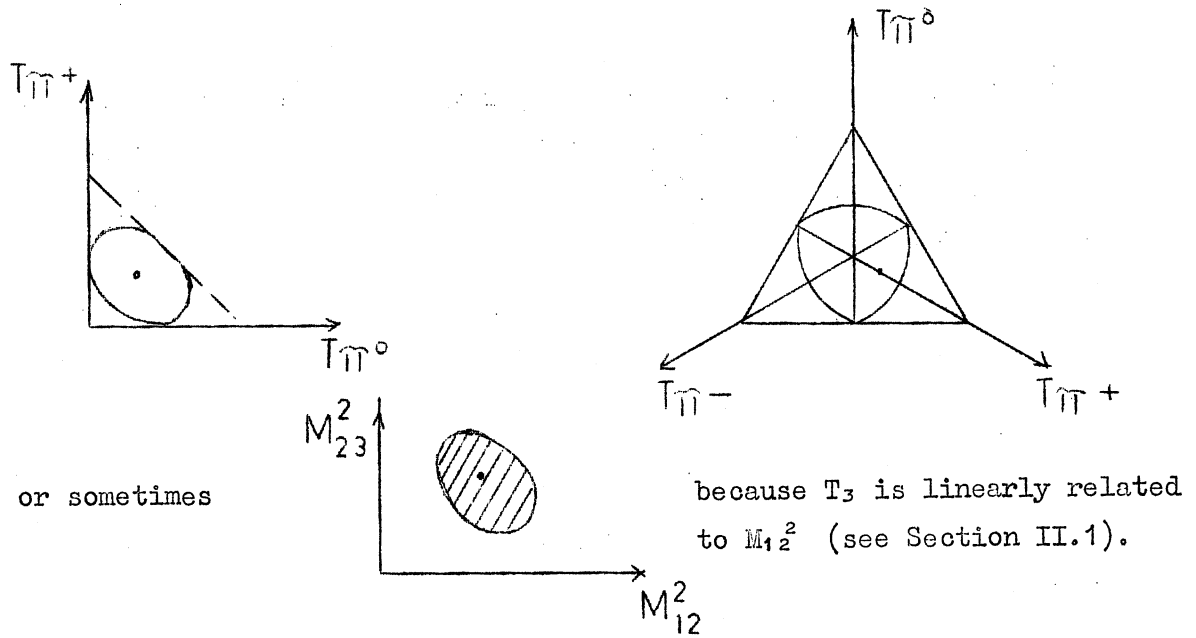


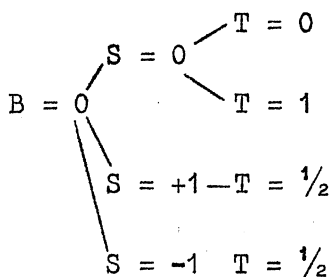
FIG. 5

- b) Each event is represented by a dot in the diagram. Only a certain area of the diagram is allowed by energy-momentum conservation.
- c) The density of points is the important thing to study, because it is proportional to the square of the matrix element for the transition. The reason that this is so is that the phase space volume is proportional to $dT_1 dT_2$, the elementary area in the diagrams. Figure 6 shows how the density may vary with spin and parity. Figure 7 is the Dalitz diagram for ω and Fig. 8 a radial plot for ω .

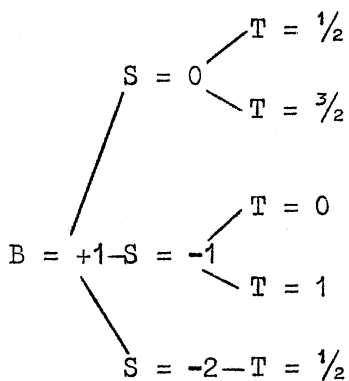
4. Particle systematics

The systematization of the particles, resonance particles and old particles together, is based on the quantum numbers. We leave the leptons ($e \mu \nu_e \nu_\mu$) out and also the photon (γ) and study only the strongly interacting particles. The start is made with the baryon number B , which is strictly conserved, then the strangeness S , the isobaric spin T and finally spin J and parity P . Table 3 shows isospin multiplets.

Table 3



J^P				
0^+	0^-	1^-	1^+	unknown
	$\eta^0(548)$	$\omega^0(782)$		$f^0(1250)$
	$\pi(140)$	$\rho(750)$		$\xi(560)$
	$K(494)$	$K^*(890)$		
	$\bar{K}(494)$	$\bar{K}^*(890)$		



$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{3}{2}^+$	unknown
$N(938)$		$N^*(1512)$		$N^*(2190)$
			$N^*(1238)$	$N^*(1920)$ $N^*(2360)$
$\Lambda^0(1115)$		$Y_0^*(1520)$		$Y^*(1405)$
$\Sigma(1190)$		$Y_1^*(1660)$		$Y_1^*(1385)$
$\Xi(1321)$				$\Xi^*(1530)$

Note how well filled are the meson columns (0^-), (1^-), leaving 0^+ , 1^+ empty. The octets (see Van Hove's lecture) are found in these columns and in the one containing the nucleon. One new octet may be formed in the $\frac{3}{2}^-$ column. This new octet should then consist of the doublet $N^*(1512)$, the singlet $Y^*(1520)$, the triplet $Y_1^*(1660)$ and a yet to be discovered doublet with strangeness $S = -2$.

5. Production channels

A large variety of reactions have been studied in which the resonances occur. In a few cases, the threshold is below the masses of the particles in the initial state, such as in the production of pion resonances (ω , ρ , K^*) in $P\bar{P}$. Also η is below the threshold, of course, but has not been observed. This fact might not exclude it being produced in the reaction because of two factors:

- i) the phase space is low in the η region;
- ii) the η is decaying to charged pions in only about 25% of the events.

The η has been observed in the following reactions.

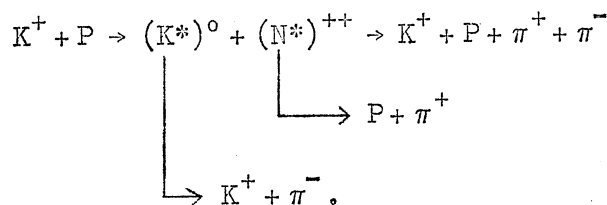
	<u>Threshold kinetic energy</u>
a) $\gamma + P \rightarrow \eta + P$	710 MeV
b) $\pi^+ + P \rightarrow \eta + P + \pi^+$	790 MeV
c) $\pi^- + P \rightarrow \eta + n$	560 MeV
d) $\pi^+ + d \rightarrow \eta + P + P$	480 MeV
e) $K^- + P \rightarrow \eta + \Lambda$	380 MeV
f) $P + P \rightarrow \eta + P + P$	1260 MeV

a similar list for ω contains:

g) $\pi^- + P \rightarrow \omega + \text{neutrals}$	960 MeV
h) $\rightarrow \omega + \pi^- + P$	1220 MeV
i) $\pi^+ + P \rightarrow \omega + \pi^+ + P$	1220 MeV
j) $\pi^+ + d \rightarrow \omega + P + P$	802 MeV
k) $K^- + P \rightarrow \omega + \Lambda^0$	825 MeV
l) $P + P \rightarrow \omega + P + P$	1890 MeV
m) $P + \bar{P} \rightarrow \omega + \pi^- + \pi^+ + i\pi^0 \quad i \geq 0$	< 0 MeV
n) $\rightarrow \omega + 2\pi^+ + 2\pi^-$	< 0 MeV
o) $\rightarrow \omega + K^+ + K^-$	< 0 MeV
p) $\rightarrow \omega + K^0 + \bar{K}^0$	< 0 MeV.

