PION PRODUCTION IN p-p COLLISIONS AT 0.8, 1.5 AND 2.75 BEV

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I. Experimental arrangement

Proton-proton scattering experiments, using external proton beams from the Brookhaven Cosmotron, were carried out at 0.8, 1.5 and 2.75 Bev. The experiments were all performed using the same hydrogen-filled diffusion cloud chamber [1] operated at 20 atmospheres in a magnetic field of $\sim$ 10,000 gauss. The exposure geometries were all essentially the same and are illustrated in fig. 1. The external beams were obtained in two ways. The internal beam either spiralled out after being "blown up" horizontally by varying the radio-frequency accelerating voltage, or, as in the 1.5 Bev exposure, the internal beam was scattered out by a carbon target. The beam kinetic energies in the cloud chamber were estimated to be 0.81 ± 0.10 Bev, 1.5 ± 0.26 and 2.75 ± 0.10 Bev. These values for the two lower energies were based on momentum of the beam tracks as measured in the chamber. The value at 2.75 Bev was deduced from the energy of the circulating beam and a knowledge of the trajectories of the

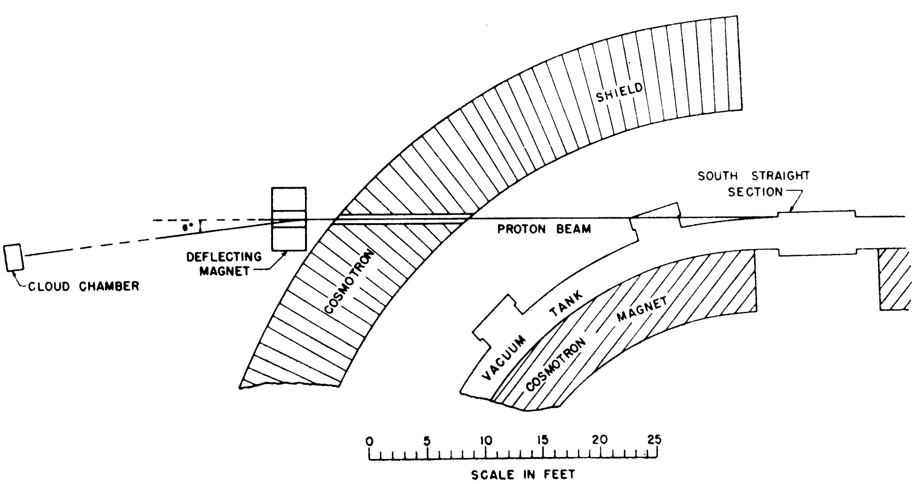

Fig. 1. A schematic view of one quadrant of the Cosmotron, showing the external proton beam geometry.

* This paper is a summary of four papers on p-p scattering submitted to the Physical Review.
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emergent protons and the amount of intervening matter before they entered the cloud chamber. A total of 244, 152 and 212 interactions were observed at 0.8, 1.5 and 2.75 Bev, respectively. The reactions were idendified by combining the knowledge obtained from the measured momenta, ionizations and spatial angles of the secondaries with an energy and momentum balance using the known primary momentum. The momenta were measured using a low power microscope and the angles were measured on a stereoscopic reprojection system. The inaccuracies in these measurements were generally due to the typically short track length of the secondaries, since the cloud chamber had a sensitive depth of approximately 1.5 inches.

II. Experimental results

A. *Total cross-sections*

The total cross-sections were obtained by scanning a selected region of the chamber and estimating the total track length of protons in this region by making a random sampling of the film. After correcting for scanning inefficiencies, the total p-p interaction cross-sections at 0.8, 1.5 and 2.75 Bev were found to be 45 $\pm$ 6 mb, 35 $^{+8}_{-5}$ mb, and 35 $^{+8}_{-5}$ mb, respectively, where the errors include an allowance for scanning bias corrections as well as a statistical uncertainty. These values are in agreement with the more accurate counter data of Shapiro, Leavitt and Chen [2].

B. *Partial cross-sections*

From consideration of coplanarity and the conservation laws, it is relatively simple to separate inelastic from elastic events quite unambigously in a hydrogen-filled cloud chamber. This is in contrast to similar emulsion experiments, where the Fermi momentum of the struck proton introduces an unknown and possibly confusing factor*. For this reason, we have been able to measure the elastic-to-inelastic ratio to an accuracy essentially limited by statistics only. Using this ratio and the more accurate counter data for a normalization of the total cross-section, we obtain the partial cross-sections shown in Table I. At both 0.8 and 1.5 Bev, the inelastic collisions produced pions only, whereas at 2.75 Bev, there were one definite and two probable collisions which produced "strange" particles.

TABLE I

Partial cross-sections

Beam energy (Bev)	Cross-sections Elastic (mb)	Cross-sections Inelastic (mb)
0.8	24 $\pm$ 3	24 $\pm$ 3
1.5	20 $\pm$ 2	27 $\pm$ 3
2.75	15 $\pm$ 2	26 $\pm$ 3

C. *Multiplicities*

At 0.8 Bev, there were no collisions observed which led to the production of two or more mesons. At 1.5 Bev, there were no collisions observed which required an interpretation as the production of 3 or more mesons. At 2.75 Bev, both single, double and triple meson production were observed in comparable amounts. The difficulty of reliably distinguishing single, double and triple meson production increases with beam energy due to the higher c.m.s. motion and consequent inaccuracies in individual momentum determinations. Further, it is very difficult, if not impossible in principle in a large number of cases, to distinguish between p-p collisions leading to final states, such as pn + 0 and pn + 00. Experimentally, because of errors in individual measurements, it is often difficult to distinguish a pn + from a pn + 0 reaction. For these reasons, the multiplicities quoted in Table II are subject to large uncertainties. In particular, the 2.75 Bev data are only an estimate, whereas the 1.5 Bev data are more reliable and the 0.8 Bev data are reasonably certain. The theoretical comparisons will be treated in the next section.

