PART III.2

A METHOD FOR DETERMINING THE SPIN AND PARITY OF THE Y_1^* †)

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The helium bubble chamber group[1] has presented evidence for the presence of the mass 1385 MeV Y_1^* state in the absorption at rest of K^- mesons in He^4. They find that the reaction channel

$$K^- + He^4 \rightarrow \pi^- + \Lambda^0 + He^3 \tag{1}$$

is dominated by

$$K^- + He^4 \rightarrow Y_1^{*-} + He^3 , \; Y_1^{*-} \rightarrow \Lambda^0 + \pi^- \; . \tag{2}$$

Also, this group[2] has reported a large yield of helium hypernuclei from the reaction

$$K^- + He^4 \rightarrow \pi^- + {}_\Lambda He^4 \; . \tag{3}$$

Dalitz and Downs[3] have successfully analysed hypernuclear states by considering, for example, that ${}_\Lambda He^4$ is a bound state of a real He^3 core, surrounded by a loosely bound (~ 2.2 MeV) Λ^0. This implies that the hypernuclei in Eq. (3) are created by the strong final state interactions causing binding between the Λ^0 and He^3 from reaction (1). Since the Y* mechanism dominates (2), this implies that hypernuclear formation in (3) is due to the reaction chain

$$K^- + He^4 \rightarrow Y_1^{*-} + He^3 \rightarrow \pi^- + {}_\Lambda He^4 \; . \tag{4}$$

This note will investigate the rate of hyperfragment production in (4) as a function of three level assignments for Y_1^*, i.e. $s_{1/2}$, $p_{1/2}$ and $p_{3/2}$.

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It is convenient to introduce the ratio

$$R = (\pi^- + {}_\Lambda He^4)/(\pi^- + {}_\Lambda He^4) + (\pi^- + \Lambda^0 + He^3) ,$$

where the symbols represent the rates of the indicated reactions. The dependence of R on the Y* spin and parity can be most clearly qualitatively seen as follows. We adopt the co-ordinate system sketched in Fig. 1, and make the following assumptions:

a) only a pure Y* state is found in reaction (1);
b) the K^- is absorbed from an s-state atomic orbit[4];
c) the K^- is pseudoscalar[2].

Since the total angular momentum J = 0, and parity is conserved in (2), it is readily seen that $\ell = L$, independent of the Y* spin or parity. If we let J_Y be the spin of the Y*, then $\vec{J}_Y = \vec{\ell} + \vec{1/2}$. For the level assignment $s_{1/2}$, we then have $\ell = 0$, $L = 0$, and $J_Y = 1/2$; for $p_{1/2}$, $\ell = 1$, $L = 1$, $J_Y = 1/2$; and for $p_{3/2}$, $\ell = 1$, $L = 1$, $J_Y = 3/2$. Thus, there is only one possible state produced in reaction (2), and it is uniquely determined by the Y* quantum numbers. A convenient axis of quantization is the normal to the production plane of (1), since $\vec{p}_\pi + \vec{p}_\Lambda + \vec{p}_3 = 0$. If we rewrite the wave function of the system, not in terms of J_Y, L, and s_3, but rather in terms of ℓ, L and $\vec{S} \equiv \vec{s}_\Lambda + \vec{s}_3$, we obtain

$$\chi = a_0 \cos \phi | S = 0, S_z = 0 > + a_1 \sin \phi | S = 1, S_z = 0 > \qquad (4)$$

where ϕ is the angle between $\vec{p}_{\Lambda\pi}$ and $\vec{p}_3$. The relative Λ-π momentum is given by $\vec{p}_{\Lambda\pi} = m_\pi \vec{p}_\Lambda - m_\Lambda p_\pi / m_\Lambda + m_\pi$, and the spinors $|S = 0, S_3 = 0>$ and $|S = 1, S_3 = 0>$ refer to the total spin S of the system, i.e. S = 0 or 1. For $s_{1/2}$, we have $a_1 = 0$; for $p_{1/2}$, $|a_0|^2 = |a_1|^2$; for $p_{3/2}$, $|a_0|^2 = 4|a_1|^2$. The corresponding angular distributions are shown in Table 1. Although the angular distributions for $s_{1/2}$ and $p_{1/2}$ are the same, they yield very different hyperfragment production rates. The spin of ${}_\Lambda He^4$ has been shown to be zero[2]. Hence, only those states with S = 0 can form hypernuclei. The probability for S = 0 is also

Table 1

Hyperfragment production rate R, as a function of the Y* state, for capture of a pseudoscalar K^- meson in an s-state orbit.

