SCATTERING OF 18.7 MEV NEGATIVE PIONS BY LIQUID HYDROGEN

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(presented by D. E. Nagle)

The scattering of 18.7 Mev negative pions was studied in the liquid hydrogen bubble chamber. The object was to obtain the S-wave phase shifts, actually the linear combination ($2\,\alpha_1 + \alpha_3$), using a pion energy for which the S-wave scattering is maximized with respect to P-wave and Coulomb effects. The construction and operation of the chamber have been described[1]. The method of extracting and focusing the pions is in general that described by Warshaw and Wright[2]. It remains to present the experimental results and to describe how they are analyzed to obtain the phase shifts.

First we discuss the scattering events themselves. Fig. 1 shows a pion-proton scattering event as seen by the scanners, who record the lengths of proton recoil and the angles of scattering of pion and of proton. From the proton range energy relation, the density of the liquid hydrogen (.060 g/cm²), and the kinematics of pion-proton scattering, we compute the energy of the incident pion and the angle of scattering in the center of mass system. These are listed in Table I. The letter F after an event indicates that it occurred in the central portion of the chamber (that is more than 4 mm. from the walls). Because only the central portion of the chamber was used in determining the total number of pions crossing the chamber, only the F-type events can be related to an absolute cross-section while the others only serve to improve the information on the angular distribution. Both types are used to determine the phase shifts as described below.

The pion beam which comes from the Chicago cyclotron consists of pions with an appreciable energy spread, of muons and of electrons. In order to determine the cross-section for pions, we need the number and energy distribution of the pions. The mean energy and fraction of pions is given approximately by a range curve taken with counters. The result is that 39 per cent of the beam is composed of pions and that these pions have a mean energy of 19.5 Mev at the center of the bubble chamber. The corresponding energy as computed from the magnetic deflection of the beam is about 18 Mev. We obtain additional information from a study of the "kinds", which are tracks which show a deflection with no visible recoil. For a beam with the composition and energy indicated in the range curve, one can show that the kinds with angles in the range 18-48 degrees come predominantly from $\pi - \mu$ decay. The kinds from 8-18 degrees may be used to estimate a small correction due to Coulomb scattering in the 18-48 degree range. This analysis was done with the help of a digital computer. Taking for the pion lifetime 2.52×10^{-8} sec.,[3] the density of the liquid hydrogen 0.06 g/cm², we find for the length of pion track in H atoms/cm² :

$$L(E) = 13.64 \times 10^{27} \times (\sqrt{2\,\pi})^{-1} \exp \tfrac{1}{2} \left(\frac{E - 18.7}{5}\right)^2$$

where E is the energy in Mev.

The observed scattering data is then analyzed by the maximum likelihood technique assuming

(*a*) the differential cross-section comes mostly from the Coulomb effect and from the S-waves, with a smaller contribution from P-waves. For the expression used see Rinehart et al.[4].

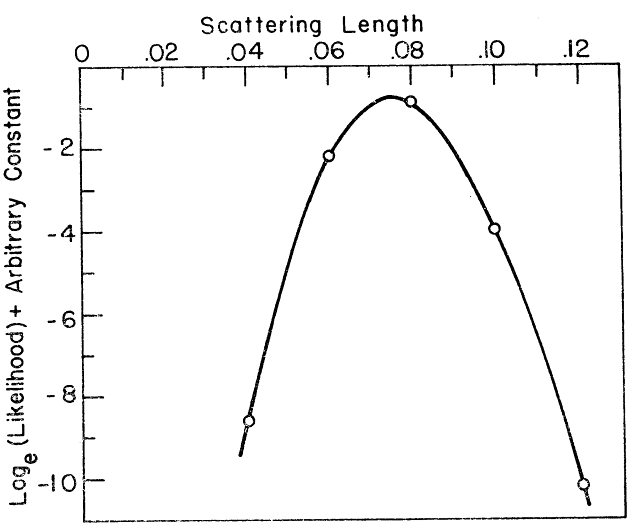

Fig. 2. Likelihood function of the scattering length $a = \tfrac{1}{3}(2\alpha_1 + \alpha_3)\,\eta^{-1}$.