TABLE II

Multiplicity of pion production as a function of beam energy compared with theory

Beam energy (Bev)	Experimental ratios (single: double: triple)	Fermi (single: double: triple)	Kovacs (single: double)
0.8	100:0 :0	100:0 :0	100:0
1.5	80:20:0	94:6 :0	55:45
2.75	36:48:16	78:20:2	28:72*

* Extrapolated linearly from Kovacs' curves.

III. Analysis

A. *Pion multiplicity in inelastic events*

In discussing inelastic events the first step is to consider the overall frequency of different pion multiplicities before going to a more detailed breakdown in terms of final charge states and distributions in energy and angle of the particles emitted. Since many different reactions are possible, the analysis of inelastic events is much more complicated than that of elastic events. No detailed theory is available for comparison, but various aspects of the data can be compared with the Fermi statistical theory, which makes no specific assumptions concerning the interaction of pion and nucleon fields [3]. If the experimental results show departures from the predictions of the statistical theory, these may give information concerning the details of the interactions involved. One detail which may be of particular interest is the strong interaction in the pion-nucleon state with T = J = 3/2 shown by

* In the Fermi theory the radius of the interaction volume in which equilibrium is supposed to take place is an unspecified parameter that can in principle be adjusted to fit experiment. The calculations quoted were made with radius equal to the Compton wave-length of the pion, as is conventional.

pion-nucleon scattering results. It is, therefore, important to determine whether pion production phenomena are influenced by such a strongly-interacting state.

The "best values" for the frequency of single, double and triple pion production are summarized in Table II, where they are compared with theoretical predictions. The third column gives the results of an evaluation of the Fermi theory * using exact relativistic expressions [4]. The experimental results at 1.5 and 2.75 Bev show considerably more double and triple meson production than predicted. This conclusion is similar to that obtained from analysis of n-p collisions [5].

A number of theoretical calculations have been made in which the Fermi theory is modified in different ways. Lepore and Neuman introduced the conservation law of relativistic center-of-energy [3]. Qualitatively, their results predict a lowering of the average momentum of the pion (in comparison with the original Fermi theory), resulting in a higher multiplicity. Since no detailed calculations have been made at our specific energies, however, it is not known whether this theory gives quantitative agreement with our results. Bocchieri and Feldman have evaluated matrix elements by a perturbation calculation and have shown that a sufficiently large coupling constant might lead to frequent double pion production [7]. The effects of final state interactions between nucleons and between pions and nucleons have been included by Kovacs [8]. He finds that the pion-nucleon interaction seriously alters the statistical results, with an enhancement of two-meson states due to resonance effects. If this effect dominates the production process, one would expect the cross-section for single meson production to rise sharply at a laboratory kinetic energy of $\sim$ 0.7 Bev, whereas double meson production should compete seriously at $\sim$ 1.5 Bev, and triple production should set in at $\sim$ 2.4 Bev. This is in good qualitative agreement with our experimental results. Kovacs' calculated results for the single : double pion production ratios are indicated in column 4 of Table II. The theoretical multiplicities are somewhat larger than the experimental, but this discrepancy might well be ascribable to the approximate calculation methods employed, in particular, the non-relativistic treatment of the nucleons.

B. Charge state ratios and charge independence

For each pion multiplicity we now investigate the relative frequencies of the various possible charge states. The concept of charge independence of nuclear forces has proved to be a valuable tool in the analysis of high energy nuclear reactions and all the following discussion is based on the charge independence hypothesis. In particular, we will develop from this hypothesis general results (independent of any specific model) for the reactions $p + p \rightarrow p + n + \pi^+$ or $p + p + \pi^0$, which can be compared with experiment. As will be indicated later, our experiments actually provide an additional independent verification of charge independence.

Since the initial p-p state of our system is in an isospin $T = 1$, $T_z = +1$ state, we require that our final system consisting of two nucleons and a pion be in a 1, + 1 state. Within the context of charge independence, the two nucleons appearing in the final state must be treated as identical particles, since the charge state of a given nucleon is merely an additional quantum number specified by its z component of isotopic spin. This requires that the total final state wave function (depending on nucleons 1 and 2 and the pion), which is a product of real space $\times$ real spin $\times$ isospin wave functions, be anti-symmetric in the exchange of nucleons 1 and 2.*

The final state isotopic spin wave function corresponding to an isospin $T = 1$ can be constructed by first coupling the isospin of the nucleons to obtain $T' = 0$ or 1 and then combining T' with the isospin of the meson to get $T = 1$. This mode of intermediate coupling insures that the final isospin state is either symmetric or anti-symmetric. We find that the final isospin wave functions corresponding to the two different intermediate values of T' are :

$$\text{(a)}\ \tau_A = \frac{1}{\sqrt{2}} \mid p_1 n_2 + > - \frac{1}{\sqrt{2}} \mid n_1 p_2 + > \text{ for } T' = 0 \tag{1}$$

$$\text{(b)}\ \tau_S = -\frac{1}{2} \mid p_1 n_2 + > - \frac{1}{2} \mid n_1 p_2 + > + \frac{1}{\sqrt{2}} \mid p_1 p_2 0 > \text{ for } T' = 1, \tag{2}$$

where for example, $\mid p_1 n_2 + >$ is the ortho-normal state function for nucleon 1 being in a proton state (i.e., $T_{z_1} = +{}^1/_2$), nucleon 2 being in a neutron state and the pion being in a π^+ state. We note that from (1) τ_A is anti-symmetric, whereas from (2) τ_S is symmetric. If we denote the (real space $\times$ real spin) portions of the outgoing wave function by φ, then the total wave function ψ (which is a function of the two nucleons and the pion) is

$$\psi = \varphi_S \tau_A + \varphi_A \tau_S \tag{3}$$

where the appropriate real space $\times$ real spin portion of τ_A must be symmetric in order to insure ψ being anti-symmetric, and for the same reason, the real space $\times$ real spin portion of τ_S must be anti-symmetric. Combining (1), (2), and (3) the total wave function can be written as

$$\psi = \left(-\frac{\varphi_A}{2} + \frac{\varphi_S}{\sqrt{2}}\right) \Big| p_1 n_2 + > + \left(-\frac{\varphi_A}{2} - \frac{\varphi_S}{\sqrt{2}}\right) \Big| n_1 p_2 + > + \frac{\varphi_A}{\sqrt{2}} \Big| p_1 p_2 0 >. \tag{4}$$

* Hereafter, the terms symmetric and anti-symmetric, and/or subscripts S and A will denote wave functions which are symmetric or anti-symmetric under the exchange of two nucleons.

