POLARIZATION EFFECTS IN PION REACTIONS

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Introduction

In this talk I want to summarize the information to be gained from a study of polarization effects in pion-nucleon processes, i.e. I shall consider :

1. scattering : $\pi + N \succ \pi + N$
2. meson production in nucleon-nucleon collisions : $N + N \succ \pi + N + N$;

here the two nucleons may be in a bound state (deuteron).

These processes are described in terms of certain parameters : phase-shifts and production amplitudes, and any theory must be able to predict these. It seems possible that meson theory, as formulated by Chew and Low [1] and more recently in the dispersion relations [2], may shortly lead to predictions of these parameters which will have to be taken seriously. It is therefore of special interest to see what information experiments can give about these parameters. As is well known, one has usually more parameters than can be determined from simple scattering experiments, etc., so that one must consider polarization effects of varying complexity. The situation is here quite analogous to that in the case of the nucleon-nucleon interaction.

Pion-nucleon scattering

Up to now all evidence [3] seems to indicate that s- and p-waves suffice to explain pion-nucleon scattering up to 250 or possibly even 300 Mev. At the USSR Conference on High Energy Physics [4], and also at this Symposium, work on pion-nucleon scattering up to 330 Mev was reported which was also fitted in terms of s-, p- and d-waves. However, it is only above 300 Mev that d-waves are required to give good agreement with the angular distributions, and even here the evidence for d-waves is not strong. It would be important to establish just where d-waves do come into play. Assume for the moment that one need only consider s- and p-waves.

Phase-shift analyses of the scattering data then show two types of uncertainty :

1. At any energy several sets of phases reproduce the cross-sections within experimental error : (i) we do not know the absolute signs of the phases; (ii) we have the Yang-Fermi ambiguity, (iii) we have the Minami ambiguity. In addition we may obtain other sets, not related to some simple symmetry property. Some of these sets can now be eliminated using the Coulomb interference or the dispersion relations.
2. Secondly, having decided on a particular set of phases, we are usually still very ignorant what the small phase-shifts in the set are. E.g. taking the Fermi set, we really only know δ_{33} with any confidence. About the other p-phases we know almost nothing. For the s-phases, the Orear fit seems to be satisfactory (though recent measurements at Rochester at 40 Mev seem to disagree strongly with Orear), but it is not clear up to what energies.

More accurate measurements of cross-sections will reduce some of these uncertainties. A more powerful method, however, is to study polarization phenomena. Fermi [5] pointed out that a study of the polarization of the recoil nucleon should distinguish between different sets of phases, giving quite different angular distributions for the polarization. In addition, having chosen one particular set, the polarization will give information about the small phases. The polarization is perpendicular to the plane of scattering; for (π^+,p)- scattering, it is proportional to

$$- 4 \sin\theta \sin(\delta_{33} - \delta_{31}) \{\sin\delta_3 \sin(\delta_{33} + \delta_{31} - \delta_3) + 3 \cos\theta \sin\delta_{33} \sin\delta_{31}\} \qquad (1)$$

For the Fermi set we believe we know δ_{33} well. Hence, if we can determine the angular distribution of the polarization well enough to separate out the $\cos\theta$ part in (1), this would give us δ_{31}. Corresponding information can be obtained from the (π^-,p)-scattering, involving the appropriate combinations of $T = {}^3/_2$ and $T = {}^1/_2$ states.

The polarization predicted by typical sets of phases is quite large. Thus, Fermi's set at 120 Mev and $\theta = 90°$ gives

$$\varepsilon = \frac{I_+ - I_-}{I_+ + I_-} = 0.66 \qquad (2)$$

for (π^+,p)-scattering. Nevertheless, the measurement of this polarization is difficult. I am certainly not qualified to talk of these experimental problems, but I should like to mention two difficulties:

1. Intensity: one is here dealing with a 'triple scattering' experiment:
   (i) meson production,
   (ii) meson scattering,
   (iii) analysis of the recoil nucleon by scattering.
2. Analysis of the proton polarization gets very difficult below 70 Mev. (Below this energy, protons are not much polarized by scattering by complex nuclei. Nothing seems to be known about pp polarization at these low energies.)

Both these difficulties are ameliorated by going to higher energies. For 250 Mev mesons, the energy of the recoil proton varies from 78 Mev to 156 Mev for CoM scattering angles varying from 90° to 180°. For 300 Mev mesons, the corresponding energy range is from 100 to 200 Mev. As stated earlier, it may be necessary to use d-waves at these energies.