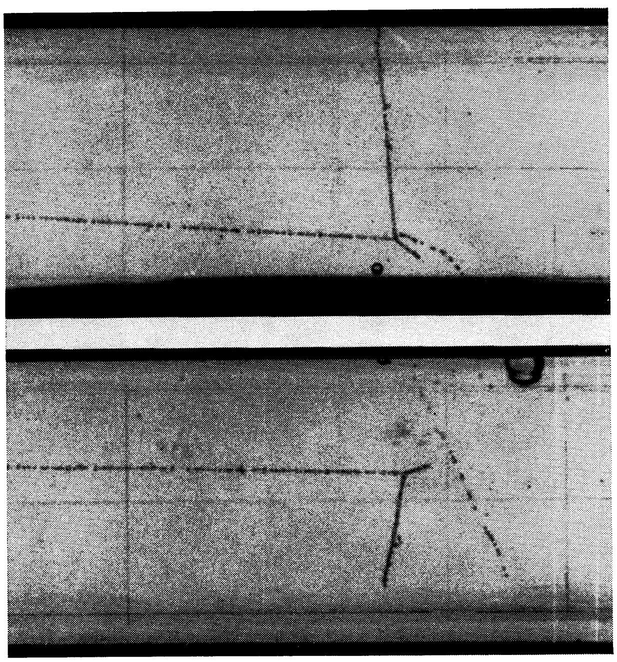

Fig. 1. Two views of a $\pi^- - p$ scattering event. The pion enters from the left; is scattered through a large angle and finally leaves the chamber. The recoil proton track is about 2.5 mm. long. An electron track can be seen near the scattering event.

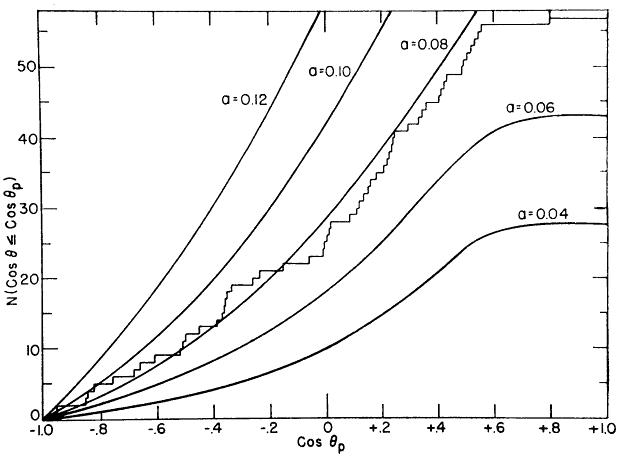

Fig. 3. Integral distribution of events compared to distributions expected for various values of the scattering length $1 = \frac{1}{3}(2\alpha_1 + \alpha_3)/\eta$. The curves become flat as $\cos\theta \to 1$ because no events are included unless the energy of the recoil proton is greater that 1.9 Mev.

(*b*) of the P-waves only α_{33} contributes and its momentum dependence is $\alpha_{33} = 0.235\eta^3$ where $\eta = p_\pi/m_\pi c$,

(*c*) the S-wave phase shifts depend linearly on momentum. The combination $(2\alpha_1 + \alpha_3)\eta^{-1}$ is treated as the parameter to be determined.

The resulting value of the likelihood function is shown in fig. 2. The function has a maximum for $(2\alpha_1 + \alpha_3)\eta^{-1} = 0.23$ and the $1/e$ values are for $\pm$ .04.

Fig. 3 shows how the observed integral distribution in angles compares with those computed for the measured pion energy spectrum for scattering lengths $a = \frac{1}{3}(2\alpha_1 + \alpha_3)\eta^{-1}$ = .04, .06, .08, .12, .14 respectively. It is seen that a value slightly less than .08 gives the best fit. In this experiment the statistical errors are regarded as dominant. Table II gives a comparison of our result with some other low energy π^- scattering experiments. The agreement is within statistics. It may also be compared with the value 0.21 suggested by Orear on the basis of a least squares fit of a number of experiments, including some at considerably higher energy.

One concludes that the agreement between the several experiments is satisfactory, and that the assumptions of linear momentum dependence for the S-wave phase shifts is at least approximately correct.