Y* level	Spin	Parity	Angular distribution	Probability of S = 0	R (hypernuclear rate), in %
$s_{1/2}$	$1/2$	-	isotropic	1	23
$p_{1/2}$	$1/2$	+	isotropic	$1/3$	7
$p_{3/2}$	$3/2$	+	$1 + 3\cos^2 \phi$	$2/3$	14

shown in Table 1. It is clear that, all other factors being equal, the $s_{1/2}$ state will yield the most hypernuclei. A detailed quantitative estimate is made below.

We will calculate the rate for the continuum reaction

$$K^- + He^4 \rightarrow Y_1^{*-} + He^3 \rightarrow \pi^- + \Lambda^0 + He^3 \quad , \tag{5}$$

using the impulse model. The final state wave function is taken to be

$$v_{\vec{k}}(\vec{r}_\Lambda - \vec{r}_3)e^{i\vec{p}_\pi \cdot \left(\vec{r}_\pi - \frac{m_3\vec{r}_\Lambda + m_3\vec{r}_3}{m_3 + m_\Lambda}\right)} ,$$

i.e. a plane wave pion and a continuum $\Lambda^0 - He^3$ wave function $v_{\vec{k}}$, labelled by its internal momentum $\vec{k}$. The co-ordinate system and coupling scheme used is illustrated in Fig. 2, where

$$\vec{k} = \frac{m_3\vec{p}_\Lambda - m_\Lambda\vec{p}_3}{m_\Lambda + m_3} .$$

The wave function for ${}_\Lambda He^4$ is taken as a gaussian, that is

$$\phi\, He^4 = N_1 \exp - \left(\beta \sum_{i,j} (\vec{r}_i - \vec{r}_j)^2\right)$$

where β is fitted to the charge radius deduced from the Hofstadter[5)] electron scattering experiments. The initial wave function for our process, $u(r)$, is thus the decomposition of the He^4 wave function into the relative motion of nucleon 1 (which is later transformed into a Λ) and the centre-of-mass of the He^3-like core. Thus

$$u(r) = N \exp - \left(\alpha (\vec{r}_1 - \vec{r}_3)^2 \right),$$

where N is the normalization factor. In units where $\hbar = c = 1$, we find that $\sqrt{\alpha} = 89$ MeV/c. Let T be defined as the transition operator for the elementary reaction $K^- + N \to Y^*$, operating on the final state wave function. Since the appropriate impulse operator also includes the contact interaction terms $\delta(\vec{r}_\pi - \vec{r}_\Lambda)\ \delta(\vec{r}_\Lambda - \vec{r}_1)$, the matrix element for the transition of reaction (5) is given by

$$M = \int \left[T v_{\vec{k}}(r) \right]^* e^{-i\vec{p}\cdot\vec{r}}\ u(r)\, d\vec{r}\ , \tag{6}$$

where

$$\vec{p} \equiv \frac{m_3}{m_\Lambda + m_3}\ \vec{p}_\pi\ .$$

If a pure Y* state is formed, the transition operator T in the $p_{\Lambda\pi}, p_3$ language (see Fig. 1) is completely specified. It is in

cases (a) and (b): $T = A_0 p_{\Lambda\pi} \cdot p_3 + A_1 (p_{\Lambda\pi} \times p_3) \cdot \sigma_\Lambda$,

if $p_{3/2}$ or $p_{1/2}$;

