12th LECTURE
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Expected frequencies of events and evaluation of data in the Spark Chamber experiment.

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With the present knowledge of the neutrino spectrum we have been trying to foresee what the events are going to look like if certain theoretical assumptions are correct.

1. Scanning and Measurement.

We have at present three scanning tables for the spark chamber photographs: two with four projectors each, and one with two projectors. The table with two projectors has adapted a digitized "coordinatograph", for automatic measurement. We have a computer programme[1)] which makes the geometrical reconstruction of the events and evaluates the energy of the particles under the assumptions that they are either muons, or pions or protons.

2. Electron Showers.

A few months ago we made several runs at the CERN PS, with the chambers exposed to electrons and π^- beams of different energies, in order to have a calibration for the energy of showers produced by electrons and π^0's. The results will appear in another report.

3. The Spark-chamber Set-up.

All our spark chambers have an area of 1.0 m x 1.6 m, and are made of plates 5 mm thick. The spark-chamber set-up consists of three regions, disposed in the following order along the neutrino beam (Figure 1):

a) production region - consists of 20.9 tons of spark chambers, part of aluminium and part of brass.

b) a magnet - to measure the sign of the charge and the momentum of muons produced in the production region.

c) range regions - consists of 53 tons of lead and iron disposed in layers with spark chamber in between.

3.1 Production region.

The spark chambers of this region are mounted on 16 carriages, disposed in two rows of 8 side by side along the neutrino beam. Following the neutrino beam, the first four carriages are equal and we call them α-carriages. Each α-carriage has 6 aluminium chambers, 3 brass chambers and 2 plastic counters, disposed as indicated in Figure 2.

The next four carriages are called β_1, β_2, β_3 and β_4. The first three consist each of a series of brass chambers, one iron plate 3 cm thick and one plastic counter; β_4 consists of brass chambers and one liquid counter 15 cm thick (see Fig. 2).

Table 1 gives the mass, the number of grams/cm^2 and the energy loss of a minimum ionizing particle, for each carriage.

TABLE I

Carriage	Mass (Kg)	g/cm^2					Energy loss for min. ion. (MeV)
		Aℓ	Brass	Fe	Counter.	Total	
α_1	1080	24	38		4	66	110
α_2	1080	24	38		4	66	110
α_3	1080	24	38		4	66	110
α_4	1080	24	38		4	66	110
β_1	1800		88	22	2	112	170
β_2	1640		81	22	2	105	160
β_3	1640		81	22	2	105	160
β_4	1050		51		15	66	100
Totals	10450	96	453	66	37	652	1030

Since we have two rows of such carriages the total mass of the production region is 20.9 tons.

Between the production region and the magnet there is an iron wall 20 cm thick. This adds 215 MeV to the energy loss of a minimum ionizing particle.

It should be remembered that the <u>geometrical mean free path</u> is:

125 g cm^{-2} in copper
95 g cm^{-2} in aluminium.

3.2 Range region.

This region consists of a series of layers of lead. In between two layers there is always a two-gap spark chamber.

The total mass of this region is 53 tons, and the total number of $g\ cm^{-2}$ is 167. The energy loss for a minimum ionizing particle is 2.3 GeV.

4. Neutrino Flux at the Spark Chamber.

Van der Meer computed, for PS proton momentum of 24.8 GeV/c

a) the ratio between neutrino flux at a point and the neutrino flux at the centre of the bubble chamber, as a function of the distance of the point from the centre of the bubble chamber[2];

b) the neutrino flux in 5 concentric circular zones, each 20 centimetres wide, at the centre of the bubble chamber[3] (Figure 3).

By combining a) and b) we obtain the neutrino flux in the five concentric circular zones at the centre of the production region of the spark chambers. The results are given in Table 2, for neutrinos coming from $\pi \rightarrow \mu + \nu$ decay, from $K \rightarrow \mu + \nu$ decay, and for both added.

TABLE 2

Intensity of the neutrino flux in the 5 concentric zones, at the centre of the production region of the spark chambers, as obtained from Van der Meer's computation for protons of 24.8 GeV/c. The numbers are:

$$\frac{\partial^2 N_\nu}{\partial E_\nu \partial S} \quad \text{in (MeV/c)}^{-1}\ \text{m}^{-2} \text{ per circulating proton.}$$

Neutrino Energy (GeV)	ZONE				
	1	2	3	4	5
2	0.376, -6 0.125, -7 0.389, -6	0.429, -6 0.125, -7 0.442, -6	0.323, -6 0.126, -7 0.336, -6	0.158, -6 0.126, -7 0.171, -6	0.074, -6 0.127, -7 0.087, -6
3	0.140, -6 0.124, -7 0.152, -6	0.080, -6 0.125, -7 0.925, -7	0.403, -7 0.125, -7 0.528, -7	0.188, -7 0.125, -7 0.313, -7	0.106, -7 0.126, -7 0.232, -7
4	0.261, -9 0.110, -7 0.112, -7	0.230, -8 0.111, -7 0.134, -7	0.472, -8 0.113, -7 0.160, -7	0.450, -8 0.114, -7 0.159, -7	0.358, -8 0.115, -7 0.151, -7
5	0.097, -7	0.096, -7	0.097, -7	0.098, -7	0.098, -7
6	0.732, -8	0.746, -8	0.730, -8	0.654, -8	0.528, -8
7	0.549, -8	0.492, -8	0.412, -8	0.314, -8	0.224, -8
8	0.283, -8	0.247, -8	0.184, -8	0.138, -8	0.109, -8
9	0.800, -9	0.770, -9	0.700, -9	0.573, -9	0.443, -9
10	0.130, -9	0.197, -9	0.230, -9	0.229, -9	0.184, -9

For energies of 2, 3 and 4 GeV the first line corresponds to neutrinos coming from pions, the second to neutrinos coming from kaons and the third to the sum of both. For energies from 5 to 10 GeV the numbers correspond to neutrinos from kaons only. The negative number after the comma is the exponent of the power of 10 by which the numbers must be multiplied.