I should now like to discuss the question of d-waves in pion-nucleon scattering and how they would show up as one goes to higher energies. For simplicity, I shall restrict myself to (π^+,p)-scattering so that I need only consider the $T = {}^3/_2$ states. Entirely analogous considerations hold for (π^-,p)-scattering, direct and exchange, only one must use the appropriate combinations of the two isotopic spin states.

Denoting the $T = {}^3/_2$ phase-shifts by δ_3 for the s-phase and by $\delta^l_{3,2j}$ for the (l,j) states, and putting

$$\mu = \exp(2i\delta) - 1,$$

with the appropriate indices, the differential cross-section for (π^+,p)-scattering, in terms of s-, p- and d-waves, is given by

$$\frac{d\sigma}{d\Omega} = (\lambda^2/4)\{|(\mu_3 - \mu^2_{33} - {}^3/_2\,\mu^2_{35}) + \cos\theta\,(\mu^1_{31} + 2\mu^1_{33}) + \cos^2\theta\,(3\mu^2_{33} + {}^9/_2\mu^2_{35})|^2 + \sin^2\theta\,|(\mu^1_{31} - \mu^1_{33}) + 3\cos\theta\,(\mu^2_{33} - \mu^2_{35})|^2\} \quad (3)$$

which is of the form

$$\frac{d\sigma}{d\Omega} = (\lambda^2/4)\{A_0 + A_1\cos\theta + A_2\cos^2\theta + A_3\cos^3\theta + A_4\cos^4\theta\} \quad (3a)$$

The polarization of the recoil proton, at right angles to the plane of scattering, is proportional to

$$\frac{d\sigma(+)}{d\Omega} - \frac{d\sigma(-)}{d\Omega} = i(\lambda/2)^2\sin\theta\,\{[(\mu_3 - \mu^2_{33} - {}^3/_2\mu^2_{35}) + \cos\theta\,(\mu^1_{31} + 2\mu^1_{33}) + \cos^2\theta\,(3\mu^2_{33} + {}^9/_2\mu^2_{35})] \times [(\bar\mu^1_{31} - \bar\mu^1_{33}) + 3\cos\theta\,(\bar\mu^2_{33} - \bar\mu^2_{35})] - \text{complex conjugate}\} \quad (4)$$

where $d\sigma(\pm)/d\Omega$ are the differential cross-sections for a meson, incident in the z-direction, to be scattered through an angle θ (CoM) into the half-plane $\varphi = 0$, the recoil nucleon having spin parallel and antiparallel to the y-axis respectively. We can write (4) as

$$\frac{d\sigma(+)}{d\Omega} - \frac{d\sigma(-)}{d\Omega} = (\lambda^2/2)\sin\theta\,\{B_0 + B_1\cos\theta + B_2\cos^2\theta + B_3\cos^3\theta\}. \quad (4a)$$

For reference we give the coefficients B_i, expressed in terms of the phases:

$$B_0 = 4\sin(\delta^1_{33} - \delta^1_{31})\,[-2\sin(\delta^1_{33} + \delta^1_{33} - \delta_3 - \delta^2_{33}) \times \sin(\delta_3 - \delta^2_{33}) + 3\sin\delta^2_{35}\sin(\delta^1_{33} + \delta^1_{31} - \delta^2_{35})]$$

$$B_1 = -12\{2\sin\delta^1_{33}\sin\delta^1_{31}\sin(\delta^1_{33} - \delta^1_{31}) - \sin(\delta^2_{35} - \delta^2_{33})\,[2\sin(\delta_3 - \delta^2_{33})\sin(\delta_3 - \delta^2_{35}) + 3\sin\delta^2_{33}\sin\delta^2_{35}]\}$$

$$B_2 = 12\{\sin(\delta^1_{33} - \delta^1_{31})\,[2\sin\delta^2_{33}\sin(\delta^2_{33} - \delta^1_{31} - \delta^1_{33}) + 3\sin\delta^2_{35}\sin(\delta^2_{35} - \delta^1_{31} - \delta^1_{33})] + 2\sin(\delta^2_{35} - \delta^2_{33}) \times [\sin\delta^1_{31}\sin(\delta^1_{31} - \delta^2_{33} - \delta^2_{35}) + 2\sin\delta^1_{33} \times \sin(\delta^1_{33} - \delta^2_{33} - \delta^2_{35})]\}$$

$$B_3 = -180\sin\delta^2_{33}\sin\delta^2_{35}\sin(\delta^2_{35} - \delta^2_{33}). \quad (5)$$

TABLE I

Angular distribution and polarization in (π^+, p)-scattering at 240 and 307 Mev, using s-, p- and d-phases. (Phases in degrees.)