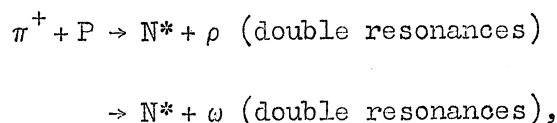
Figure 9 shows results of reactions (d) and (j). Figure 10 on reaction (o) shows how free from background ω is produced, e.g. nearly all pions produced come via ω (~ 90%).

Nature seems to favour the production of a few particles, which then decay to a few particles giving a final state of larger multiplicity. A striking example is the following reaction reported by W. Chinowsky et al.²⁾



The total cross-section for K^+P is at the energy of this experiment 18 ± 3 mb. The channel with $K^+P \pi^+ \pi^-$ corresponds to about 2 mb and of this about 80% goes through the double resonance K^*N^* . In addition some cases correspond to $K^*P \pi^+$ and some to $K^+ \pi^- N^*$. Thus only a few per cent of the cases proceed without involving a resonance particle (Fig. 11).

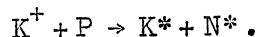
Another example of interest is the reaction $\pi^+ + P \rightarrow P + \text{several } \pi$ as studied by Alff et al.³⁾. Here the three pionic resonances ρ , η and ω were all found and with considerable intensities for the ρ and ω . Also the cross-section is appreciable for



where N^* is the pion-nucleon first resonance ($\frac{3}{2}, \frac{3}{2}$). The intensities for ω are impressive. Thus ω is produced in 50% of the sample with $\pi^+ \pi^- \pi^0$ present. Of this sample 10-15% seems to go via $N^* + \omega$.

About ten years ago the multiple production of particles, mainly pions, was thought of as a statistical mechanics problem. The Fermi theory for evaporation of many pions from the heated interaction volume of two colliding high-energy particles was used during the 1950's. The situation has now changed and the multiple particle states are far more interesting than anybody could dream of ten years ago.

The observed production dynamics of the resonances can be used to determine their intrinsic properties. As an example we can take the reaction mentioned above:



In condensed form the following holds:

- i) the K^* is emitted strongly forward;
- ii) an Adair analysis was made on the forward events to obtain information on the spin of K^* ;
- iii) the angle α is the angle between K^+ outgoing in the c.m.s. of K^* and the incident K^+ direction. The distribution of α was found to be $\cos^2 \alpha$;
- iv) the anisotropy rules cut $J(K^*) = 0$;
- v) spin of K^* is from $\cos^2 \alpha$ evidence $J(K^*) \geq 1$.

Alston et al.⁴⁾ have evidence that $J(K^*) < 2$ from which it is concluded that

$$J(K^*) = 1 .$$

As for the parity, it is odd on the basis of $P(K^*)$ is odd and spin $J(K) = 0$, because then $K + \pi$ is in a p-state which is odd (the odd intrinsic parities of K and π cancel). K^* is thus $J^P = 1^-$, nicely fitting in the octet of vector mesons with ω and ρ and \bar{K}^* . The CERN study [Armenteros et al.⁵⁾] of $\bar{P} + P \rightarrow K^* + K$ together with arguments by M. Schwarz also is evidence for the assignment of vector character to the K^* .

6. Decay channels

A new state of resonance (particle) is discovered by the study of a particular combination of decay products. The next important question that arises is whether there are other decay modes of the same state. The identification of these presents new difficulties. The branching ratios are of great importance for the understanding of their properties. The following discusses what is at present known in this field.

η (548) For the η meson several groups have reported a large amount of neutral decays of unidentified nature. Thus in the reaction $K^- + P \rightarrow \Lambda + \text{neutrals}$, the missing mass of η occurred⁶⁾. Likewise in $P + P \rightarrow P + P + \text{neutrals}$ ⁷⁾, in $\pi^+ + d \rightarrow P + P + \text{neutrals}$ ⁸⁾ and $\pi^+ + P \rightarrow \pi^+ + P + \text{neutrals}$ ³⁾. Events with η mass were found in such quantity that the branching ratio is

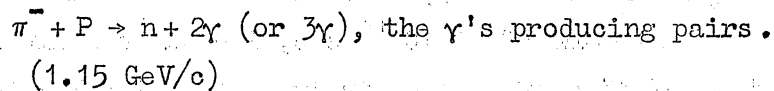
$$\frac{\eta \rightarrow \text{neutrals}}{\eta \rightarrow \pi^+ \pi^- \pi^0} = 3 \pm 0.5 .$$

The nature of the neutral modes are important as can be seen from the following Table 4.

Table 4

J^{PG} of η	Expected dominant decay mode
1^{--}	$\pi^0 + \gamma$
0^{--}	2γ and $3\pi^0$
1^{++}	$\pi^0 + 2\gamma$ and $3\pi^0$

If the 2γ mode is observed it fixes $J = 0$ for the η . It has been observed by Chrétien et al.⁹⁾ and more recently by Behr et al. at the Ecole Polytechnique¹⁰⁾. The last-mentioned group worked with a heavy liquid bubble chamber at "Saturne". In such a chamber the photons have a good chance to produce electron pairs. The reaction was:



The mass of the parent of 2γ is found from

$$M^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 = 2E_1 E_2 - 2p_1 p_2 \cos \Phi$$

$$= 2E_1 E_2 (1 - \cos \Phi) = 4E_1 E_2 \sin^2 \Phi/2 \text{ (note } E = p \text{ for photons).}$$

The background caused by unrelated γ 's from two π^0 in $\pi^- + P \rightarrow n + \pi^0 + \pi^0$ could be subtracted and what remained was as

shown in Fig. 12. They also find no evidence for

$$\eta \rightarrow \pi^0 + \gamma \quad (\rightarrow \gamma + \gamma + \gamma).$$

A further study of η by Fowler et al.¹¹⁾ has shown that $\eta \rightarrow \pi^+ + \pi^- + \gamma$ occurs (Figs. 13 and 14). The branching ratio they obtain is:

$$\frac{\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma)}{\Gamma(\eta \rightarrow \pi^+ \pi^- \pi^0)} = 0.26 \pm 0.08 .$$

The branching ratio for $\eta \rightarrow 3\pi^0$ is not experimentally known and is more difficult to study. Wali²¹⁾ predicts that the ratio depends on the π^0 energy spectrum in $\eta \rightarrow \pi^+ \pi^- \pi^0$. This spectrum is now known (Dalitz plot of η) and in the paper by Fowler et al.¹¹⁾ the predicted value is

$$\frac{\Gamma(\eta \rightarrow 3\pi^0)}{\Gamma(\eta \rightarrow \pi^+ \pi^- \pi^0)} = 1.68 \pm 0.05 .$$