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5. The Reaction $\nu + n \rightarrow p + \mu^-$

Løvseth is computing differential cross sections for this reaction, assuming the exchange of a virtual intermediate boson, as functions of the boson mass[4]. He is also going to integrate the differential cross sections over the neutrino spectrum of our present experiment.

If we consider total cross sections for point interaction computed by Lee and Yang[5], and Yamaguchi[6], we obtain the following rates for this reaction:

In the bubble chamber: 1.2 events/ton x day x 10^{11} circulating protons.

In the spark chamber: 0.9 events/ton x day x 10^{11} circulating protons.

The computation was done assuming Van der Meer's neutrino spectrum for 24.8 GeV/c circulating protons, machine efficiency 100%, detector efficiency 100%, machine repetition rate $(3\ \text{sec})^{-1}$. For the bubble chamber the neutrino flux was averaged in a circle of radius 40 cm, and for the spark chamber the spectrum was taken in 5 zones as given by Table 2.

Nothing is known for sure about the cross section for pion production in this reaction. Bell and Berman[7] estimated roughly that the pion production cross section might be from 1 to 3 times the "elastic" process cross sections for neutrino energies up to ~ 10 GeV.

6. The Reactions $\nu + \binom{Z}{p} \rightarrow \binom{Z}{p} + W^+ + \mu^-$.

6.1 Cross Sections.

The differential and total cross sections for these reactions, for aluminium and copper, have been calculated by Veltman[8]. For completeness we reproduce in Tables 3, 4 and 5 the values of the total cross sections found by Veltman for an intermediate vector boson of magnetic moment 1 (no anomalous magnetic moment) and two values of the boson mass: 0.6 of the proton mass and the proton mass.

TABLE 3

Total Cross Sections σ_i for the Incoherent Process

$\nu + p \rightarrow p + W^+ + \mu^-$

obtained by Veltman.

Neutrino Energy (GeV)	σ_i in 10^{-38} cm^2 per proton	
	W mass = 0.6 p mass	W mass = p mass
2	3.7	0.15
3	8.8	2.0
4	13.6	4.8
5	17.8	7.9
6	21.5	10.9
7	24.8	13.8
8	27.7	16.5
9	30.3	19.0
10	32.7	21.4

The magnetic moment of the boson was taken equal to 1, i.e. no anomalous magnetic moment.

TABLE 4

Total Cross Sections for the Process

$\nu + A\ell \rightarrow A\ell + W^{+} + \mu^{-}$

in Aluminium, obtained by Veltman.

Neutrino Energy (GeV)	Total cross section in 10^{-38} cm^2 per aluminium nucleus					
	W mass = 0.6 p mass			W mass = p mass		
	σ_{coh} = coherent process	$13\sigma_i$	σ_{coh} + $13\sigma_i$	σ_{coh} = coherent process	$13\sigma_i$	σ_{coh} + $13\sigma_i$
2	10.4	48.1	58.5	0.01	2.0	2.0
3	94.5	114.4	208.9	0.23	26.0	26.2
4	274.5	176.8	451.3	3.5	62.4	65.9
5	529.6	231.4	761.0	18.4	102.7	121.1
6	839.1	279.5	1118.6	52.3	141.7	194.0
7	1193.4	322.4	1515.8	107.9	179.4	287.3
8	1593.6	360.1	1953.7	185.3	214.5	399.8
9	2046.2	393.9	2440.1	283.4	247.0	530.4
10	2560.7	425.1	2985.8	401.0	278.2	679.2

The magnetic moment of the boson was taken equal to 1, i.e., no anomalous magnetic moment.

TABLE 5

Total Cross Sections for the Process

$\nu + Cu \rightarrow Cu + W^+ + \mu^-$

in Copper, obtained by Veltman.

Neutrino Energy (GeV)	Total cross section in 10^{-38} cm^2 per copper nucleus.					
	W mass = 0.6 p mass			W mass = p mass		
	σ_{coh} = coherent process	$29\sigma_i$	σ_{coh} + $29\sigma_i$	σ_{coh} = coherent process	$29\sigma_i$	σ_{coh} + $29\sigma_i$
2	11.4	107.3	118.7	0.0014	4.95	4.35
3	186.0	255.2	441.2	0.67	58.0	58.67
4	688.1	394.4	1082.5	3.5	139.2	142.7
5	1506.5	516.2	2022.7	21.4	229.1	250.5
6	2584.3	623.5	3207.3	80.6	316.1	396.7
7	3891.5	719.2	4610.7	203.2	400.2	603.4
8	5429.6	803.3	6232.9	401.2	478.5	879.7
9	7221.4	878.7	8109.1	678.9	551.0	1229.9
10	9301.3	948.3	10249.6	1037.7	620.6	1658.3

The magnetic moment of the boson was taken equal to 1, i.e., no anomalous magnetic moment.

6.2 General expression of the rate

Call:

$\frac{\partial^2 N}{\partial E \partial S}$ = the differential spectrum of the neutrinos, per circulating protons;

$\partial(E)$ = the cross section per nucleus, in 10^{-38} cm^2, for a given reaction produced by neutrinos with energy E;

M = detector mass;

A = detector mass number;

N = Avogadro's number;

I_p = number of circulating protons per pulse of the machine;

K = number of machine pulses per day;

η_m = machine efficiency;

η_d = detector efficiency;

Make $B = \frac{1}{A} M N I_p K \eta_m \eta_d 10^{-38}$

and $I = \int_{E_{min}}^{E_{max}} \frac{\partial^2 N}{\partial E \partial S} \sigma(E)\, dE$

Then, the rate of reactions of cross section $\sigma(E)$, per day, is given by:

$$R = I\,B$$

For:

$M = 10^6$ g

$I_p = 10^{11}$ protons $(\text{pulse})^{-1}$

$K = 2.88 \times 10^4$ pulses $(\text{day})^{-1}$

$\eta_m = 1$

$\eta_d = 1$

we obtain:

$B = 6.4 \times 10^5$ for **aluminium**

and $B = 2.7 \times 10^5$ for copper

Values of the integral I are given in tables 6 and 7.