From (4), it can be shown that the ratio

$$R = \frac{d\sigma(p + p \rightarrow p + n + \pi^+)}{d\sigma(p + p \rightarrow p + p + \pi^0)}$$

is given by

$$R = 1 + 2\frac{|\varphi_S|^2}{|\varphi_A|^2}. \quad (5)$$

Thus an experimental determination of R fixes the ratio of $\frac{|\varphi_S|^2}{|\varphi_A|^2}$. In particular, from (5), if $\varphi_S = 0$, then $R = 1$, whereas if $\varphi_A = 0$, $R = \infty$, i.e., $1 \leqslant R \leqslant \infty$.

If we consider the case $R = \infty$, (4) can be written as

$$\psi = \frac{\varphi_S}{\sqrt{2}} | p_1 n_2 + > - \frac{\varphi_S}{\sqrt{2}} | n_1 p_2 + >. \quad (6)$$

It can readily be shown from (6) that the probability that the nucleon in position 1 is in the proton charge state is equal to the probability that it is in the neutron charge state. In other words, in the reaction $p + p \rightarrow p + n + \pi^+$, the spatial distributions for neutrons and protons must be *identical* for the final state neutrons and protons. This conclusion remains unaltered for the case $R = 1$, as is seen from inspection of (4). Thus, in summary, if experiment were to show that R is given by either $R = \infty$ or $R = 1$, then it is predicted as a consequence of charge independence that the angular distributions, momentum distributions, etc. of the protons and neutrons in the aforementioned reaction would be identical.

Experimentally, our most reliable charge state ratios are obtained from the above reactions, which only involve single meson production. Whereas in principle analogous ratios could be obtained for both double and triple meson producing processes, these ratios are exceedingly uncertain, as mentioned earlier. In particular, our best value of R (for single production) is given by the 0.8 Bev data, since at this energy no evidence was found for multiple production and thus there is less chance for misidentification. The actual raw data obtained at 0.8 Bev yield 84 pn+, 5 ppo and 28 events that could be either pn+ or ppo. From the definitely identified events, $R = 17$. From an estimate of the number of ppo events in the unidentified fraction and statistical uncertainties, it is concluded that $R = 17 \pm 8$. At 1.5 Bev, there were 19 identified pn+ and zero identified ppo. The corresponding 2.75 Bev value of R, which is apparently also very large, is unfortunately too unreliable to be of use. From (5), using the above range of R values, it is found that between 80 and 95% of the cross-section for single meson production is due to φ_S, the symmetric (real space × real spin) wave function. Therefore, the major effects in this experiment can be attributed to φ_S, and we neglect φ_A in comparison. It, therefore, follows from the preceding arguments that charge independence requires that the proton and neutron have the same spatial distributions (if φ_A is set equal to zero). As will be shown in later sections, we find that, experimentally, the proton and neutron do indeed have similar distributions, within experimental error. Thus, this result provides an independent check on the hypothesis of charge independence of nuclear forces for reactions occuring at 0.8 Bev, an energy considerably higher than those at which other checks have been made. For a summary of charge independence experiments, see Henley and others [9].

We further conclude from the above remarks that if the R values at 1.5 and 2.75 Bev are very large, as they indeed appear to be, then if charge independence holds at these higher energies, the charge labels of proton and neutron in pn+ reactions should be interchangeable and thus, no significant difference should be observed between the neutron and proton. This means of course that, if φ_A could be completely neglected, not only must the proton and neutron angular and momentum distributions be identical, but also the proton and the neutron must have the same correlation angles and Q plots with respect to the π^+ mesons. Therefore, in the approximation $\varphi_A = 0$, any differences observed should be due to statistical fluctuations, experimental errors and experimental bias. Indeed, such biases must play an important role since the proton and the pion have their momenta and angles directly measured, whereas the neutron values are inferred from these measurements by means of conservation laws. These effects will be discussed in more detail later. At the higher beam energies, the large experimental measurement errors (due to the average momenta of the reaction products increasing rapidly with beam energy) make interpretation difficult in some cases, and indeed, impossible in a very large number of cases. Among the uninterpretable cases may lie the missing events necessary to insure that the neutron and proton distributions be essentially the same. In practice, since φ_A is small, but non-vanishing, small asymmetries might be expected from its interference with φ_S. As a consequence of the foregoing, it is observed that any detailed model which is invoked to explain these reactions cannot contain features leading to any appreciable asymmetry between the proton and neutron distributions.

Lindenbaum and Yuan (private communication) have suggested that meson formation at Cosmotron energies proceeds via a two-step process (1) excited isobars are formed by the colliding nucleons, and (2) the isobars subsequently decay into a pion and nucleon. Peaslee [10] has investigated the case where only excited nucleons with an isospin of $^3/_2$ are formed. In order to bring these interpretations within the general framework of charge independence (as expressed in (3) and (4) for the reaction $p + p \rightarrow p + n + \pi^+$ or $p + p + \pi^0$), we consider a general model for single meson production which includes the formation of both the $^3/_2$ and $^1/_2$ isobars. It is now assumed that nucleon 1 and the pion are coupled together in either a $^3/_2$ or $^1/_2$ isospin state, which resultant is then coupled to nucleon 2 to lead to a final isospin state for the 3 particles of $T = 1$, $T_z = +1$. These isospin wave functions are labelled by the value of the intermediate state, and are given by

$$\tau_{3/2}(1,2) = -\sqrt{\frac{1}{6}}\,|p_1p_2 0> - \frac{1}{\sqrt{12}}|n_1p_2+> + \sqrt{\frac{3}{4}}\,|p_1n_2+>, \quad (7)$$

$$\tau_{1/2}(1,2) = -\frac{1}{\sqrt{3}}\,|p_1p_2 0> + \sqrt{\frac{2}{3}}\,|n_1n_2+>. \quad (8)$$