case (c) : $T = B_0$,

if $s_{1/2}$,

with $A_0 = a_0/(p_{\Lambda\pi}^2/2\mu - p_{\Lambda\pi}^{*2}/2\mu) + i\Gamma/2$, $A_1 = a_1/(p_{\Lambda\pi}^2/2\mu - p_{\Lambda\pi}^{*2}/2\mu) + i\Gamma/2$, and $B_0 = b_0/(p_{\Lambda\pi}^2/2\mu - p_{\Lambda\pi}^{*2}/2\mu) + i\,\Gamma/2$, i.e. the coefficients are given by Breit-Wigner resonance amplitudes, with $p_{\Lambda\pi}^{*2}/2\mu$ being the resonant energy ($\mu = m_\Lambda m_\pi/m_\Lambda + m_\pi$), and $\Gamma/2$ being the level half-width. We assume the a's and b's are constants, and for $p_{3/2}$, $|a_0|^2 = 4|a_1|^2$, and for $p_{1/2}$, $|a_0|^2 = |a_1|^2$.

In order to discuss hypernuclear formation, we must use the co-ordinate scheme of Fig. 2. We carry out this co-ordinate change by transforming the transition operator T to the $\vec{k}, \vec{p}_\pi$ language by noting that

$$\vec{p}_{\pi\Lambda} = -\frac{m_\pi}{m_\Lambda + m_k}\vec{k} + \left[1 - \frac{m_\pi m_3}{(m_\pi + m_\Lambda)(m_3 + m_\Lambda)}\right]\vec{p}_\pi \qquad (7a)$$

and

$$\vec{p}_3 = -\vec{k} - \frac{m_3}{m_3 + m_\Lambda}\vec{p}_\pi \ . \qquad (7b)$$

Since m_π is small, we simplify our relations (7a), (7b) to be

$$\vec{p}_{\pi\Lambda} = \vec{p}_\pi \qquad (8a)$$

$$\vec{p}_3 = -\vec{k} - \frac{m_3}{m_3 + m_\Lambda}\vec{p}_\pi \ . \qquad (8b)$$

Using Eqs. (8a) and (8b) we obtain the transformed T operator,

cases (a) and (b): $T = -A_0\left[\frac{m_3}{m_3 + m_\Lambda}p_\pi^2 + \vec{p}_\pi \cdot \vec{k}\right] - A_1\left[\vec{p}_\pi \times \vec{k}\right] \cdot \vec{\sigma}_\Lambda$,

for $p_{3/2}$ or $p_{1/2}$, where $\vec{k}$ represents the gradient operator on the relative co-ordinate $\vec{r}_\Lambda - \vec{r}_3$;

case (c) : $T = B_0$,

for $s_{1/2}$.

Using partial integration, it can be readily shown that the matrix element of (6) is given by

$$M = -A_0 p_\pi \int v_{\vec{k}}^*(r) e^{-i\vec{p}\cdot\vec{r}} \cos\Theta \frac{d}{i\,dr} u(r)\,d\vec{r}$$
$$-A_0 p_\pi \int v_{\vec{k}}^*(r) e^{-i\vec{p}\cdot\vec{r}} \sin\Theta \frac{d}{i\,dr} u(r)\,d\vec{r}\,, \qquad (9)$$

where Θ is the angle between $\vec{p}_\pi$ and $\vec{r}$ for the $p_{1/2}$ and $p_{3/2}$ cases. For $s_{1/2}$ we obtain

$$M = B_0 \int v_{\vec{k}}^*(r) e^{-i\vec{p}\cdot\vec{r}} u(r)\,d\vec{r}\,. \qquad (10)$$