6.3 Expected rates of the reaction $\nu + \binom{Z}{p} \rightarrow \binom{Z}{p} + W^+ + \mu^-$ at the production region of the spark chamber.

Table 6 gives the expected number of events per day, per ton of aluminium and of copper, computed under the following assumptions:

(a) cross sections computed by Veltman and given in Tables 4 and 5;

(b) neutrino spectrum obtained by Van der Meer for circulating protons of momentum 24.8 GeV/c, with the spark chambers divided into 5 circular zones, as given in Table 2;

(c) the values of B given above.

TABLE 6

R = Events/(ton x day x 10^{11} circulating protons), at the spark chamber, assuming $\eta_m = \eta_d = 1$

W-MASS	PROCESS	ALUMINIUM		COPPER	
		I	R = I B	I	R = I B
0.6 p mass	Coherent Incoherent Total	2.82×10^{-6} 1.83×10^{-6}	1.8 1.2 3.0	7.85×10^{-6} 4.10×10^{-6}	2.1 1.1 3.2
p mass	Coherent Incoherent Total	1.43×10^{-7} 1.09×10^{-6}	0.1 0.7 0.8	6.50×10^{-7} 2.38×10^{-6}	0.2 0.6 0.8

6.4 Expected rates of the reaction $\nu + \binom{Z}{p} \to \binom{Z}{p} + W^+ + \mu^-$ at the bubble chamber.

For comparison, we computed the rates for this reaction at the position of the bubble chamber, under the same assumptions used in the computation for the spark chamber, with the only difference that Van der Meer's neutrino spectrum was taken as an average over a circle of radius 40 cm.

TABLE 7

R = Events/(ton x day x 10^{11} circulating protons), at the bubble chamber, assuming $\eta_m = \eta_d = 1$.

W-MASS	PROCESS	ALUMINIUM		COPPER	
		I	R = I B	I	R = I B
0.6 p mass	Coherent	4.83×10^{-6}	3.1	2.24×10^{-7}	0.1
	Incoherent	3.73×10^{-6}	2.4	9.76×10^{-7}	0.6
	Total		5.5		0.7
p mass	Coherent	1.29×10^{-5}	3.5	4.15×10^{-7}	0.1
	Incoherent	8.32×10^{-6}	2.4	2.18×10^{-6}	0.6
	Total		5.9		0.7

We see that, with the assumptions of Veltman's computations, the rate is practically the same per ton of aluminium and per ton of copper, a result which is not obvious because of the influence of the form factors in the coherent process.

6.5 Differential cross sections integrated over the neutrino spectrum.

Let us consider the reaction

$$\nu + \binom{Z}{p} \rightarrow \binom{Z}{p} + W^{+} + \mu^{-} ,$$

with the subsequent decay of the W^+:

$$W^{+} \rightarrow \mu^{+} + \nu .$$

Call: Θ_W, Θ_{μ^-} and Θ_{μ^+} the angles of the W^+, the μ^- and the μ^+ respectively, with the direction of the incident neutrino, and P_W, P_{μ^-} and P_{μ^+} the momenta of these particles.

Veltman[8] computed the differential cross sections $\frac{\partial^2 \sigma}{\partial\Theta\partial P}$, $\frac{\partial\sigma}{\partial\Theta}$ integrated over all momenta and $\frac{\partial\sigma}{\partial P}$ integrated over all angles, for those three particles. His computations were done for W magnetic moment equal to 1, for W masses equal to 0.6 and 1 proton mass, for the coherent and incoherent processes, and for aluminium and copper.

These differential cross-sections were integrated over the neutrino spectrum obtained by Van der Meer at the position of the bubble chamber.

Figures 4 and 5 show the integration of $\frac{\partial^2 \sigma}{\partial\Theta_{\mu^-}\partial P_{\mu^-}}$ and $\frac{\partial^2 \sigma}{\partial\Theta_W \partial P_W}$ for constant values of Θ_{μ^-} and Θ_W , respectively, for the <u>coherent</u> process in aluminium and for W mass = proton mass. Figure 6 shows the integration of $\frac{\partial^2 \sigma}{\partial\Theta_{\mu^+}\partial P_{\mu^+}}$ for constant values of Θ_{μ^+} , for <u>coherent + incoherent</u> processes in aluminium, also for W mass = proton mass. For comparison with aluminium, Figure 7 shows the integration of $\frac{\partial^2 \sigma}{\partial\Theta_{\mu^-}\partial P_{\mu^-}}$ for <u>copper</u> and W mass = proton mass.

We have available a collection of curves of these types for aluminium and copper, for W mass equal to 0.6 and 1 proton mass.

REFERENCES

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2. C. Vahlbruch and S. Van der Meer - NPA Internal Report 63-11.
3. S. Van der Meer - private communication.
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5. T. D. Lee and C. N. Yang - Phys. Rev. Letters 4, 307 (1960).
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8. M. Veltman - Papers in this report and private communication.

FIGURE CAPTIONS

Figure 1 General set-up of the spark chamber neutrino experiment. The neutrino beam goes from right to left. P is the production region; M is the magnet; R is the range region.

Figure 2 The different types of spark chambers carriages in the production region.