The indices 1,2 refer to the spatial positions of nucleons 1 and 2, and for simplicity, the pion position is omitted throughout. If nucleons were *non-identical* particles, the total wave function would be given by

$$\psi(1,2) = \varphi_{3/2}(1,2)\,\tau_{3/2}(1,2) + \varphi_{1/2}(1,2)\,\tau_{1/2}(1,2), \quad (9)$$

where the φ's correspond to the (real space $\times$ real spin) portion of the outgoing wave function. Since nucleons are identical particles (within the framework of charge independence), and we thus require

$$\psi(1,2) = -\psi(2,1),$$

(9) is clearly an inadmissable wave function in general. To give it the necessary anti-symmetry, we form

$$\psi(1,2) = \varphi_{3/2,S}(1,2)\,\tau_{3/2,A}(1,2) + \varphi_{3/2,A}(1,2\,\tau)_{3/2,S}(1,2) + \varphi_{1/2,S}(1,2)\,\tau_{1/2,A}(1,2) + \varphi_{1/2,A}(1,2\,\tau)_{1/2,S}(1,2), \quad (10)$$

where

$$\tau_{3/2,\,^S_A}(1,2) = \frac{1}{\sqrt{2}}\left[\tau_{3/2}(1,2) \pm \tau_{3/2}(2,1)\right], \quad (11)$$

and

$$\varphi_{3/2,\,^S_A}(1,2) = \frac{1}{\sqrt{2}}\left[\varphi_{3/2}(1,2) \pm \varphi_{3/2}(2,1)\right], \quad (12)$$

and similarly for the $^1/_2$ state. These are functions of manifest symmetry and obviously lead to (10) being anti-symmetric. We find from (11) and its analogue for the $^1/_2$ state that

$$\tau_{3/2,S} = -\sqrt{\frac{2}{3}}\,\tau_S, \quad (13)$$

$$\tau_{3/2,A} = -\sqrt{\frac{4}{3}}\,\tau_A, \quad (14)$$

$$\tau_{1/2,S} = -\sqrt{\frac{4}{3}}\,\tau_S, \quad (15)$$

$$\tau_{1/2,A} = -\sqrt{\frac{2}{3}}\,\tau_A, \quad (16)$$

where τ_S and τ_A have been defined in (1) and (2).

Using (13) - (16) and (10), the total wave function is found to be

$$\psi = \left(\sqrt{\frac{4}{3}}\,\varphi_{3/2,S} - \sqrt{\frac{2}{3}}\,\varphi_{1/2,S}\right)\tau_A + \left(-\sqrt{\frac{2}{3}}\,\varphi_{3/2,A} - \sqrt{\frac{4}{3}}\,\varphi_{1/2,A}\right)\tau_S. \quad (17)$$

Comparison of (16) with (3) indicates that the wave functions φ_A and φ_S previously obtained can be interpreted, in the excited nucleon model, as being given by

$$\varphi_S = \sqrt{\frac{4}{3}}\,\varphi_{3/2,S} - \sqrt{\frac{2}{3}}\,\varphi_{1/2,S}, \quad (18)$$

and

$$\varphi_A = -\sqrt{\frac{2}{3}}\,\varphi_{3/2,A} - \sqrt{\frac{4}{3}}\,\varphi_{1/2,A}. \quad (19)$$

Under Peaslee's assumption,

$$\varphi_{1/2,A} = \varphi_{1/2,S} = 0.$$

Further it is implicit and necessary in his calculation that the isobar be assumed to have a sufficiently long lifetime so that on the average it decays when it is quite separated from the other nucleon, and thus is not interacting with it. Under these conditions, the spatial portion of the wave function containing the two nucleons can be separated into forms of the type $\frac{1}{\sqrt{2}}(f(1)\ g(2) \pm f(2)\ g(1))$. If nucleon 1 (the one assumed to be coupled to the pion in the case of *non-identical* nucleons) is sufficiently localized so that it has an appreciable wave function only in the neighborhood of position 1, then

$$\varphi_{3/2,S}(1,2) \approx \varphi_{3/2,A}(1,2) \approx \sqrt{\frac{1}{2}}\,\varphi_{1/2,S}(1,2),$$

where $\varphi_{3/2}(1,2)$ is the unsymmetrized wave function in (9). In this limit,

$$\varphi_S \approx \sqrt{\frac{2}{3}}\,\varphi_{3/2}, \text{ and } \varphi_A \approx -\sqrt{\frac{1}{3}}\,\varphi_{3/2},$$

Therefore, from (5), we obtain $R(pn+/ppo) = 5{:}1$, the result obtained by Peaslee. If we were to have made the opposite assumption that only the $^1/_2$ state isobar contributes to the process, the corresponding value of R would be 2:1. Values of R larger than 5 and smaller than 2 can only occur because of interference terms between the $^3/_2$ and $^1/_2$ states. Because of the $^1/_2$ state contributions and the consequent large interference terms that are present for the large values of R found in this experiment, the excited nucleon model loses its simplicity and most of its predictive powers, and for these reasons it will not be included further in our analysis.

The Fermi statistical theory has been applied to the analysis of nucleon-nucleon collisions at Cosmotron energies [3]. This theory satisfies our above-mentioned requirement of symmetrical behavior between neutron and proton, and is therefore applicable to our data. In particular, Fermi predicts that the charge state ratio of pn+/ppo in p-p collisions is R = 3. However, it should be pointed out that this is not a unique result. The statistical theory predicts no difference in spatial behavior between proton and neutron. We can use this property, conversely, to define what is meant by a statistical theory. If we do so and require that the neutron and proton enter symmetrically, it follows from (4) that

$$\left| -\frac{\varphi_A}{2} + \frac{\varphi_S}{\sqrt{2}} \right|^2 = \left| -\frac{\varphi_A}{2} - \frac{\varphi_S}{\sqrt{2}} \right|^2, \quad (20)$$

if the neutron and proton are to have the same distribution in the reaction $p + p \rightarrow p + n + \pi^+$. Since φ_A and φ_S are complex quantities, (20) can be satisfied in three ways, namely:

(a) $\varphi_A = 0$

(b) $\varphi_S = 0$

(c) $\varphi_S = 0$ in regions where $\varphi_A \neq 0$, and $\varphi_A = 0$ in regions where $\varphi_S \neq 0$.