To evaluate Eqs. (9) and (10), we employ closure. We calculate the total transition rate which is proportional to

$$\iint |M|^2 \, d\vec{k}\, d\vec{p}_\pi$$

by allowing $\vec{k}$ to range from zero to infinity (ignoring momentum conservation) and replacing the integration over p_π by its maximum value, i.e. the value of p_π corresponding to $K^- + He^4 \rightarrow \pi^- + {}_\Lambda He^4$. This procedure in general tends to overestimate the rate, an effect which can reasonably be neglected if the allowed region of integration is dominated by the final state interactions. Thus we use the relation

$$\frac{1}{(2\pi)^3} v_{\vec{k}}^*(r) v_{\vec{k}}(\vec{r}\,')\,d\vec{k} = \delta(\vec{r}-\vec{r}\,') - v_B^*(\vec{r}) v_B^*(\vec{r}\,')\,,$$

where v_B is the bound state <u>singlet</u> hyperfragment wave function. We thus obtain

$$R = \frac{\left| \int v_B(r)\, e^{-i\vec{p}\cdot\vec{r}} \cos\frac{du}{dr}(r)\,d\vec{r}\right|^2}{\left[1 + 2\frac{|a_1|^2}{|a_0|^2}\right] \langle q^2 \rangle} \qquad (11)$$

for $p_{1/2}$, and $p_{3/2}$, where $< q^2 >$

$$< q^2 > \equiv \int u^*(r) \frac{d^2}{dr^2} u(r) d\vec{r} = 3\alpha .$$

In a similar way, for $s_{1/2}$,

$$R = |I|^2 , \tag{12}$$

where

$$I = \int v_B^*(r) e^{-i \vec{p}\cdot\vec{r}} u(r) d\vec{r} . \tag{13}$$

Dalitz and Downs[3] have numerically evaluated I as a function of p, using the gaussian wave function for u(r) and a numerical solution of the Schroedinger equation for v_B, i.e.

$$I(p) = N \int v_B^*(\vec{r}) e^{-i \vec{p}\cdot\vec{r}} e^{-\alpha r^2} d\vec{r} . \tag{14}$$

Letting

$$J(p) \equiv \int v_B^*(r) e^{-i \vec{p}\cdot\vec{r}} \cos\Theta \frac{d}{dr} u(r) d\vec{r} , \tag{15}$$

we note that

$$J(p) = \frac{2\alpha}{i} \frac{dI}{dp} . \tag{16}$$

Table 1 gives the numerical evaluation of the three cases. We observe the large difference between the rates for $s_{1/2}$ and $p_{1/2}$, in spite of their identical angular distribution.

Angular distributions can be markedly changed from these predictions by a small admixture of background terms, e.g. terms arising from the strong $\Lambda^0 - He^3$ interactions. This is clear because interference terms, etc., arise from the amplitudes, whereas rates go as the square of amplitudes and hence are rather insensitive to small admixtures of contamination terms. Thus, one should expect to have much greater confidence in the predictive powers of a rate calculation than in angular distributions. The large difference predicted between the three cases of Y* level assignments should provide us with a valuable tool for assigning these quantum numbers.

In order to check the sensitivity of the calculation to the assumption of p-state capture, the following transition operators were used:

cases (a) and (b): $$T = A_0 \vec{p}_{\pi\Lambda} \cdot \emptyset(0) + A_1 \left[\nabla\emptyset(0) \times p_{\pi\Lambda} \right] \cdot \sigma_\Lambda + C_0 p_3^2 \, \vec{p}_{\pi\Lambda} \cdot \nabla\emptyset(0) + C_1 p_3^2 \left[\nabla\emptyset(0) \times \vec{p}_{\pi\Lambda} \right] \cdot \vec{\sigma}_\Lambda \quad (17)$$

where $\nabla\emptyset(0)$ is the gradient of the $K^- - He^4$ atomic wave function, evaluated at the origin. It is now assumed that the C terms will be neglectable, since they correspond to L = 2 recoil terms and are probably suppressed by the angular momentum barrier. Therefore,

$$T \to A_0 \vec{p}_{\pi\Lambda} \cdot \nabla\emptyset(0) + A_1 \left[\nabla\emptyset(0) \times \vec{p}_{\pi\Lambda} \right] \cdot \vec{\sigma}_\Lambda \, , \quad (18)$$

where again the A's are resonant amplitudes. The corresponding operator for $s_{1/2}$ is given by

case (c) : $$T \to B_0 \, p_3 \cdot \nabla\emptyset'(0) \, . \quad (19)$$

The rates for hyperfragment production are given by

$$R = \frac{|I|^2}{1 + 2\frac{|a_1|^2}{|a_0|^2}} \quad (20)$$

where $|a_0|^2 = |a_1|^2$ for $p_{1/2}$,

and $|a_0|^2 = 4|a_1|^2$ for $p_{3/2}$,

and $$R = \frac{|J|^2}{< q >^2} \text{ for } s_{1/2} \, . \quad (21)$$

Table 2 summarizes these rates, which are approximately equal to those of s-state capture.