If no other neutral modes occur apart from $3\pi^0$ and 2γ we have

$$\Gamma(\eta \rightarrow 3\pi^0) = 1.68 \cdot \Gamma(\pi^+ \pi^- \pi^0) \quad (\text{predicted})$$

$$\Gamma(\eta \rightarrow 2\gamma) = 1.33 \cdot \Gamma(\pi^+ \pi^- \pi^0) \quad (\text{if } 2\gamma \text{ and } 3\pi^0 \text{ only})$$

$$\Gamma(\eta \rightarrow \pi^+ \pi^- \gamma) = 0.26 \cdot \Gamma(\pi^+ \pi^- \pi^0) \quad (\text{directly measured}).$$

The total width of η is not known (the present experimental limit of a few MeV is set by errors of measurement). From a theoretical estimate of $\Gamma(\eta \rightarrow 2\gamma)$, made with the same method as that used for $\pi^0 \rightarrow 2\gamma$, a total width of about 190 eV is deduced and a lifetime of $4 \cdot 10^{-18}$ sec, indeed in a difficult time region. This lifetime of η is entirely a theoretical value.

ρ (750)

For this state only 2π decays are known. A search for $\rho^0 \rightarrow \pi^+ \pi^-$ in antiproton-proton reactions [Chadwick et al.¹²⁾] gave the limit of less than 2.5%. The ρ mass distribution is very wide and the corresponding peak in histograms like a jelly. A splitting of ρ into two peaks (ρ_1 ρ_2) has been discussed but the evidence is not yet convincing.

ω (782) The dominant mode is 3π , namely $\omega^0 \rightarrow \pi^+ \pi^- \pi^0$. The mode $\omega \rightarrow 3\pi^0$ is forbidden. The other modes possible for ω violate G parity. The neutral mode expected to dominate is $\omega \rightarrow \pi^0 + \gamma$ and here the branching ratio from several experiments [Pevsner et al.¹³⁾, Alff et al.⁵⁾ and Armenteros et al.¹⁷⁾] seem to point to about 10%. The $\omega \rightarrow 2\pi$ is interesting, but difficult because of the nearby $\rho^0 \rightarrow 2\pi$ and interference between the two. Of the order of at the most a few per cent has been reported; but the situation is not at all clear.

$K^*(888 \text{ MeV})$ goes 100% to $K + \pi$.

Y_1^* (1385) is dominated by the decay $Y_1^* \rightarrow \Lambda + \pi$. The expected $Y_1^* \rightarrow \Sigma + \pi$ is weak or absent; < 4%.

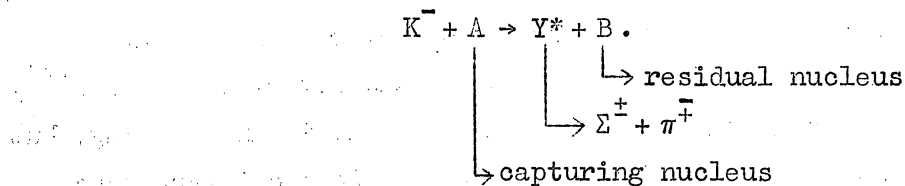
Y_0^* (1405) is observed as $Y_0^* \rightarrow \Sigma^0 + \pi^0$. No other decay is known.

III. STUDY OF Y^* (1405 MeV) BY NUCLEAR EMULSIONS

The rest of my lecture will be devoted to what has been done on Y^* (1405) in nuclear emulsion studies. Besides the research on nucleon isobars by the Copenhagen group, this resonance is the only one studied by the emulsion technique. If the resonance is very narrow as some available results indicate, it is well suited to such studies.

We note that the mass of Y_0^* reported by the bubble chamber groups $M(Y^*) = 1405(\pm 5) \text{ MeV}$ is below the sum of masses of the particles that can produce it (K^- and proton), $M(K^-) + M(P) = 1432 \text{ MeV}$. The difference is 27 MeV. With free protons only the reaction $K^- + P \rightarrow Y^* + \gamma$ is energetically possible. It has, however, not been observed, but, as far as I know, has not been seriously searched for either. The energy of the γ ray and the width of the γ ray line would give the mass of Y^* , respectively its width (Γ). The many competing reactions $K^- + P \rightarrow \Sigma\pi, \Lambda\pi$, its small phase space and the electromagnetic coupling constant, will together make it a rare reaction, unfortunately.

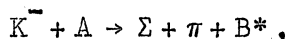
On bound protons, the residual nucleus can take up the recoil momentum so one would get



The first study of this type was undertaken by Eisenberg et al.¹⁶⁾ with K^- stopping in nuclear emulsions. From the measured energies of the Σ and the π they obtained a distribution of the energy sum $E = (E_\Sigma + E_\pi)$ and of the invariant mass

$$M = \sqrt{(E_\Sigma + E_\pi)^2 - (\bar{p}_\Sigma + \bar{p}_\pi)^2}.$$

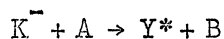
Their M distribution was somewhat narrower than the E distribution, which they claimed was evidence for Y^* production. This argument was questioned by Burhop at the Aix-en-Provence Conference (1961). He claimed that this could happen even if Y^* is not produced. Burhop's argument is apparently based on an infinitely large nucleus. For a light nucleus where you have well defined energy states of the residual nucleus



the E distribution consists of fairly sharp lines, broadened only by the kinetic energy variation of B (0-4 MeV) [one line for each excited state of B]. On the other hand the distribution of $\bar{p}_\Sigma + \bar{p}_\pi$, which reflects the internal momentum distribution of the protons in the capturing nucleus is rather wide. So, contrary to Burhop's argument, the expected M distribution should be wider than the E distribution (provided Y^* is not produced). The findings of Eisenberg et al.¹⁶⁾ are, however, not confirmed by Frisk¹⁴⁾ who finds that the M distribution is - if anything - wider than the E distribution. This can be seen in Figs. 15 and 16. Both the experimental situation and the theoretical arguments on this point are therefore somewhat confused. However, in what follows, the whole point may be forgotten. As I just said, the distribution in $|\bar{p}_\Sigma + \bar{p}_\pi|$ expected, if

the Σ and the π are directly produced by $K^- + P \rightarrow \Sigma + \pi$ on a bound proton, reflects the internal momentum distribution of the bound protons (modified by phase space, by surface absorption of the K and by angular momentum conservation).

If a Y^* is formed this is not so any longer. Let us consider a two-body process



where the residual nucleus is in a certain state (ground state or excited state). Let p be the momentum of Y^* , energy-momentum conservation gives

$$m_K + m_A - B_{\text{Atomic K}} = M(Y^*) + M_B + \frac{p^2}{2M_{Y^*}} + \frac{p^2}{2M_B}$$

$$\therefore p^2 \left(\frac{1}{2M_Y} + \frac{1}{2M_B} \right) = 27 - B_{\text{proton}} - B_A - E_{\text{exc.}}(B) + [1405 - M(Y^*)].$$

For a given capturing nucleus, the proton binding energy B_{proton} and B_A are fixed. Therefore p is constant (a line) widened only by the variation of the mass of Y^* .