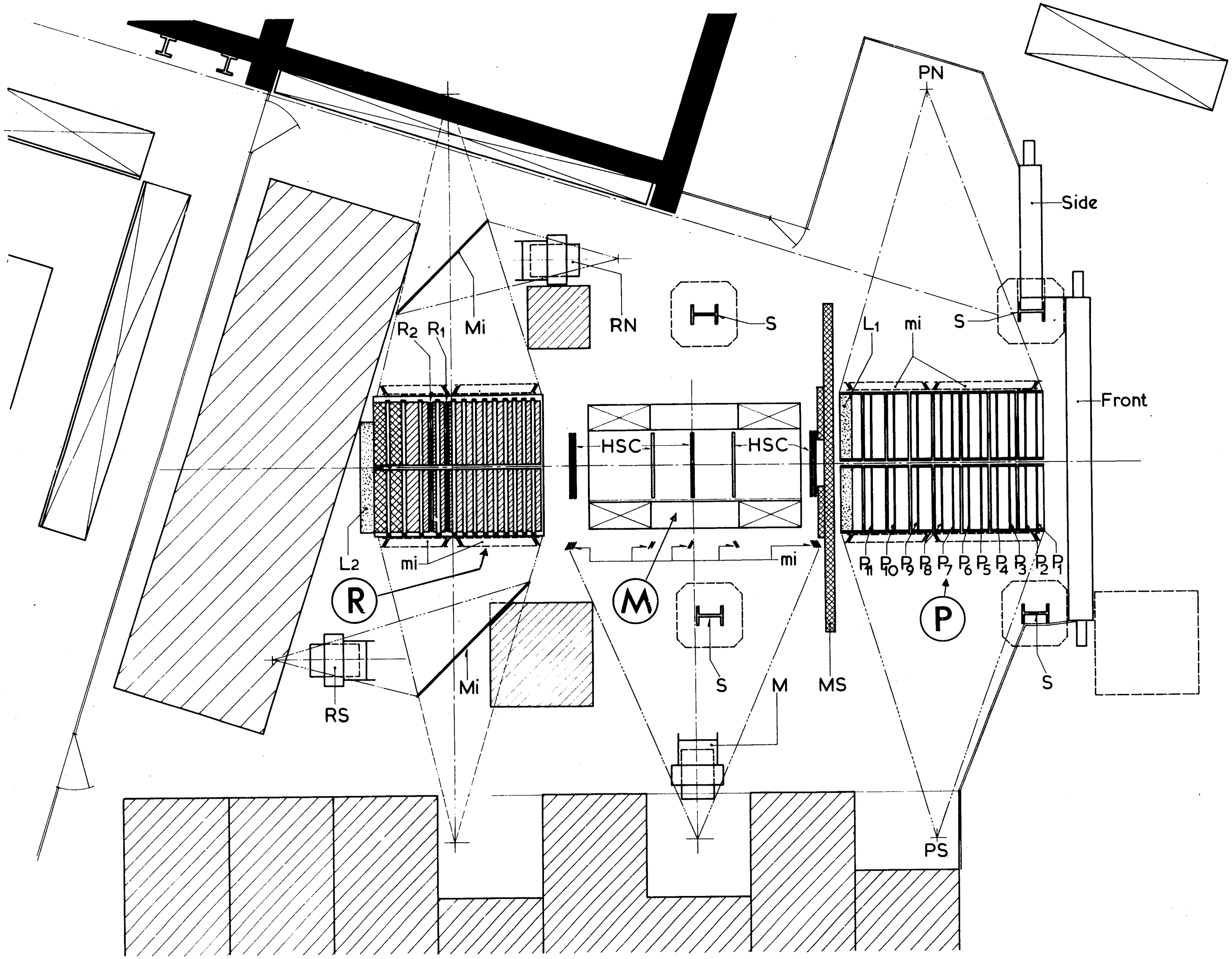
Figure 3 The five concentric zones where the neutrino flux was calculated by Van der Meer.

Figure 4 Curves corresponding to $\int \frac{\partial^2 \sigma}{\partial \Theta_{\mu^-} \partial P_{\mu^-}} \times \frac{\partial^2 N_\nu}{\partial E_\nu \partial S} \, dE_\nu$ for constant values of Θ_{μ^-} for the coherent process in aluminium, for W mass = proton mass and W magnetic moment = 1.

Figure 5 Curves corresponding to $\int \frac{\partial^2 \sigma}{\partial \Theta_W \partial P_W} \times \frac{\partial^2 N_\nu}{\partial E_\nu \partial S} \, dE_\nu$ for constant values of Θ_W, for the coherent process in aluminium, for W mass = proton mass and W magnetic moment = 1.

Figure 6 Curves corresponding to $\int \frac{\partial^2 \sigma}{\partial \Theta_{\mu^+} \partial P_{\mu^+}} \times \frac{\partial^2 N_\nu}{\partial E_\nu \partial S} \, dE_\nu$ for constant values of Θ_{μ^+}, for coherent + incoherent processes in aluminium, for W mass = proton mass and W magnetic moment = 1.