SET	ENERGY MEV	δ_3	δ^1_{31}	δ^1_{33}	δ^2_{33}	δ^2_{35}	A_0	A_1	A_2	A_3	A_4	B_0	B_1	B_2	B_3
I	240	−14	−2	114.4	0	0	3.4	2.3	10.3	0	0	1.4	0.68	0	0
II	240	−14	−2	114.4	1	$-{}^5/_3$	3.4	2.5	10.5	0.17	−0.05	1.2	0.62	−1.3	−0.004
III	240	−14	−2	114.4	3	−5	3.3	2.8	11.3	0.80	−0.47	0.92	0.55	−4.0	−0.11
IV	307	−13	−4	133.7	9.5	−10	1.9	4.4	11.5	0.75	−3.5	0.60	1.0	−7.9	−1.7

The terms A_3 and B_2 arise from the interference of p- and d-waves; A_4 and B_3 from d-d interference. Hence in the energy region where d-waves first become important, whereas the p-phases (in particular δ^1_{33}) are already large, one expects d-waves to show up as a $\cos^3\theta$-term in the cross-section and a $\sin\theta\cos^2\theta$-term in the polarization. As the d-waves become larger this is of course no longer the case.

To obtain a more quantitative idea I have calculated the above angular distributions for the phase-shifts recently proposed by Mukhin et al.[4], at 240 and 307 Mev. The results are shown in Table I, sets III and IV respectively. One sees that at 307 Mev, where the d-phases are quite large, the d-d interference term A_4 in the cross-section is actually larger than the p-d interference term. For the purpose of seeing what the effect of d-phases is, we also give in Table I two fictitious sets of phases, differing from set III only in having different d-phases, namely set I: d-phases equal to zero, and the intermediate set II: 'small' d-phases. Perhaps the most interesting feature of these results is that the polarization is very sensitive to d-waves, i.e. the coefficient B_2 of the $\sin\theta\cos^2\theta$-term is comparable or large compared to B_0 and B_1, even for quite small d-phases. Hence if the actual phases show a behaviour similar to that of the above sets, one would expect polarization experiments, if practicable at all, to give considerable information about d-waves. Analysis of (π^+,p)-scattering in terms of s-, p- and d-waves requires 5 phase-shifts. Cross-section and polarization experiments can provide 9 data, so that in principle the phases could be determined from these experiments. In practice this would presumably be very difficult. (Compare the difficulty of determining the three phase-shifts at lower energies.)

Pion production in nucleon-nucleon collisions

I shall consider the bound-state reaction

$$p + p \rightarrow \pi^+ + D; \tag{6}$$

being a 2-body process, it is simplest experimentally and theoretically: fewer final channels are involved, i.e. fewer parameters to be determined.

So far all evidence[6] seems to indicate that up to quite high energies (600 to 800 Mev) one need only consider s- and p-wave pion production. Again, one of the things one would like to know is where d-waves first become important. For s- and p-waves, 3 channels i.e. 5 parameters are involved. For s-, p- and d-waves, 7 channels, i.e. 13 parameters are required. Hence in this case it will probably be necessary to work at sufficiently low energies to be able to neglect d-waves if one wants to have any hope of determining these parameters. A serious weakness of the usual analysis of pion production is that one must assume the energy dependence of the production cross-sections in the various channels. This is based on a phenomenological analysis by Rosenfeld[7]. Its main justification is if it leads to good agreement with experiment. But there is no very basic justification for the assumed simple energy dependence and it can presumably only hold over a limited range of energy. So this energy dependence is another feature one would like to test.

The angular distribution of the meson production in the case of s-, p- and d-wave production has been considered by Wolfenstein[8] and by Mandl and Regge[9]. For an incident proton beam of polarization (0,p,0) it is given by

$$\frac{d\sigma(\theta,\varphi)}{d\Omega} = \frac{1}{32\pi}\{\gamma_0 + \gamma_2\cos^2\theta + \gamma_4\cos^4\theta - p\sin\theta\cos\varphi\,[(\lambda_0 + \lambda_2\cos^2\theta) + (\lambda_1\cos\theta + \lambda_3\cos^3\theta)]\} \tag{7}$$

Here the γ's and λ's are parameters expressible in terms of the various production amplitudes. For s- and p-wave production, only γ_0, γ_2 and λ_0 are non-zero. d-waves show up as a $\cos^4\theta$ term in the unpolarized cross-section. A more sensitive test for the presence of d-waves may result from the part of the polarization term which is asymmetric about $\theta = 90°$, i.e. the terms in λ_1 and λ_3; it arises from interference of s- and d-wave mesons.