Fermi has apparently chosen condition (c) and, to satisfy the statistical requirement, has taken $|\varphi_A|^2 = |\varphi_S|^2$. However, it is equally consistent with the statistical hypothesis to choose either condition (a) or (b). We then interpret the exclusion of the other symmetry state as being due to a new selection rule. In summary, it is found that there are *three a priori* predictions of R possible from the statistical theory, corresponding to assumption (a), (b) and (c), respectively, $R = \infty$, $R = 1$ and $R = 3$. It should be observed that the large experimental values of R are in agreement with condition (a), and therefore we conclude that these results are in essential agreement with the statistical theory, even though they disagree with the R value derived from the more restrictive conditions chosen by Fermi. These same remarks would apply to the modified statistical theory of Lepore and Neuman[6], in which the relativistic center-of-energy is conserved.

C. Angular distributions for (pn+)

Since the (pn+) reaction is the most frequent, it is the only one having a sufficient number of cases for angle and momentum distributions to be meaningful. There are few definite theoretical predictions concerning angular distributions with which the experimental data can be compared. Of course all c.m.s. angular distributions should show forward-backward symmetry about 90°, since the two colliding protons are identical particles. The simple statistical model (in which angular momentum is not conserved) would predict isotropy for all of the particles. Fermi has shown that at very high energies, where there is an appreciable Lorentz contraction of the sphere into which energy is dumped, the conservation of angular momentum requires a strong forward-backward peaking in the c.m.s[3]. It is not clear whether, at the rather low energies of these experiments, this effect would lead to appreciable anisotropy in the c.m.s. Near threshold it is usually assumed that the nucleons are emitted in S-states and pions in P-states, and one might expect such states to predominate at higher energies, as was assumed by Kovacs [8]. In such a case the nucleons should be fairly isotropic, but the pions peaked forwards and backwards. The argument presented earlier leads to the conclusion that the symmetric function φ_S predominates, and therefore, neutron and proton should have similar angular distributions, whatever each may be.

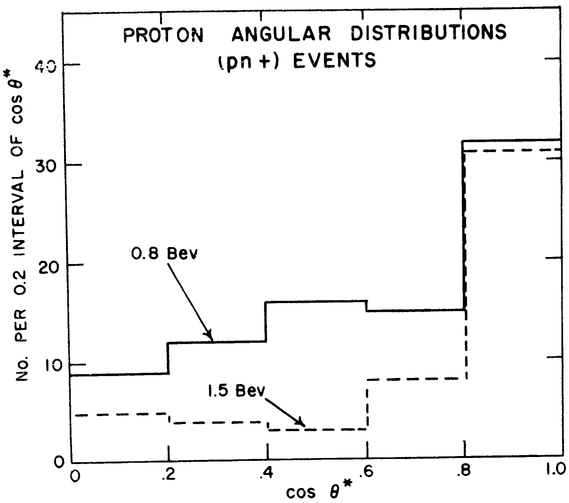

Fig. 2. Angular distributions in the c.m.s. of the protons from pn+ reactions. The solid histogram is the experimental result at 0.8 Bev; the dotted histogram corresponds to 1.5 Bev.

When the experimental angular distributions found at 0.8, 1.5 and 2.75 Bev are tested for symmetry about 90°, we find appreciable asymmetries which are greatest for 2.75 Bev and least for 0.81 Bev. As previously pointed out, these must be ascribed to bias arising largely from the fact that events with charged particles emitted backward in the c.m.s. are easiest to identify. It is difficult to determine the extent to which this bias will modify other distributions. Since the bias is worst for 2.75 Bev and the data fewest, distributions at this energy will not be discussed. In order to reduce the bias at the lower energies, all distributions were folded about 90°. The resulting angular distributions are plotted in figs. 2, 3 and 4. The nucleon distributions show a marked forward-backward preference in all cases. They are certainly not consistent with S-wave or other isotropic emission, but suggest a collision with low momentum transfer that is quasi-elastic in character. At each energy proton and neutron distributions are similar and could be identical within the experimental errors, as predicted. On the other hand the data certainly do not rule out the possibility of minor differences

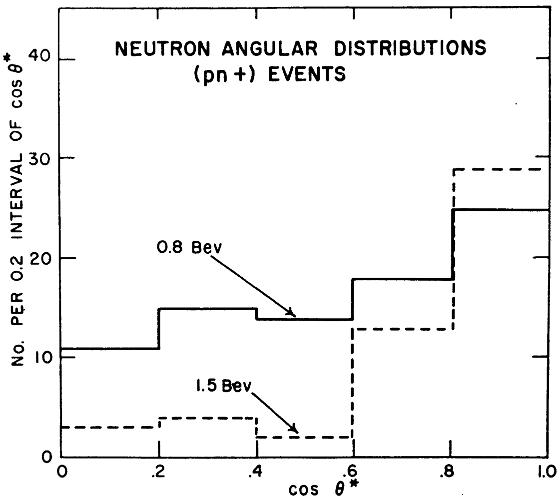

Fig. 3. Angular distributions in the c.m.s. of the neutrons from pn+ reactions. The solid histogram is the experimental result at 0.8 Bev; the dotted histogram corresponds to 1.5 Bev.

between neutron and proton distributions; such differences might arise from interference between φ_A and φ_S even though φ_A is relatively small.*

The pion angular distribution at 0.81 Bev does indeed resemble a P-wave angular distribution, having a minimum at 90°. The 1.5 Bev distribution, however, appears essentially isotropic.

In principle one should be able to obtain information on the values of the angular momentum of the system by an examination of the angular distributions. It was shown earlier that the major portion of the (real space × real spin) part of the total wave function is symmetric in the interchange of the nucleons. If analysis of the spatial distributions where to show that the real space portion was of only one symmetry, e.g. symmetric, then we would be able to conclude that the two nucleons were interacting in a particular real spin state, e.g. the triplet state. In practice the analysis of the data from this point of view presents formidable difficulties and has not been attempted.