The effect of a wide Y^* -mass distribution has now to be worked out. According to bubble chamber results on Y^* (1405), it has been reported that $\Gamma(Y_0^*) \simeq 50 \text{ MeV}^{20}$. This means that in our formula the term $(1405 - M)$ is often at least 25 MeV, which has a large effect on p . It spreads p from its central value down to 0 and up to 300 MeV/c with a tail even higher up. Many such curves will overlap, one curve for each capturing nucleus and each final state nuclide. Qualitatively, the expected distribution in p is like the one expected for direct production of Σ and π ; so it would seem as if nothing could be learned from a p plot.

It was, therefore, very surprising when Frisk (1962) found a sharp line in the momentum distribution. He studied events in nuclear emulsion of the type $K^- + A \rightarrow \Sigma^\pm + \pi^\mp + B + (\text{ev. neutrals})$ [Fig. 17]. The peak in the region 160-180 MeV/c is highly significant. A search for systematic errors has not led to any explanation, nor has a search for nuclear physics effects.

The peak has been identified with the existence of a $Y_0^* \rightarrow \Sigma^{\mp} + \pi^{\pm}$. From the foregoing arguments, the narrowness of the momentum line requires the Y^* to have a width much smaller than the 50 MeV reported from bubble chamber experiments. The observed width of the line at 170 MeV/c is ± 5 MeV/c (standard error) which corresponds to the computed errors of measurements from $p^2 = p_{\pi}^2 + p_{\Sigma}^2 + 2p_{\pi}p_{\Sigma} \cos \Phi$, where Φ is the space angle between \bar{p}_{π} and \bar{p}_{Σ} .

$$\frac{p \delta p}{M_{\text{red}}} = \delta M(Y^*),$$

where M_{red} is the reduced mass of Y^* and the residual nucleus B;

$$\frac{p}{M_{\text{red}}} = \frac{1}{7}$$

is the relative velocity, β , of Y^* and residual nucleus,

$$\therefore \delta M = \frac{1}{7} \cdot \delta p = \pm \frac{1}{7} \cdot 5 = \pm 0.7 \text{ MeV.}$$

Thus $\Gamma < 1.4$ is the result.

In a later work by Frisk and myself¹⁵⁾ we could make a more detailed study of the line and confirm the suggestion made by Frisk that the line corresponds to captures in carbon.

If the line is caused by captures in light elements (C,N,O), the momentum of 170 MeV/c is sufficient to render the recoil track of the residual nucleus visible (2-3 μm). Captures in Ag, Br would not make the recoil visible. The events were divided into two subsamples:

- i) those with no recoil and no Auger electron;
- ii) those with a possible recoil.

The graphs Figs. 18 and 19 show that the peak at 170 MeV/c fell into the second sample, but is absent from the no-recoil sample. In the second sample a further subdivision of events was undertaken. This time we required the recoil to be measurable and not obscured by other tracks, be it the K meson track or Auger electron or the outgoing Σ or π tracks.

This procedure introduces a bias, corresponding so that:

- i) no heavy recoil nucleus can be included (no captures in Ag and Br);
- ii) light nuclei with momenta below about 100 MeV/c are absent.

For the region of interest, around momenta of 170 MeV/c of light nuclei, the bias is thus not serious. In the sample so selected we measured the angle between $\overline{p_{Y^*}} = \overline{p_{\Sigma}} + \overline{p_{\pi}}$ and the recoil track. The plane angle was in nearly all cases $180^\circ (\pm 20^\circ)$ and in dip angle we only could require that they were approximately opposite.

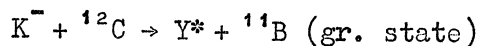
The momentum distribution of this highly selected sample is shown in Fig. 20. The peak is still in the same position sitting on a low background. The position of this peak (172 MeV/c) was used to compute the Y^* mass from the formula

$$m(Y^*) = m_K + m_p - B_{\text{proton}} - B_{\Lambda} - E_{\text{exc.}}(B) - p^2 \left(\frac{1}{2m_{Y^*}} + \frac{1}{2m_B} \right)$$

or the equivalent relativistic expression for the kinetic energy.

The mass one obtains depends on the nature of the capturing nucleus (B_{proton}) and the state of excitation ($E_{\text{exc.}}$) of the final nucleus. To help in fixing these things we did the following:

- i) computed the invariant mass for the peak events with the result $\langle M_{\text{inv}} \rangle = 1404 \pm 1$ MeV where ± 1 is the standard error in the weighted mean;
- ii) measured the ranges of the recoil tracks. The mean range is (after correction for dip) only consistent with the lightest possible recoil nucleus, ${}^1\text{B}$ (from Barkas range energy relation for heavy nuclei, extrapolated down to our region). One would like to have ranges for light nuclei down to about 1 MeV kinetic energy (the present limit is 4 MeV). We, finally, find all evidence points to the reaction



as mainly responsible for the line. The results are: mass of $Y^* = 1404.2 \pm 0.4$ MeV [invariant mass of $Y^* = 1404 \pm 1$ MeV], width (Y^*) is $\Gamma < 1.4$ MeV.

The line has been reported also by Eisenberg¹⁸⁾. In a study of S. White and C. Gilbert (unpublished) it was not seen in the overall ($\Sigma\pi$) sample, but was clear in the sample with recoils. They obtained two lines one at the position expected for carbon captures, the other (weaker line) corresponding to oxygen captures. The Brussels group has also reported the line, whereas still others are working on it. (Note added in proof Recently at Berkeley A. Barbaro-Galtieri, F.M. Smith and J.W. Patrick, Phys. Letters 5, 63 (1963), reported results which do not confirm the existence of a line in the momentum spectrum. The reason for this situation with conflicting results is not at present known.) Frisk measured the pion energies mostly by grain-counting, by range only for pions that stopped in his rather small stack. In order to get better resolution range measurements are preferable; but then, one must have a large stack as the pion ranges go up to 8 cm. If one selects only stopping pions in a small stack, a bias is introduced.

Another systematic error which may wash out a sharp peak is emulsion distortion which has an effect on $p(Y^*)$ because of the error it introduces in the angle Φ

$$p^2 = p_\pi^2 + p_\Sigma^2 + 2p_\pi p_\Sigma \cos \Phi$$

$$\frac{\partial p}{\partial \Phi} = - \frac{p_\pi p_\Sigma}{p} \sin \Phi d\Phi.$$

The distortion error in Φ comes in via errors in dip angles and the plane angle between the π and the Σ . A Monte Carlo calculation of the expected broadening of an initially sharp peak has been carried out by P. Carlson (unpublished). The full width at half maximum of the line is about $60 \cdot K$ MeV, where K is the distortion vector. So if $K = 20 \mu\text{m}/600 \mu\text{m} = 1/30$, only 2 MeV/c broadening is expected. The calculation was made only for a simple combination of first and second order distortion.

Future lines of research have obviously first to deal with the problem of whether there is a line as reported by Frisk or not. With a large enough sample one should be able to find lines at other momenta,

corresponding to captures in other nuclei than carbon and to reactions with the final nucleus left in an excited state. The finding of such lines would constitute strong evidence in favour of the existence of a Y^* with a narrow width. The existence or non-existence of a line in the momentum plot of the $\Sigma\pi$ system is always based on statistical tests and it seems now as if more data are needed.