Table 2

Hyperfragment production rate for pseudoscalar K capture in p-wave and scalar K capture in s-wave states

Y* level	Spin	Parity	R (hypernuclear rate in %) for p-wave capture, pseudoscalar K	R (hypernuclear rate in %) for s-wave capture, scalar K
$s_{1/2}$	½	-	21	$\lesssim 5$
$p_{1/2}$	½	+	7.5	$\lesssim 5$
$p_{3/2}$	3/2	+	15	$\lesssim 5$

We further test the hypothesis that the K^- is scalar, and is captured in an s-state. This would require that the hyperfragment be produced in a spin 1 excited state, and decay into the spin zero ground state. Dalitz and Downs[3)] have estimated that if such an excited state is found, its binding energy is $\lesssim$ 0.1 MeV.

The matrix elements for the transition are given by

$$T = A_1 \vec{p}_{\Lambda\pi} \cdot \vec{\sigma}_\Lambda \text{ for } p_{1/2} \text{ and } p_{3/2} , \qquad (22)$$

and

$$T = B_1 \vec{p}_3 \cdot \vec{\sigma}_\Lambda \text{ for } s_{1/2} . \qquad (23)$$

The corresponding rates are given by

$$R = |I'|^2 \quad \text{for} \quad p_{1/2} \quad \text{and} \quad p_{3/2} , \qquad (24)$$

and by

$$R = |J'|^2 < q^2 > . \qquad (25)$$

The primes refer to the hyperfragment wave function corresponding to 0.1 MeV binding energy.

For our purposes, it is sufficiently accurate to assume that both I and J are proportional to $E^{1/2}$, where E is the binding energy. This proportionality comes from the normalization of the hypernuclear wave function, which for sufficiently small E, satisfies the above. These numerical results are indicated in Table 2. In summarizing, we see from Tables 1 and 2 that hypernuclear production is severely limited if the K is scalar, whereas the rates are insensitive to whether pseudoscalar K is captured from either an s or a p orbit.