With the restriction to s- and p-wave production, we have only 3 possible transitions, from the pp-states $^1S_0$, $^1D_2$ and $^3P_1$. The first two lead to p-wave, the last one to s-wave pions. We have then five parameters to determine from the following experimental results:

(i) the absolute differential cross-section (on the angular distribution and the total cross-section) gives us two data: γ_0 and γ_2.

(ii) The asymmetry, using polarised protons, gives us λ_0. Incidentally, λ_0 is a direct measure of s-wave production, as it arises from s-p interference. (Marshak and Messiah[10]). This is of special interest as the angular distribution is not much different from that for pure p-waves. (For pure p-waves one gets $1/3 + \cos^2\theta$. Experimentally one finds $\alpha + \cos^2\theta$ with $\alpha = 0.2$ to 0.4.)

(iii) The energy-dependence of the total cross-section gives us a fourth datum, if we assume it to be of the form

$$\sigma_T = \alpha\eta + \beta\eta^3 \tag{8}$$

(η = meson momentum; α, β parameters expressible in terms of the production amplitudes), the first term being due to s-wave production, the second to p-waves. We have already discussed this point.

(iv) To proceed any further we must study the polarization of the recoil deuteron. Since the deuteron has spin 1, this is a rather complicated matter and we shall not go into details which are given in the papers referred to above[11]. One obtains different information according as to whether one uses polarized or unpolarized protons. In the former case the deuteron polarization depends only on the p-wave amplitudes, in the latter it depends also on the s-wave amplitudes. Using polarized protons, requires a triple scattering experiment and the resultant loss of intensity makes this a much more difficult experiment. It would of course be nice to do both experiments. One could then determine all parameters without assuming the energy dependence of the production amplitudes, and one would hence obtain an independent check on this point.

Measurement of the deuteron polarization presents its own problems. In particular, it seems very difficult to separate the contributions from the tensor components $\langle T_1^1 \rangle$ and $\langle T_2^1 \rangle$ which have the same angular dependence, and which are unfortunately the components of interest. Lakin[12] has pointed out that in principle one could use a magnetic field to cause different precessions in the two components. This idea has not been received enthusiastically by experimentalists.

The polarization of the recoil deuteron has recently been measured by Tripp[13], using unpolarized 340 Mev protons. For this purpose he had to assume that $\langle T_2^1 \rangle \cong 0$. All that one can deduce from experiments, which measure $\left\{ |\langle T_1^1 \rangle|^2 - |\langle T_2^1 \rangle|^2 \right\}$, is that $\langle T_1^1 \rangle \gg \langle T_2^1 \rangle$. Some calculations by Stapp[14] suggest that $\langle T_2^1 \rangle$ is negligible. However, I feel this is one of the more unsatisfactory features of this analysis. Together with other measurements of the reaction (6), indicated above, Tripp obtains a complete determination of the production amplitudes. Actually four possible sets are found; this ambiguity arises from the fact that the experiments determine only trigonometric functions of the required quantities. Again, two of these solutions are of the Fermi type and two of the Yang type. (For the Fermi type of pion-nucleon interaction, production from the $^1D_2$ state is much more important than from the $^1S_0$ state. For the Yang type, the situation is reversed.)

Lastly, I want to mention that the phases of the pion production amplitudes from the various pp-states, can be related to the scattering phase-shifts of these states[15]. Berkeley from a very complete set of experiments have also determinated these and Tripp finds good agreement with one of the four sets of production amplitudes he obtained, which is of the Fermi type.