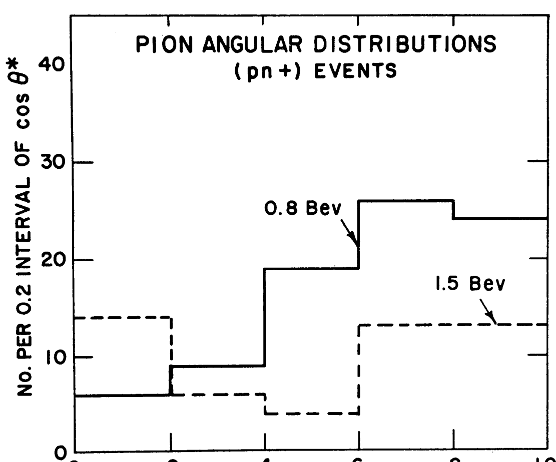

Fig. 4. Angular distributions in the c.m.s. of the pions from pn+ reactions. The solid histogram is the experimental result at 0.8 Bev; the dotted histogram corresponds to 1.5 Bev.

D. Momentum distribution for (pn+)

The momentum distributions expected on the basis of the Fermi statistical theory have been calculated using exact relativistic expressions[4]. The curves given in figs. 5, 6 and 7 show the results for 0.8 Bev and 1.5 Bev.

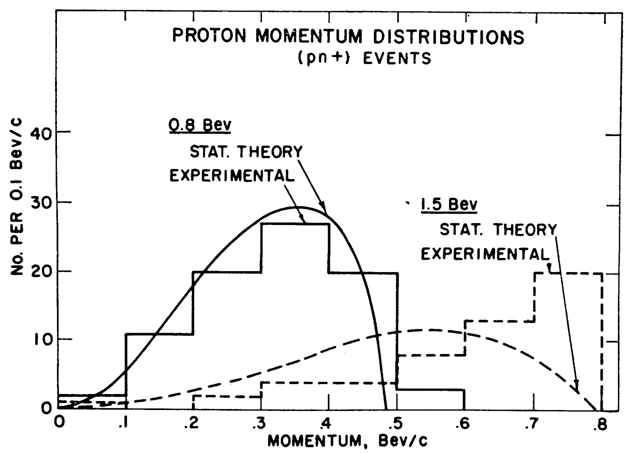

Fig. 5. Momentum distributions in the c.m.s. of the protons from pn+ reactions. The solid histogram is the experimental distribution at 0.8 Bev, whereas the solid curve is the theoretical distribution predicted by the Fermi statistical model; the corresponding quantities at 1.5 Bev are the dotted histogram and dotted curve.

It should be emphasized that the shape of the momentum distribution for a given multiplicity does not depend on the interaction volume. The discussion on charge independence indicates that proton and neutron should have approximately the same momentum distributions as well as angular. Other predictions can be made only qualitatively. In the work of Lepore and Neuman, the pion would be emitted with lower average momentum than that predicted by the Fermi theory. The calculations of Kovacs also give lower pion momenta than the Fermi theory for incident proton energies greater than about 1 Bev.

The experimental momentum distributions are also plotted in figs. 5, 6 and 7. The results at 0.8 Bev fit well

* Another condition also has to be fulfilled if the neutron and proton are to be interchangeable. It is the symmetry about the 0° – 180° axis of the distribution of the function $\alpha_P - \alpha_N$, where α is the azimuthal angle of emission ($0 \leqslant \alpha \leqslant 2\pi$). It is found that this condition is fulfilled within the accuracy of the experiment.

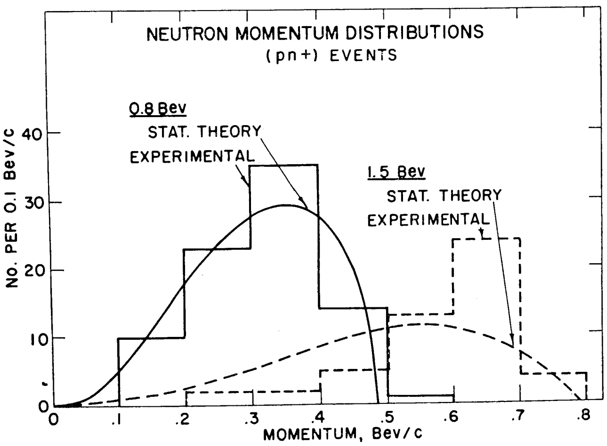

Fig. 6. Momentum distributions in the c.m.s. of the neutrons from pn+ reactions. The solid histogram is the experimental distribution at 0.8 Bev, whereas the solid curve is the theoretical distribution predicted by the Fermi statistical model; the corresponding quantities at 1.5 Bev are the dotted histogram and dotted curve.

with Fermi theory curves for both proton, and π^+. Differences between proton and neutron, if any, are small. The experimental distributions at 1.5 Bev, however, are not consistent with the Fermi predictions. The statistical factor predicts a peak at too low a momentum for the nucleon distribution and at much too high a momentum for the pion. The results of Lepore and Neuman and

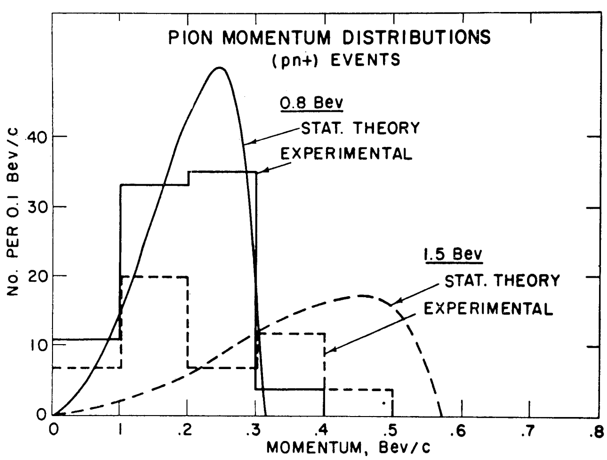

Fig. 7. Momentum distributions in the c.m.s. of the pions from pn+ reactions. The solid histogram is the experimental distribution at 0.8 Bev, whereas the solid curve is the theoretical distribution predicted by the Fermi statistical model; the corresponding quantities at 1.5 Bev are the dotted histogram and dotted curve.