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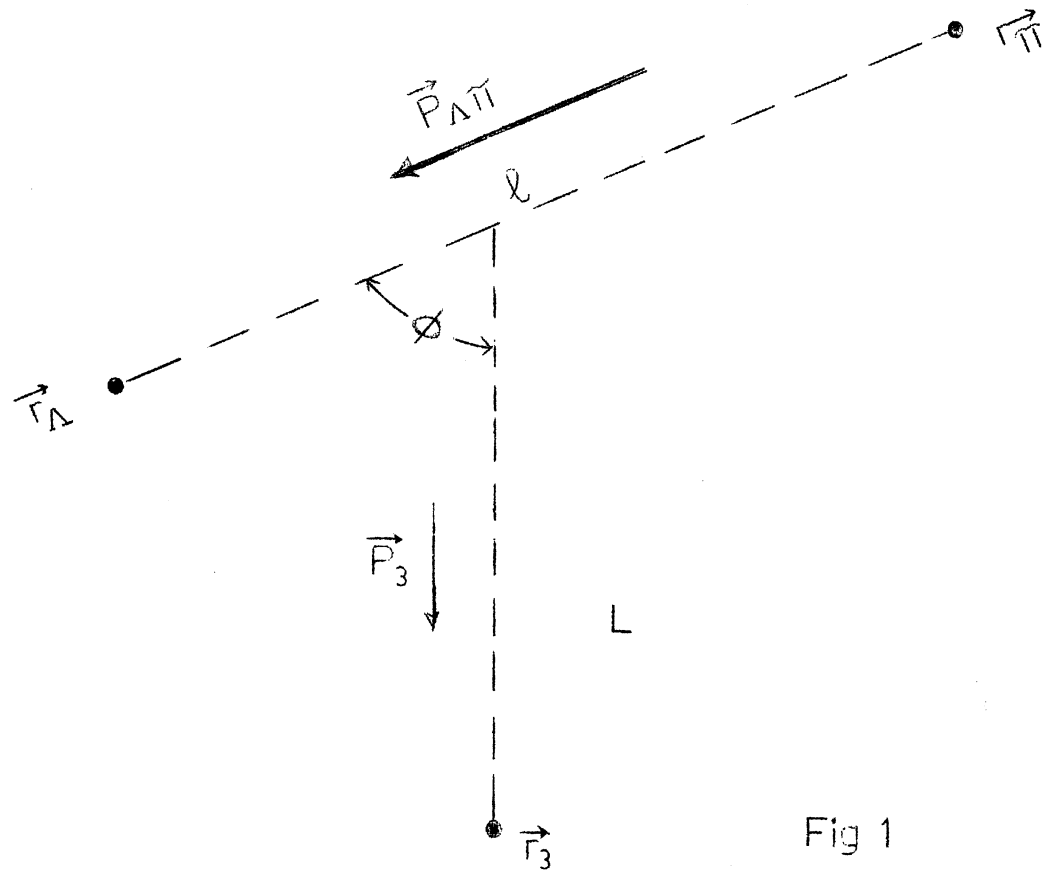

Fig 1

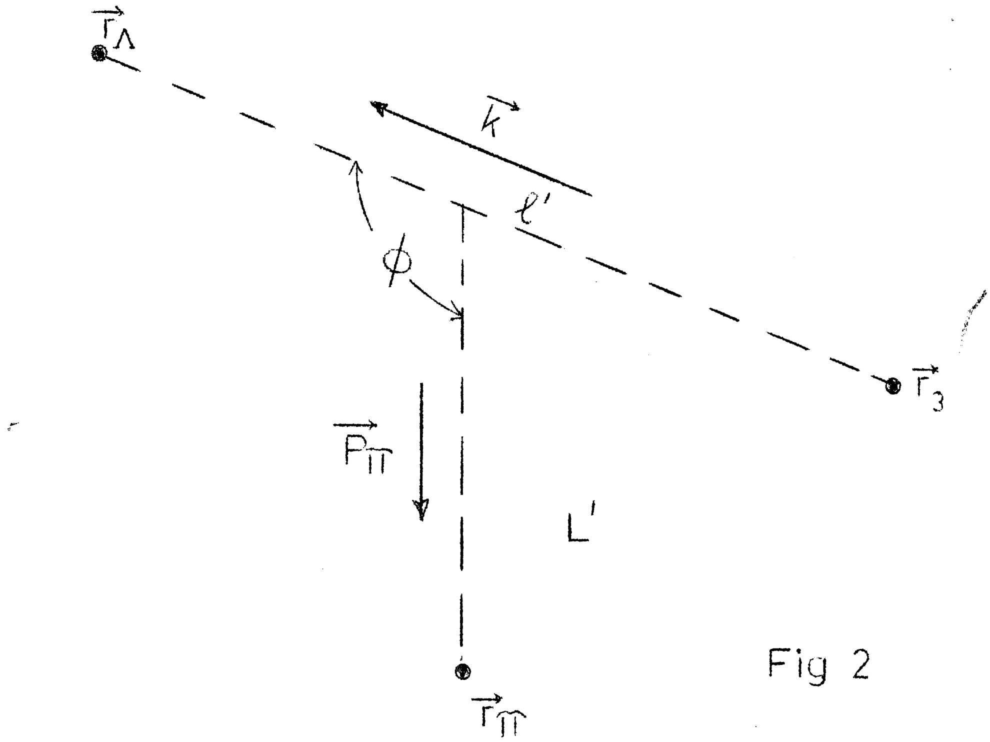

Fig 2

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