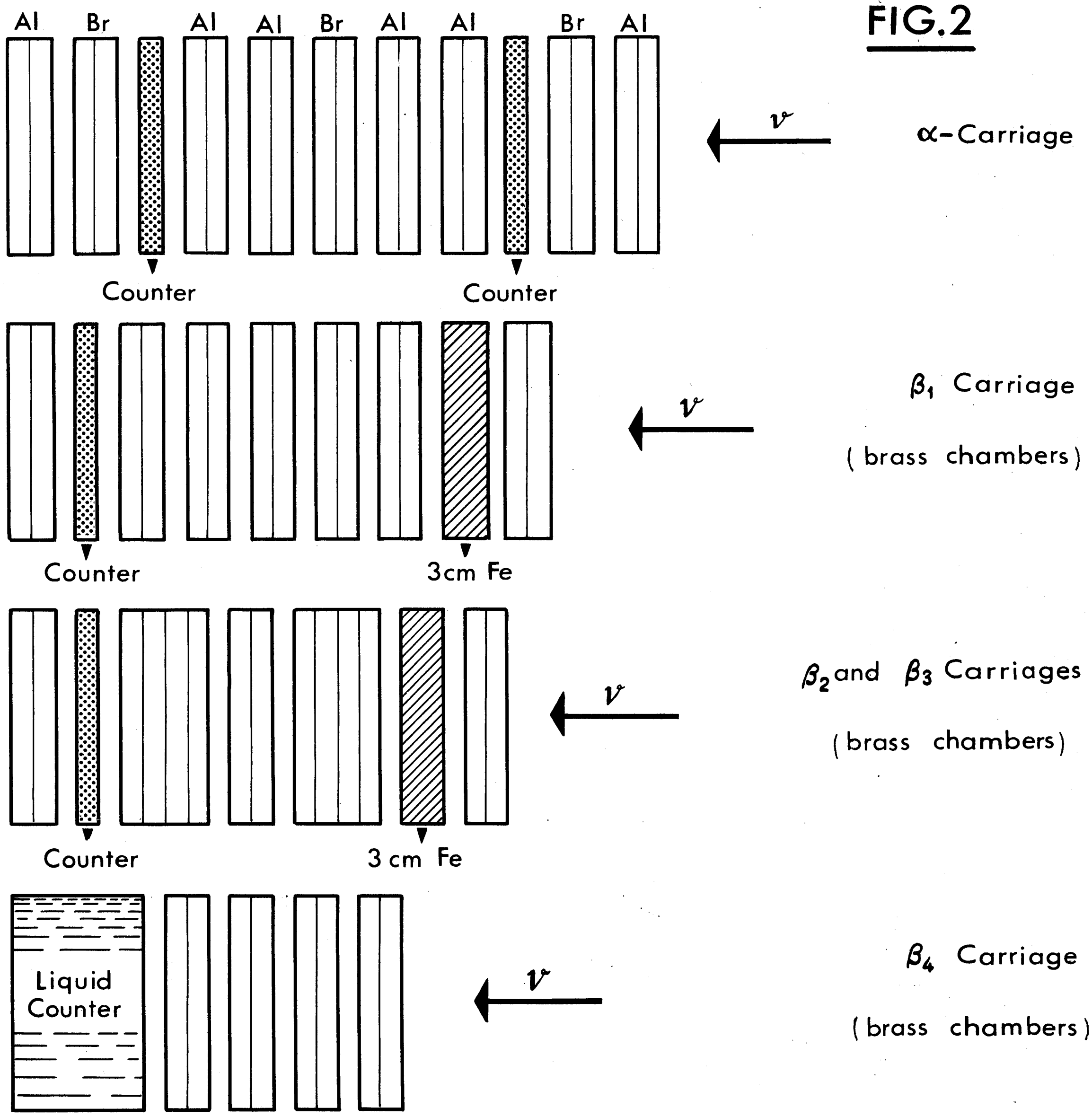
Figure 7 Curves corresponding to $\int \frac{\partial^2 \sigma}{\partial \Theta_{\mu^-} \partial P_{\mu^-}} \times \frac{\partial^2 N_\nu}{\partial E_\nu \partial S} \, dE_\nu$ for constant values of Θ_{μ^-}, for the coherent process in copper, for W mass = proton mass and W magnetic moment = 1.



$P_1 - P_{11}$, R_1, R_2 — Plastic Counters
L_1, L_2 — Liquid YES Counters
Front, Side — Anti Coincidence Counters
+ — Virtual Camera Positions
PN, PS — Camera Pairs (production region north and south respectively)
RN, RS, M — Stereo Cameras (range chamber north, south and magnet)
MS — Magnetic Shielding Wall
HSC — Hodoscope Spark Chambers
mi — Small Mirror Pairs
Mi — Large Mirrors
S — Roof Support
▨ — Pb Walls
▩ — Fe Walls

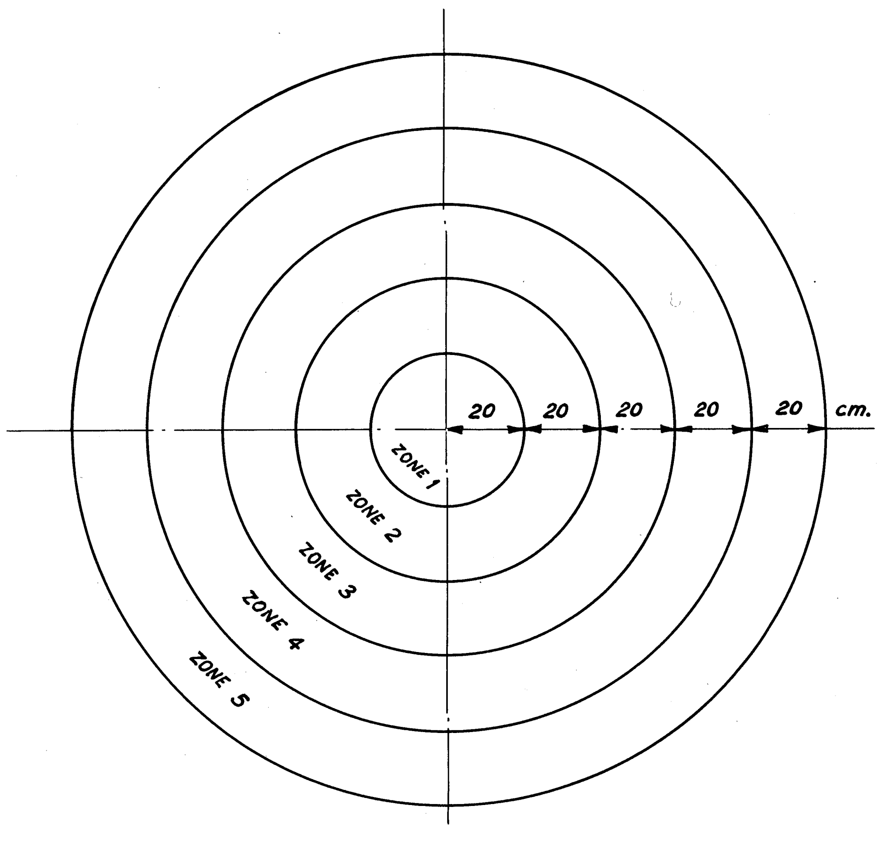
FIG. 1

FIG.2



Al
Br
Al
Al
Br
Al
Al
Br
Al
Counter
Counter
v
α-Carriage
Counter
3cm Fe
v
β_1 Carriage
(brass chambers)
Counter
3 cm Fe
v
β_2 and β_3 Carriages
(brass chambers)
Liquid
Counter
v
β_4 Carriage
(brass chambers)