Kovacs would give better agreement with experiment at 1.5 Bev. Proton and neutron spectra are again similar at 1.5 Bev, so that the predictions of charge independence are confirmed.

If one compares pion distributions at 0.8 and 1.5 Bev, it is evident that the difference between them is much less than one predicts on the basis of Fermi theory. Yuan and Lindenbaum have noted that the c.m.s. pion energy spectrum from nucleon-nucleon collisions does not change much with incident nucleon energy [11]. They have interpreted this in terms of a strong effect of the pion-nucleon resonance. Our momentum spectra are in rough agreement with their observation, but the (pn+) / (ppo) ratio of 17 indicates that pion production does not occur entirely through a $T = J = {}^3/_2$ intermediate state.

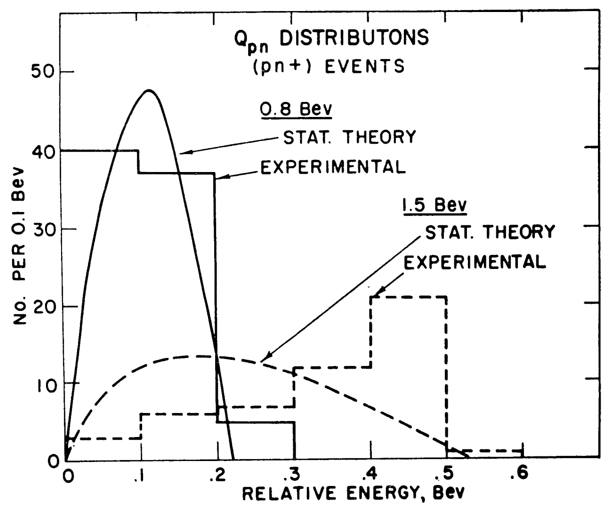

Fig. 8. The Q-value distributions between proton and neutron in pn+ reactions. The solid histogram is the experimental distribution at 0.8 Bev, whereas the solid curve is the theoretical distribution predicted by the Fermi statistical theory; the corresponding quantities at 1.5 Bev are the dotted histogram and the dotted curve.

E. Q-values between pairs of particles

Interactions between pairs of particles, such as a pion-nucleon interaction, might be expected to be shown most directly by an effect on the distribution of relative energy, or Q-value, between pairs of particles.* For example, a resonant $T = {}^3/_2$ interaction would strongly affect the Q_{p+} distribution. Calculations based on the Fermi theory have been made to predict Q-value distributions that would exist in the absence of important 2-particle interactions [4].

Figs. 8, 9 and 10 show Q-value distributions for (p+), (n+), and (pn) pairs from 0.8 and 1.5 Bev (pn+) events

* It should be pointed out that for collisions with three-body final states and unique c.m.s. energy, such as (pn+), the Q-plots are not kinematically independent of the momentum distributions but rather are alternative means of analyzing the same data. In particular, a specification of the neutron momentum distribution automatically determines the Q_{p+} distribution. However, the Q-plots do serve as independent approaches for collisions involving 4 or more final state particles.

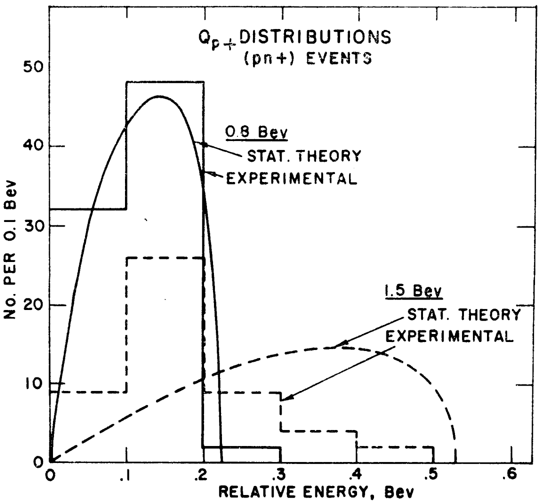

Fig. 9. The Q-value distributions between proton and pion in pn+ reactions. The solid histogram is the experimental distribution at 0.8 Bev, whereas the solid curve is the theoretical distribution predicted by the Fermi statistical theory; the corresponding quantities at 1.5 Bev are the dotted histogram and the dotted curve.

as observed experimentally and as predicted from the Fermi theory. At 0.8 Bev reasonable agreement is obtained. Also the agreement between Q_{p+} and Q_{n+} distributions is support of our earlier conclusion that charge independence is satisfied. At 1.5 Bev the statistical predictions are in marked disagreement with the data. There appears to be a difference between Q_{p+} and Q_{n+} distributions, but it is difficult to be sure whether it is significant because of the large number of uncertain classifications, statistical fluctuations, measurement errors, and possible biases.

F. Angular correlations between pairs of particles

The distributions of included angles in c.m.s. between particle pairs from (pn+) reactions have been examined. A stricking feature at all beam energies is the strong back-to-back correlation between proton and neutron. This of course mainly reflects the fact that the average nucleon momentum is considerably higher than that of the average pion; consequently, momentum conservation in the c.m.s. requires the nucleons be emitted almost anti-parallel.

There appears to be a strong preference for large pion-proton correlation angles in (pn+) events at 0.8 Bev, which is somewhat less pronounced for the pion-neutron pair. However, these apparent differences are probably too small to be conclusive. It should be noted that the kinematics of a 3-body final state are such that a specification of two correlation angles suffices to specify the magnitudes of all momenta. Thus, if the (π^+, p) and (π^+, n) angular correlation distributions are dissimilar, this would result in a dissimilarity between the proton and neutron *momentum* distributions. Since we had concluded that there were no obvious differences in the momentum spectra of the two nucleons, the small apparent asymmetry in angular correlations may be spurious. On the other hand, there may in fact be a small difference between neutron and proton momentum distributions.