* * *

REFERENCES

1. A.R. Erwin, R. March, W.D. Walker and E. West, Phys.Rev.Letters 6, 628 (1961).
2. W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee and T. O'Halloran, Proc.of Int.Conf.on High-Energy Phys., CERN 1962, p. 380 and Phys.Rev.Letters 9, 330 (1962).
3. C. Alff, D. Berley, D. Colley, N. Getfand, U. Nauenberg, D. Miller, J. Schultz, J. Steinberger, T.H. Tan, H. Brugger, P. Kramer and R. Plano, Phys.Rev.Letters 9, 322 (1962).
4. M. Alston, L.W. Alvarez, P. Eberhard, M.L. Good, W. Graziano, H.K. Ticho and S.G. Wojcicki, Phys.Rev.Letters 6, 300 (1961).
5. R. Armenteros, L. Montanet, D.R.O. Morrison, S. Nilsson, A. Shapira, J. Vandermeulen, Ch. d'Andlau, A. Astier, C. Ghesquière, B.P. Gregory, D. Rahm, P. Rivet and F. Solmitz, Proc.of Int.Conf.cn High-Energy Phys., CERN 1962, p. 295.
6. P.L. Bastien, J.P. Berge, O.I. Dahl, M. Ferro-Luzzi, D.H. Miller, J.J. Murray, A.H. Rosenfeld and M.B. Watson, Phys.Rev.Letters 8, 114 (1962).
7. E. Pickup, D.K. Robinson and E.O. Salant, Phys.Rev.Letters 8, 329 (1962).
8. M. Meer, R. Strand, R. Kraemer, L. Madansky, M. Nussbaum, A. Pevsner, C. Richardson, T. Toohig, M. Block, S. Orenstein and T. Fields, Proc.of Int.Conf.on High-Energy Phys., CERN 1962, p. 103.

9. M. Chrétien, F. Bulos, H.R. Crouch, R.E. Lanou, J.T. Massimo, A.M. Shapiro, J.A. Averell, C.A. Bordner, A.E. Brenner, D.R. Firth, M.E. Law, E.E. Ronat, K. Strauch, J.C. Street, J.J. Szymanski, A. Weinberg, B. Nelson, I.A. Fless, L. Rosenson, G.A. Salandin, R.K. Yamamoto, L. Guerriero and F. Waldner, Phys.Rev.Letters 9, 127 (1963).
10. L. Behr, P. Mittner and P. Musset, Phys.Letters 4, 22 (1963).
11. E.C. Fowler, F.S. Crawford, L.J. Lloyd, R.A. Grossman and LeRoy Price, Phys.Rev.Letters 10, 110 (1963).
12. G.B. Chadwick, W.T. Davies, M. Derrick, C.J.B. Hawkins, P.B. Jones, J.H. Mulvey, D. Radojicic, C.A. Wilkinson, M. Cresti, A. Grigoletto, S. Limentani, A. Loria, L. Peruzzo and R. Santangelo, Proc.of Int. Conf.on High-Energy Phys., CERN 1962, p. 73.
13. A. Pevsner, R. Kraemer, M. Nussbaum, C. Richardson, P. Schlein, R. Strand, T. Toohig, M. Block, A. Engler, R. Gessaroli and C. Meltzer, Phys.Rev.Letters 7, 421 (1961).
14. ^oA. Frisk, Ark.Fys. 24, 221 (1963).
15. ^oA. Frisk and G. Ekspong, Phys.Letters 3, 27 (1962).
16. Y. Eisenberg, G. Yekutieli, P. Abrahamsson and D. Kessler, Nuovo Cimento 21, 563 (1961) and The Aix-en-Provence Conf.Proc., Vol. I, 389 (1961).
17. R. Armenteros, R. Budde, L. Montanet, D.R.O. Morrison, S. Nilsson, A. Shapira, J. Vandermeulen, C. d'Andlau, A. Astier, C. Ghesquière, B. Gregory, D. Rahm, P. Rivet and F. Solmitz, Proc.of Int.Conf.on High-Energy Phys., CERN 1962, p. 90.
18. Y. Eisenberg, Proc.of Int.Conf.on High-Energy Phys., CERN 1962, p. 791.
19. B.C. Maglić, L.W. Alvarez, A.H. Rosenfeld and M.L. Stevenson, Phys.Rev.Letters 7, 178 (1961).
20. M.H. Alston, L.W. Alvarez, M. Ferro-Luzzi, A.H. Rosenfeld, H.K. Ticho and S.G. Wojcicki, Proc.of Int.Conf.on High-Energy Phys., CERN 1962, p. 311.
21. K.C. Wali, Phys.Rev.Letters 9, 120 (1962).

* * *

DISCUSSION

Harmsen : How large is the error in the extrapolation of the range-energy-relation which you have used for the mass determination of the recoils?

Ekspong : The error is hard to estimate. Barkas and his collaborators have made a universal curve for the ranges of slow heavy particles (carbon, oxygen etc. up to argon). Their actual measurements for light nuclei (carbon) go down to about 4 MeV and we need ranges for 1-2 MeV particles. We used their data for heavier atoms for this extrapolation, thus assuming the universal curve of Barkas et al. to hold. The actual average range for our boron atoms are about 0.5 microns longer than expected from the extrapolation. When we first started to study the recoil ranges we did not know how good a job one can do. It seems, however, that the ranges come out rather constant, but as I said about 0.5 microns too long (the range is 2.9 μm). In the future we hope to be able to use the range measurements in the analysis to separate captures in carbon from captures in oxygen. Therefore, one would like to know the range-energy curves for light nuclei in the region 1-2 MeV from calibration experiments.

Hoogland : Do you know why in your emulsion experiment the Y^* is produced mostly in ^{12}C ?

Ekspong : I think it is only that carbon is very favourable, so when you first see the thing it is in carbon. I think it should be produced in all the elements including oxygen unless angular momentum conservation prevents it from going to the groundstate of ^{15}N from ^{16}O .

Key : Wouldn't the uncertainties in momentum in those cases which had to be grain-counted, swamp the 170 MeV/c peak?

Ekspong : I don't think so. By using checks on the grain density-range curve, counting as much as 1000 grains, you will end up with a standard error of about 9% in the pion energy. How large an error you get in the $(\Sigma\pi)$ momentum from this depends on the configuration of the event. You have to compute the error event by event. Frisk put everything in a computer which calculated the momentum with its errors and also the invariant mass and its errors. He fed the computer with all information from the measurements including errors of measurements. Most of the time the error on the momentum of the $(\Sigma\pi)$ system around 170 MeV/c was ± 5 MeV/c, but sometimes larger, up to about ± 15 MeV/c. If one could get the pion energy by range in all cases and also the Σ energy and have accurate angles, then one would certainly measure better, say ± 3 MeV/c; but the present method does not swamp the peak.

Hoogland : You spoke about the fact that interference could be an explanation for the differences in the width of resonances in different experiments with bubble chambers. Is it also possible that these interferences can give rise to different masses?