FIG. 3



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ZONE 1
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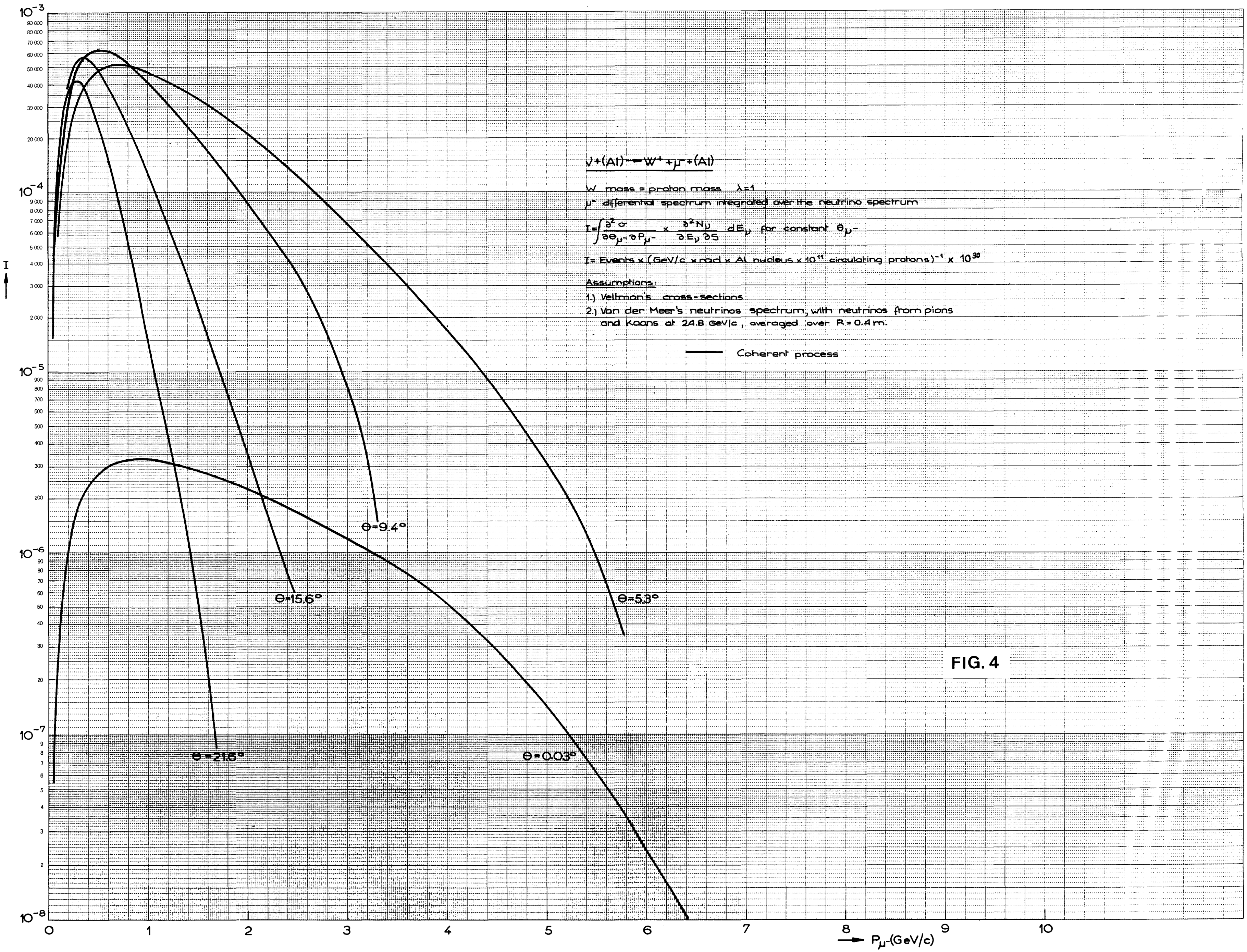