The angular correlation data at 1.5 Bev for (pn+) events show that the (π^+, p) pair has a small peak at large angles whereas the (π^+, n) pair peaks slightly at small angles. A similar difference was noted in the Q-value distributions, but the statistical accuracy of these results is low. Charge independence predicts that for a (pn+)/(ppo) ratio of ∞, the proton and neutron would have the same behavior. It is possible that the asymmetry in these observations is due to experimental errors. It is also possible that a small but non-vanishing φ_A would give rise to small asymmetries by interference with the dominant φ_S.

G. Conclusions

The rise in total cross-section for p-p interactions occurring between 0.4 and 0.8 Bev is due to inelastic events leading to the production of one or more pions, the cross-section for such processes being about 26 millibarns for energies of 0.8 and above. In this energy range the elastic cross section apparently drops slowly with increasing energy.

At 0.8 Bev only one pion appears to be produced, but at higher energies multiple pion production becomes increasingly frequent. At 2.75 Bev the percentage of single, double, and triple pion production are estimated to be 36%, 48% and 16%. Multiple pion production is

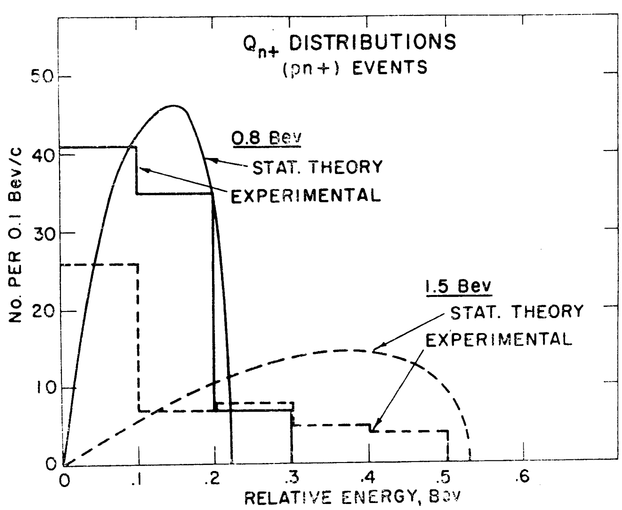

Fig. 10. The Q-value distributions between neutron and pion in pn+ reactions. The solid histogram is the experimental distribution at 0.8 Bev, whereas the solid curve is the theoretical distribution predicted by the Fermi statistical theory; the corresponding quantities at 1.5 Bev are the dotted histogram and the dotted curve.

thus considerably more common than predicted by the Fermi statistical theory, but not quite as common as the predictions of Kovacs, whose calculation takes into account a strong pion nucleon final state interaction. Qualitatively, however, the Kovacs theory leads to predicted energies (at which double and triple pion production should begin to become appreciable) that are in rough agreement with experiment.

In single pion production events the (pn+) reaction predominates at all energies. The (pn+) / (ppo) ratio is 17 ± 8 at 0.81 Bev and remains high at the higher energies. An analysis in terms of charge independence shows that the predominant final state has (real space) × (real spin) wave function, φ_s, that is symmetric under exchange of the two nucleons, and corresponds to final state nucleons couple with isotopic spin $T' = 0$. Accordingly, proton and neutron should exhibit approximately identical spatial behavior. The momentum spectra, Q-value distributions, and angular distributions of the proton and neutron are in reasonable agreement, which provides a test of charge independence at energies considerably higher than those of previous ests. At 1.5 and 2.75 Bev charge state ratios and spatial distributions are less well determined. The data involve no contradictions of charge independence.

Nucleons from inelastic collisions tend to be emitted forwards and backwards in the c.m.s. so that relatively low momentum transfer is involved. Their angle and momentum distributions show marked deviations from the predictions of a single statistical theory at 1.5 Bev.

Correspondingly, pions tend to be emitted with low momenta which are in marked disagreement with the predictions of statistical theory at 1.5 Bev.

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DISCUSSION

on papers by

I. S. Hughes, p. 344, M. G. Meshcheriakov, p. 347 and M. M. Block, p. 374.

W. O. Lock. Dr. Merrison has summarised our present knowledge of meson production in p-p collisions at energies below 700 Mev. Above this energy the data is meagre but one has information from the cloud chamber work carried out by several groups using protons from the Brookhaven Cosmotron (Block et al.)[1] and from the emulsion work carried out by Hughes et al. [2] using protons from the Birmingham synchrotron. The results so obtained are summarised in the following Table, which also includes the 660 Mev data of Meshcheriakov et al. [3] for comparison purposes.

One can make the following comments :

1. The cross-section for $p + p \rightarrow d + \pi^+$ appears to fall rapidly with energy above about 650 Mev and to be effectively zero above 1 Bev. It thus exhibits a strong resonance.

Process	660 Mev[3] σ in mb	800 Mev[1] σ in mb	925 Mev[2] σ in mb	1500 Mev[1] σ in mb
$p + p \rightarrow d + \pi^+$	3.1 ± 0.2	~ 0.3 (1 event)	0.6 ± 0.4 (2 events)	No certain events
$p + p \rightarrow p + p + \pi^0$	3.4 ± 0.4	1.4 ± 0.6 (5 events)	6 ± 2 (22 events)	No certain events
$p + p \rightarrow n + p + \pi^+$	10.2 ± 1.2	22.6 ± 3 (4 events)	27 ± 3 (92 events)	~ 22 } 91 events
$p + p \rightarrow 2\pi +$ nucleus	—	no events	no events	~ 5 }

2. The data on the process $p + p \rightarrow p + p + \pi^0$ at 800 and 925 Mev are conflicting and further information is clearly required. Above 1 Bev the cloud chamber work suggests that the cross-section is effectively zero although at 3 Bev Cester et al. [4] found a value of about 1 mb. for single π^0 production (3 events). However, at energies above 1 Bev the difficulties of identification are considerable, both in cloud chamber and in emulsion work.

3. The cross-section for $p + p \rightarrow n + p + \pi^+$ rises rapidly with energy, reaches a maximum around 900 Mev and then decreases slowly, presumably due to the onset of multiple meson production. The total inelastic cross-section is sensibly constant from 800 Mev to at least 3 Bev as is shown by the cloud chamber work and by the counter work of Shapiro et al[5].

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