Ekspong : Well, in the case of the ω and the ρ , the closeness of their masses together with the large width of the ρ , can give rise to interference between $\omega^0 \rightarrow \pi^+ \pi^-$ and $\rho^0 \rightarrow \pi^+ \pi^-$. Such an interference could change the observed distribution of the $\pi^+ \pi^-$ masses in the neighbourhood of the ω mass to one with fluctuations up and down there. As for final-state interactions they might broaden the observed mass distributions or change their shapes.

Hoogland : I thought there was some indication that in a special case of experiment the mass of the resonance was lower than that hitherto found.

Ekspong : I don't know of any clear experiment on this.

Hoogland : Can you give an explanation for the value of the branching ratio

$$\frac{\eta \rightarrow \pi^+ \pi^- \gamma}{\eta \rightarrow \pi^- \pi^+ \pi^0} = \frac{1}{4} ?$$

Ekspong : I haven't done any calculation on that at all; I don't know if anybody else has done it. Of course, one could try the simple thing with an energy independent matrix element to see whether it comes close or not to the experimental value, which is 0.26 ± 0.08 . The only thing which I know has been theoretically predicted by Wali is the ratio of $3\pi^0$ mode versus the $\pi^+ \pi^- \pi^0$. The prediction has not been experimentally confirmed as the $3\pi^0$ mode has not been measured.

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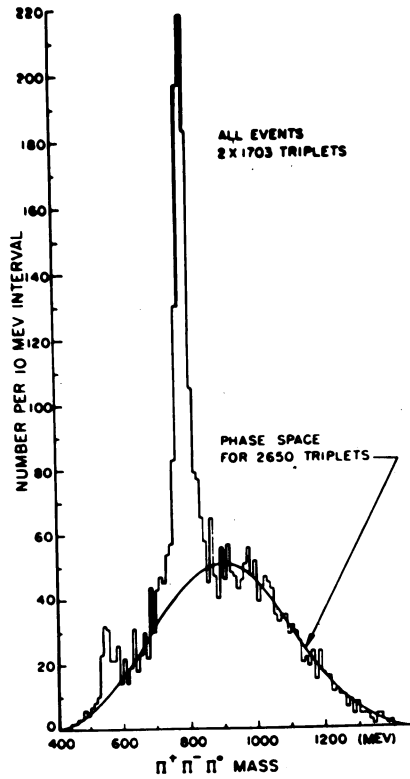


Fig. 3 Histogram with very strong evidence for ω -production. The data are from an experiment by C. Alff et al. (ref. 5), on the reaction $\pi^+ + p \rightarrow \pi^+ + p + \pi^+ + \pi^- + \pi^0$. Besides the strong ω -peak at 780 MeV, another peak at 550 MeV corresponding to η -production, is evident.

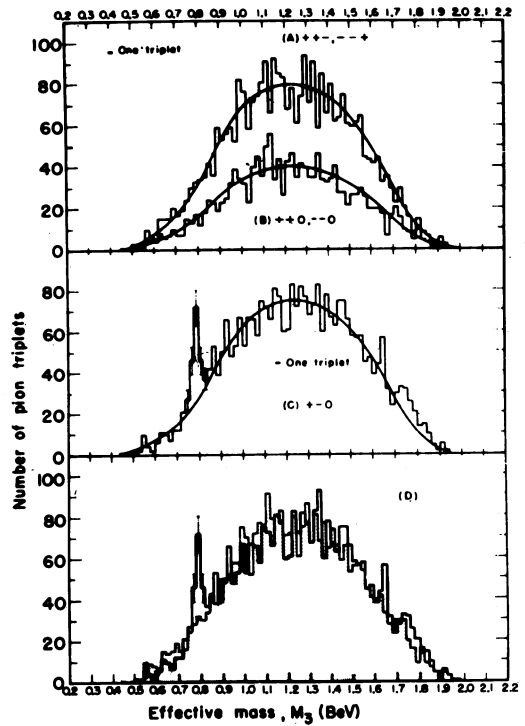


Fig. 4 Original evidence for ω -production in anti-proton-proton annihilation by Maglic et al. (ref. 19).

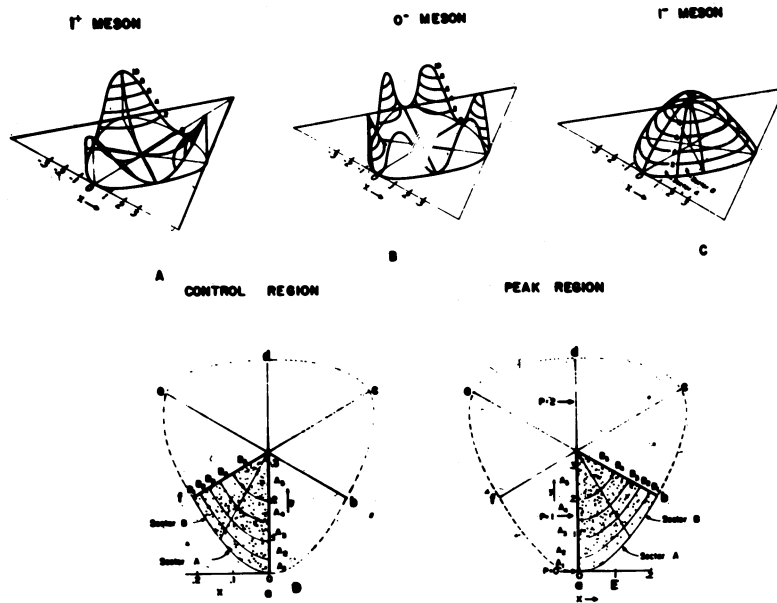


Fig. 6 Three-dimensional diagrams showing expected intensity distribution in Dalitz plots for 3π -decay of mesons with spin-parity 1^+ , 0^- and 1^- .

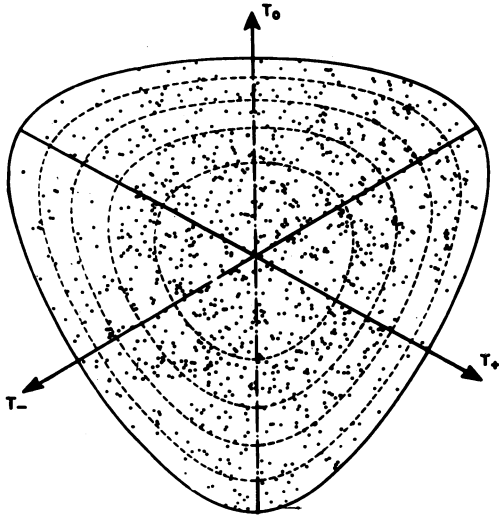


Fig. 7 Dalitz-diagram for ω -decay (1100 events) showing higher point density near the center region with a fall of intensity towards the outer sides, typical for a meson with spin parity assignments 1^- .

Fig. 8 Intensity of ω -decay as a function of the "radius" of the Dalitz plot (fig. 7). Solid lines show prediction for mesons 1^- , 0^- and 1^+ . Experimental histogram does not fit 0^- and 1^+ mesons.