FIG. 4

SIS/R/7482

SIS/R/7481

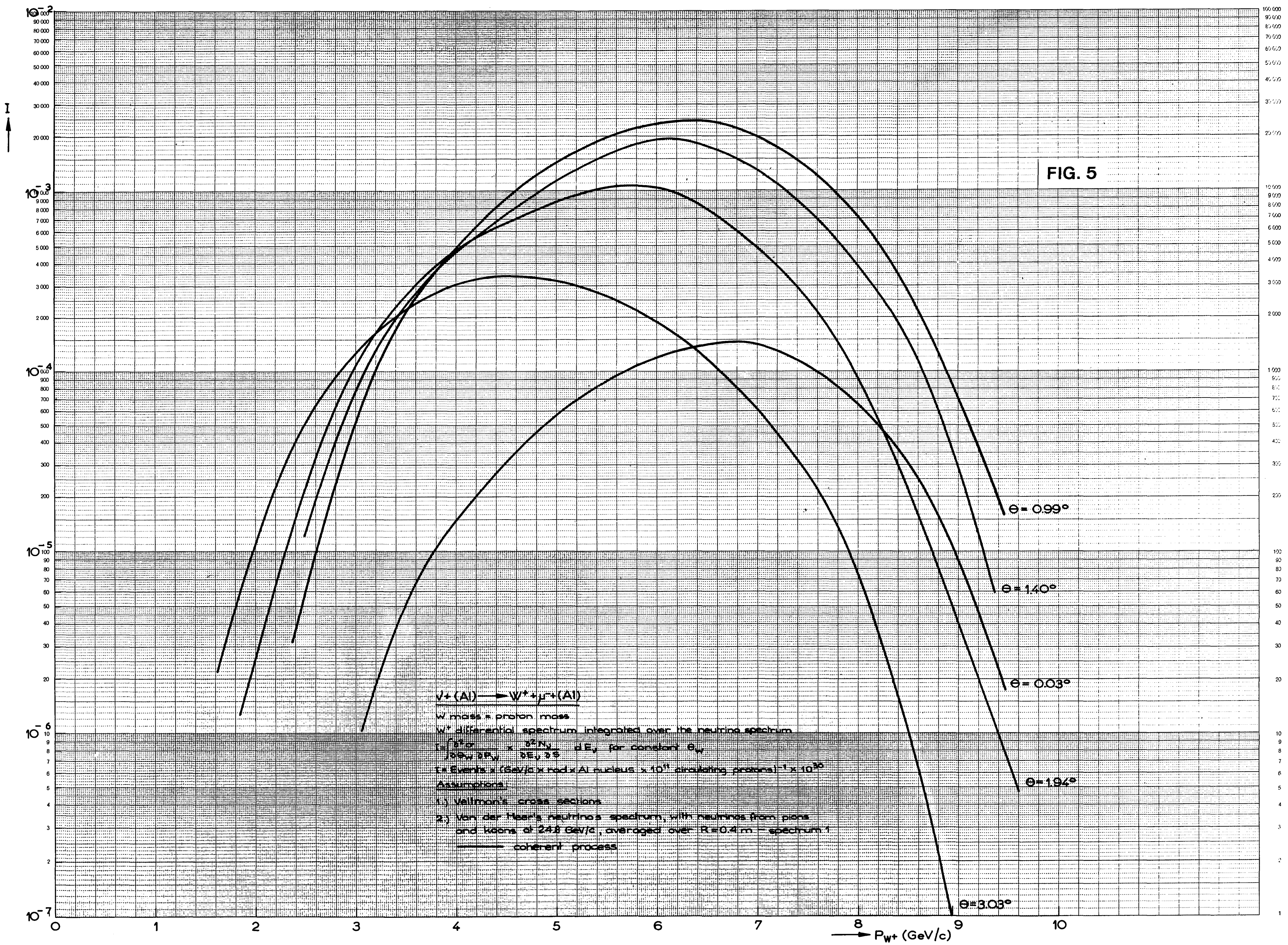
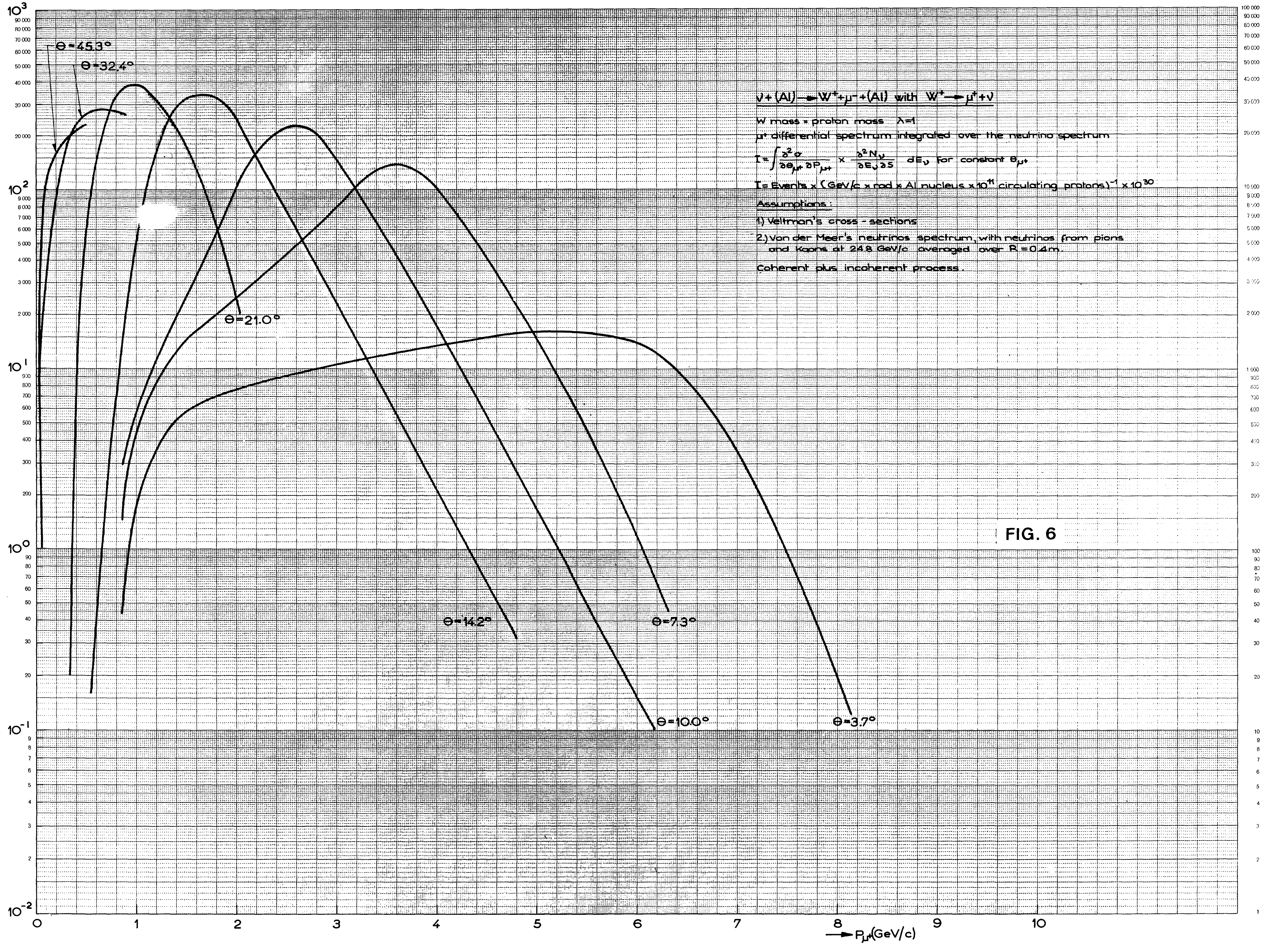