LIST OF REFERENCES

1. *a*) Low, F. E. Boson-Fermion scattering in the Heisenberg representation. Phys. Rev., *97*, p. 1392-8, 1955.
   b) Chew, G. F. and Low, F. E. Effective-range approach to the low energy p-wave pion-nucleon interaction. Phys. Rev., *101*, 1570-9, 1956.
2. *a*) Goldberger, M. L. Causality conditions and dispersion relations. I. Boson fields. Phys. Rev., *99*, p. 979-85, 1955.
   b) Goldberger, M. L., Miyazawa, H. and Oehme, R. Application of dispersion relations to pion-nucleon scattering. Phys. Rev., *99*, p. 986-8, 1955.
   c) Anderson, H. L., Davidon, W. C., Glicksman, M. and Kruse, U. E. Scattering of positive pions by hydrogen at 189 Mev. Phys. Rev., *100*, p. 279-87, 1955.
   d) Salam, A. On generalised dispersion relations. Nuov. Cim., *3*, p. 424-9, 1956., *3*, p. 607-11, 1956.
3. Rochester Conference on high energy nuclear physics. 6th. Proceedings, 1956.
4. Mukhin, A. I. and others. Abstracts of papers presented at the USSR Conference on high energy physics, p. 75-76, 1956.
5. Fermi, E. Nucleon polarization in pion proton scattering. Phys. Rev., *91*, p. 947-8, 1953.
6. Meshcheriakov, M.G. and Neganov, B.S. (Formation of mesons in the reaction $p + p \to d + \pi^+$ in the region 510-660 Mev.) Doklady Akad. Nauk SSSR, *100*, p. 677-9, 1955. Also Abstracts of papers presented at USSR Conference on High energy physics, 1956, p. 75-76.
7. Rosenfeld, A. H. Production of pions in nucleon-nucleon collisions at cyclotron energies. Phys. Rev., *96*, p. 139-49, 1954.
8. Wolfenstein, L. Polarization effects in the reaction $p + p \to \pi^+ + d$. Phys. Rev., *98*, p. 766-9, 1955.
9. Mandl, F. and Regge, T. Reaction $p + p \to \pi^+ + d$ with polarized protons. Phys. Rev., *99*, p. 1478-83, 1955.
10. Marshak, R. E. and Messiah, A. M. L. Pion production by polarized beams of nucleons. Nuov. Cim., *11*, p. 337-41, 1954.
11. Lakin, W. Spin polarization of the deuteron. Phys. Rev., *98*, p. 139-44, 1955.
12. Tripp, R. D. Polarization of the deuteron in the reaction $p + p \to \pi^+ + d$. Phys. Rev., *102*, p. 862-6, 1956.
13. Stapp, H. P. The theory and interpretation of polarization phenomena in nuclear scattering. (UCRL 3098). Thesis, University of California, 1955. (unpublished).
14. Gell-Mann, M. and Watson, K. M. The interactions between π mesons and nucleons. Annual review of nuclear science, *4*, p. 219-270, 1954.
15. Fermi, E. Lectures on pions and nucleons. Nuov. Cim. Suppl. *2*, p. 17-95, 1955.

FINAL DISCUSSION
on
pion-nucleon scattering

E. Segrè made two remarks : 1. If you want to make a polarization measurement on protons, you must face the fact that their energy must be at least 150 Mev, otherwise the analysers do not work. 2. Tripp has tried the experiment $p + p \rightarrow \pi + d$ but the analysis of the polarization of the deuteron is very complicated and he did not get very far.

G. H. Stafford. "p-p polarization effects are such that it should be possible to obtain measurable asymmetries appreciably below the 150 Mev suggested by E. Segrè. With results already obtained at A.E.R.E. it is possible to make a guess, from the nucleon-nucleon phase shifts, that there will still be some polarization at an incident proton energy of say 70 Mev. Carbon is also a good analyser down to about the same energy."

N. P. Klepikov : "I would like to draw your attention to the results of the discussion sponsored on the Moscow Conference by the report of Clementel. He encountered the difficulty not to be able to obtain any phase shifts for pion-proton scattering unless the expansion coefficients were subjected to some compatibility conditions. So did I, using a mechanical phase shift analyser. There are two conditions :

$$A_0^2 \leqslant A_0 + A_1 + A_2 \, (A_0 - A_1)^2 \leqslant A_0 - A_1 + A_2$$

involving the coefficients of the Legendre polinomial expansion

$$d\sigma/d\Omega = 1/K^2 \{A_0 + A_1 P_1 + A_2 P_2\}$$

These conditions are connected with the forward and backward scattering amplitudes. It is quite necessary to account for these conditions in the energy region of 150-200 Mev. In order to overcome the difficulty of readjustment of experimental data one should use the whole totality of the experimental data available to date. There is no such difficulty, if one uses electronic computers to find the phase shifts and does not calculate at all the coefficients, but in the case of incompatibility (in the mentioned sense) there is a great doubt about the reliability of the obtained phase shifts."