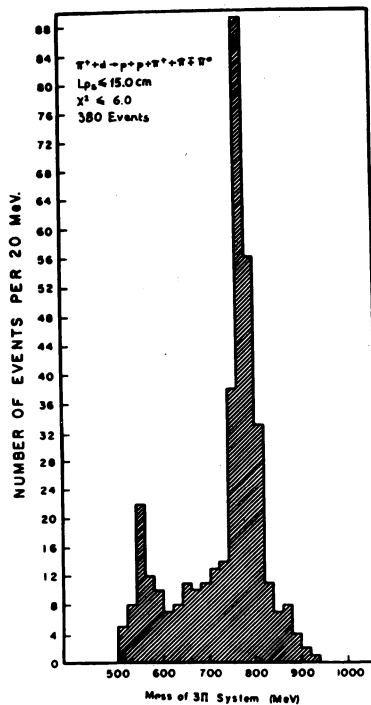
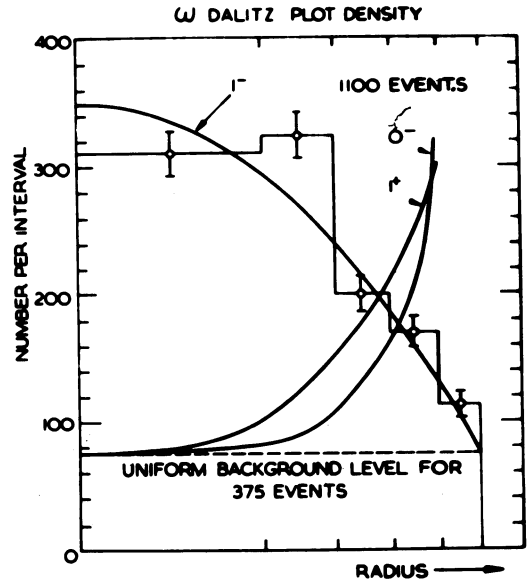


Fig. 9 The reaction $\pi^+ + d \rightarrow p + p + \pi^+ + \pi^- + \pi^0$ shows strong evidence for both ω (780 MeV) and η (550 MeV) production (ref. 13).

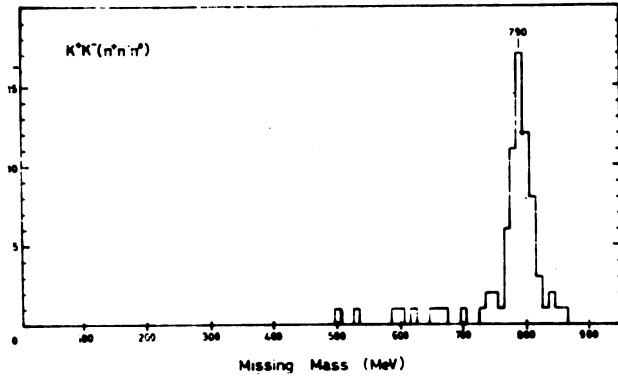


Fig. 10 In antiproton proton annihilations at rest into a pair of K mesons and three pions the pions are to a high proportion (> 90%) produced via the ω particle. The sample of ω is unusually free from background events (data by Armenteros et al. ref. 17).

Fig. 11 Simultaneous production of both K^* (890 MeV) and N^* (1238 MeV) in the reaction by 2.6 GeV/c K^+ mesons
 $K^+ + p \rightarrow K + \pi + \pi + p$
 data by W. Chinowsky et al. (ref. 2).

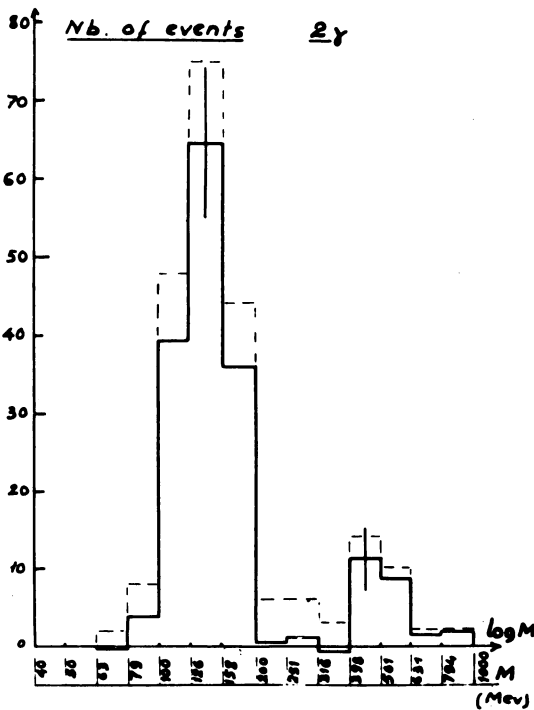
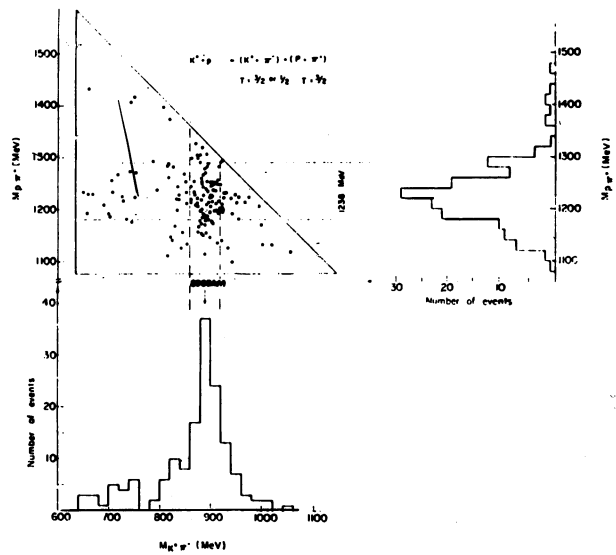


Fig. 12 Observations of two simultaneous γ -rays in a heavy liquid chamber (L. Behr et al., Saclay, ref. 10) give two peaks, one for π^0 the other corresponding to η^0 (550 MeV). Gives direct evidence that the decay $\eta \rightarrow 2\gamma$ exists, implying spin 0 for η .

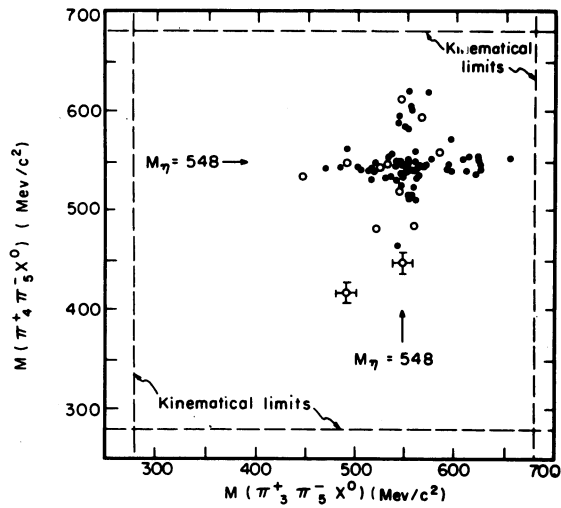


Fig. 13 Data by E. C. Fowler et al. (ref. 11) shows that the decay $\eta^0 \rightarrow \pi^+ \pi^- \gamma$ exists (open circles) besides $\eta \rightarrow \pi^+ \pi^- \pi^0$ (black dots).