FIG. 5



$\nu + (Al) \rightarrow W^+ + \mu^- + (Al)$ with $W^+ \rightarrow \mu^+ + \nu$
W mass = proton mass $\lambda = 1$
μ^+ differential spectrum integrated over the neutrino spectrum
$I = \int \frac{\partial^2 \sigma}{\partial \theta_{\mu^+} \partial P_{\mu^+}} \times \frac{\partial^2 N_\nu}{\partial E_\nu \partial S} \, dE_\nu$ for constant θ_{μ^+}
I = Events x (GeV/c x rad x Al nucleus x 10^{11} circulating protons)$^{-1}$ x 10^{30}
Assumptions :
1) Veltman's cross - sections
2) Van der Meer's neutrinos spectrum, with neutrinos from pions and Kaons at 24.8 GeV/c averaged over R = 0.4 m.
Coherent plus incoherent process.
θ=45.3°
θ=32.4°
θ=21.0°
θ=14.2°
θ=10.0°
θ=7.3°
θ=3.7°
I
P_{μ^+}(GeV/c)

FIG. 6

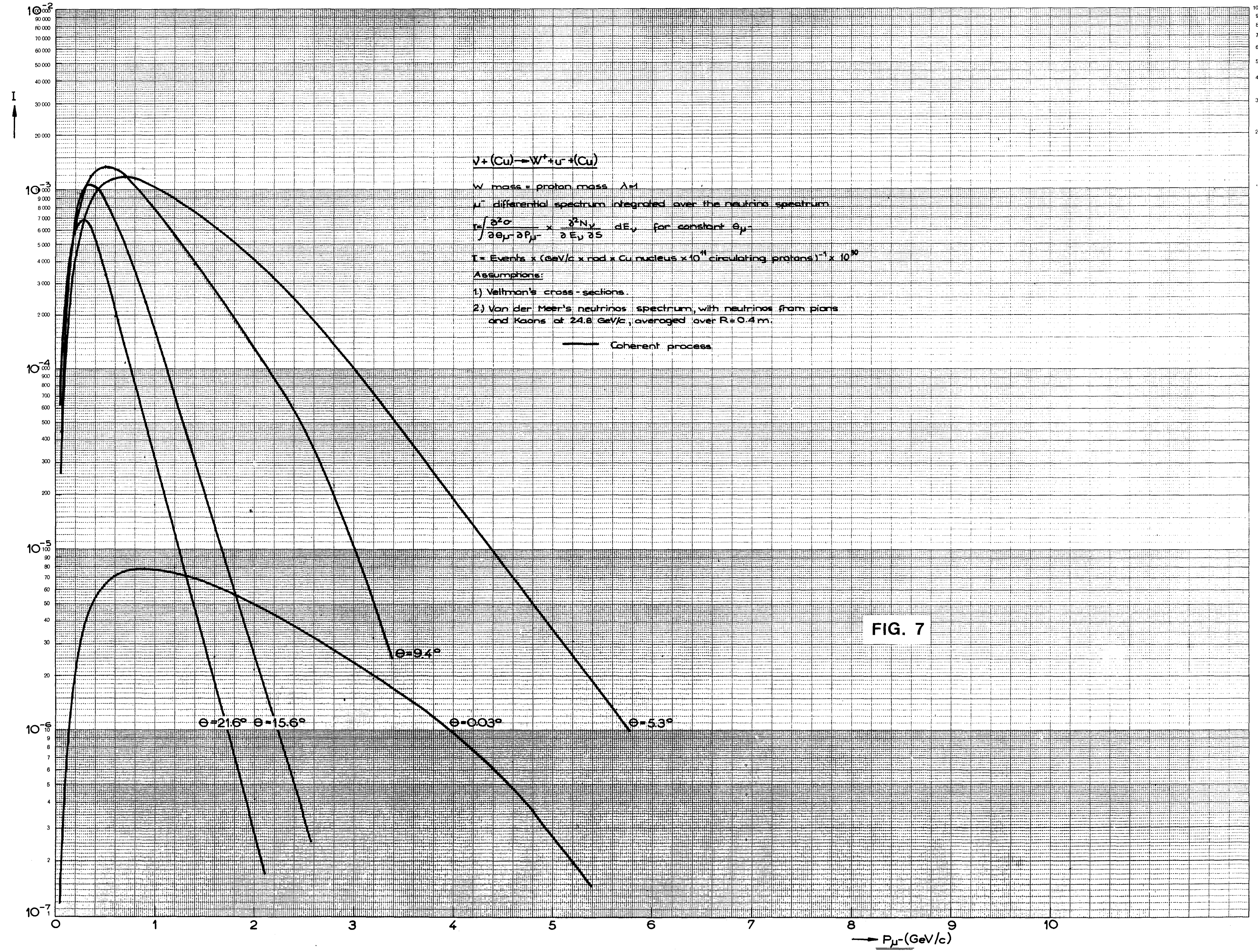

ν + (Cu) → W⁺ + u⁻ + (Cu)
W mass = proton mass λ=1
μ⁻ differential spectrum integrated over the neutrino spectrum
I = ∫ ∂²σ/∂θμ⁻ ∂Pμ⁻ × ∂²Nν/∂Eν ∂S dEν for constant θμ⁻
I = Events × (GeV/c × rad × Cu nucleus × 10¹¹ circulating protons)⁻¹ × 10³⁰
Assumptions:
1.) Veltman's cross-sections.
2.) Van der Meer's neutrinos spectrum, with neutrinos from pions and Kaons at 24.8 GeV/c, averaged over R = 0.4 m.
Coherent process
θ=9.4°
θ=21.6°
θ=15.6°
θ=0.03°
θ=5.3°
I
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
0
1
2
3
4
5
6
7
8
9
10
Pμ⁻ (GeV/c)

FIG. 7