TABLE I

Pion-proton scattering events

Pion C.m. scattering angle (degrees)	Pion C.m. energy (Mev)	Type	Pion C.m. scattering angle (degrees)	Pion C.m. energy (Mev)	Type	Pion C.m. scattering angle (degrees)	Pion C.m. energy (Mev)	Type	Pion C.m. scattering angle (degrees)	Pion C.m. energy (Mev)	Type
31° 45′	46.5	F	57° 40′	23.9	F	71° 40′	20.0	F	102° 30′	14.0	
39° 0′	24.0	F	60° 15′	10.8	F	72° 25′	16.5		102° 50′	13.0	F
43° 0′	20.0		60° 15′	16.0	F	73° 0′	10.0		104° 10′	9.5	F
45° 50′	24.0		60° 45′	11.0		73° 15′	16.5	F	108°40′	11.0	
47° 30′	18.0		61° 40′	17.5	F	75° 45′	23.0		111° 35′	18.2	
47° 30′	22.0		62° 5′	21.0		79° 40′	19.7		112° 15′	7.9	
48° 45′	25.0		64° 30′	15.0	F	80° 45′	26.0		112° 30′	26.0	
49° 0′	14.5		67° 0′	10.0		80° 0′	24.0	F	119°50′	22.5	F
49° 30′	23.0		67° 5′	15.2		81° 0′	25.7	F	124° 0′	16.8	
51° 20′	17.2		67° 10′	20.3	F	89° 40′	16.0		125° 10′	22.0	F
53° 10′	30.0	F	67° 45′	22.0		94° 30′	17.2	F	133° 5′	18.0	
53° 15′	19.5	F	68° 15′	15.8	F	96° 0′	19.9		139° 35′	12.0	
54° 30′	21.0		68° 45′	13.0	F	101° 5′	6.5		142° 15′	16.8	
56° 0′	30.5	F				101° 30′	18.5		142° 40′	14.0	
57° 30′	16.2					102° 0′	14.3		159° 0′	14.0	F

The letter F indicates that the event is a "flux" event and may be used in determining the total cross-section.

TABLE II

Results of low energy π^--P scattering measurements

	Mean energy of pion beam (Mev)	$2\alpha_1 + \alpha_3$
Orear, Slater, Lord, Eilenberg and Weaver [5]	26	$(0.15 \pm 0.09)\,\eta$ *
Rinehart, Rogers and Lederman [4]	15	$(0.25 \pm 0.05)\,\eta$
Nagle, Hildebrand and Plano	18.5	$(0.23 \pm 0.04)\,\eta$
Combined result		$(0.235 \pm 0.030)\,\eta$

* The published result $4.7^0 \pm 2.7^0$ at 26 Mev. η is the center of mass momentum in units of μc.

LIST OF REFERENCES

1. Nagle, D. E., Hildebrand, R. H. and Plano, R. J. Hydrogen bubble chamber used for low-energy meson scattering. Rev. sci. Instrum., *27*, p. 203-7, 1956.
2. Warshaw, S. and Wright, S. Rev. sci. Instrum. (to be published.)
3. Barkas, W. H., Birnbaum, W. and Smith, F. M. Mass-ratio method applied to the measurement of L-meson masses and the energy balance in pion decay. Phys. Rev., *101*, p. 778-95, 1956.
4. Rinehart, M. C., Rogers, K. C. and Lederman, L. M. Diffusion chamber study of very slow mesons. Phys. Rev., *100*, p. 883-5, 1955.
5. Orear, J. et al. Elastic scattering of 26 Mev negative pions by hydrogen in emulsion. Phys. Rev., *96*, p. 174-6, 1954.

DISCUSSION

on papers by

M. H. Alston, p. 236 and D. E. Nagle, p. 238.

G. C. Wick asked D. Nagle what his favourite values of scattering length were.

D.E. Nagle said that by considering the results of Columbia, of J. Orear and of himself, one got the value 0.235.

A. Roberts: "How do these results combine with charge exchange results to give α, α_3 individually?"

L. M. Ledermann: "If we combine the new Chicago π^- S-wave of $0.23\,\eta = 2\,\alpha_1 + \alpha_3$ with the Columbia result of $0.25\,\eta$ - and use the old Rochester charge exchange results of Spry and of J. Tinlot and A. Roberts, the resulting individual phase shifts are :

$$\alpha_1 = -.10\,\eta$$
$$\alpha_3 = +.17\,\eta$$

The errors are $\sim 15\%$ but depend on the assumption of known P-wave of $0.235\,\eta^3$ and complete neglect of small phase shifts."

R. H. Hildebrand: "If you take the result of W. H. Evans of $\alpha_3 = .13$, then you get $\alpha_1 = .18$."