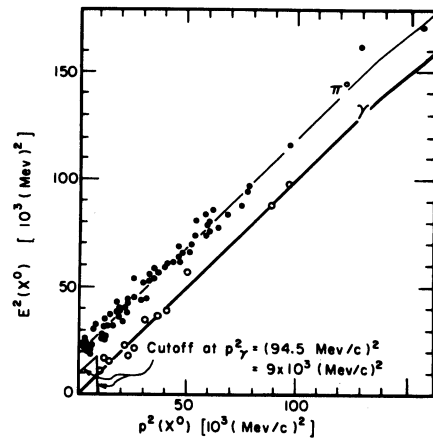


Fig. 14 Identification of neutral particle in η -decay by its mass as either π^0 (full circles) or γ (open circles). The relation $E^2 = p^2 + m^2$ ($c=1$) plotted. Same data as in fig. 13.

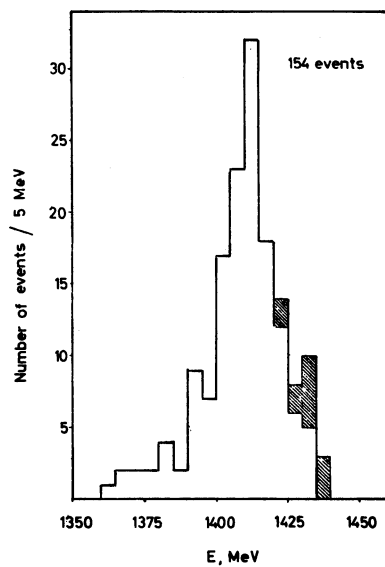


Fig. 15 Distribution of energy sum for Σ and π emitted in nuclear absorption reactions by K^- (A. Frisk, ref. 14). Shaded events correspond to reactions with free protons in nuclear emulsions.

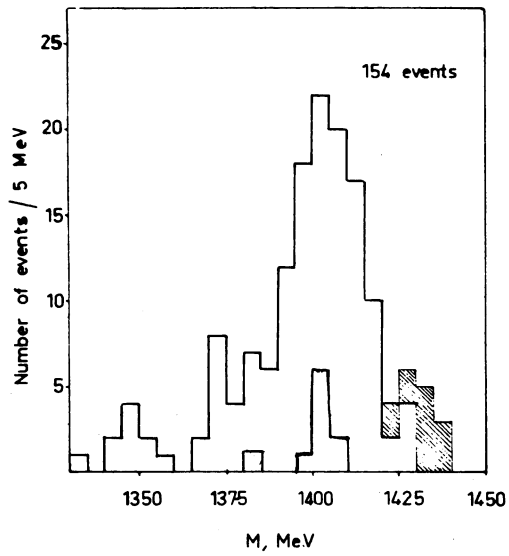


Fig. 16 Distribution of the invariant mass of 154 $\Sigma\pi$ events. Same data as in fig. 15. The 9 events close to 1405 MeV are interpreted as being examples of the reaction $^{12}\text{C}(K^-, Y^*)^{11}\text{B}$.

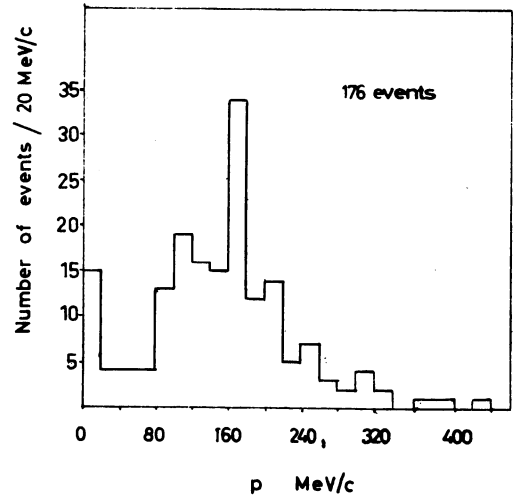


Fig. 17 Distribution of momentum of the $\Sigma\pi$ system (same data as figs. 15 & 16 showing unexpected sharp peak close to 170 MeV/c).

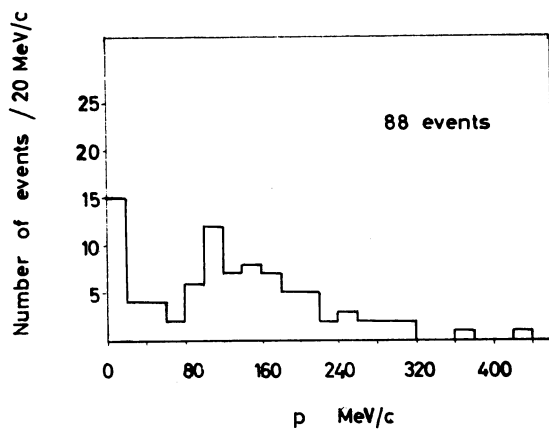


Fig. 18 Subsample of data in fig. 17 consisting of events of the type $K^- + A \rightarrow \Sigma + \pi + B$ where the residual nucleus (B) does not show any visible recoil track. Line at 170 MeV/c is absent.

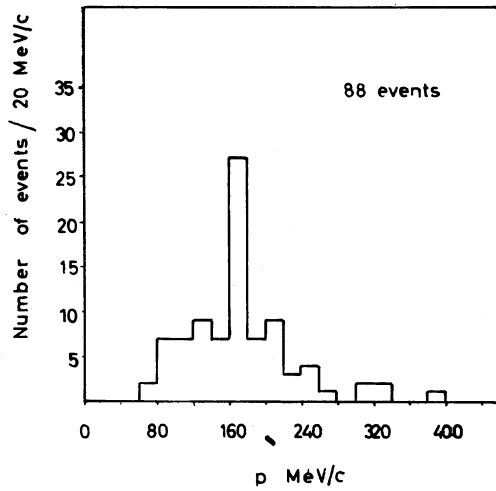


Fig. 19 Remaining subsample consisting of events enriched in visible recoils of nucleus B in reactions $K^- + A \rightarrow \Sigma + \pi + B$. Line at 170 MeV/c as strong as in the total sample of events; background reduced to about 50% of that in fig. 17.

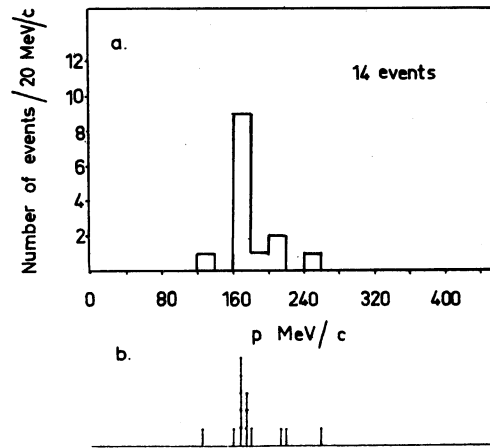


Fig. 20 Events of the type $K^- + A \rightarrow \Sigma + \pi + B$ with a measurable recoil track attributed to nucleus B. In (b) individual events are plotted, showing close grouping of events at 172 MeV/c.