A MECHANICAL ANALOGUE FOR THE STUDY OF BETATRON OSCILLATIONS

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In the CERN-PS group is was felt desirable to have an analogue model for studying betatron oscillations in 2 dimensions and in non-linear fields; in particular for studying the dynamic behaviour of the beam when sweeping through sub-resonance lines.

The construction of a model suited to imitate the focusing forces in an alternating gradient machine would require a great deal of work.

It is, however, sufficient to consider only the "smooth" part of the particle trajectory for all problems concerning the influence of non-linearity. (This follows from the theory outlined in the preceding papers.)

Thus the model could be realised in the form of an elastic pendulum. Linear and non-linear perturbations are simulated by electrostatic forces acting on it.

The elastic pendulum consists of a thin thread of fused quartz, the end of which widens out into the form of a little ball. It oscillates by flexion in vacuum, and has a linear dependence of restoring force on displacement.

The surface of the pendulum is metal coated and when connected to a voltage source, the ball carries an electrical charge.

Non-linear forces and perturbations can now be applied in the form of electric fields, produced by a set of cylindrical electrodes parallel to the axis of the quartz thread and arranged on a circle around it (fig. 1.). The potential produced is of the form

$$V(r, \varphi, z) = \sum_n V_n \left(\frac{r}{R}\right)^n \cos n\varphi$$

where r, φ, z are the coordinates, R the radius of the cylinder on which the electrodes are placed, and V_n the voltage harmonic of the multipole components of the field.

In its plane of oscillation the ball is subject to electrical forces.

$$F_r = -q \frac{\partial V}{\partial r} = -q \sum_n \frac{nV_n(t)}{R} \left(\frac{r}{R}\right)^{n-1} \cos n\,\varphi$$

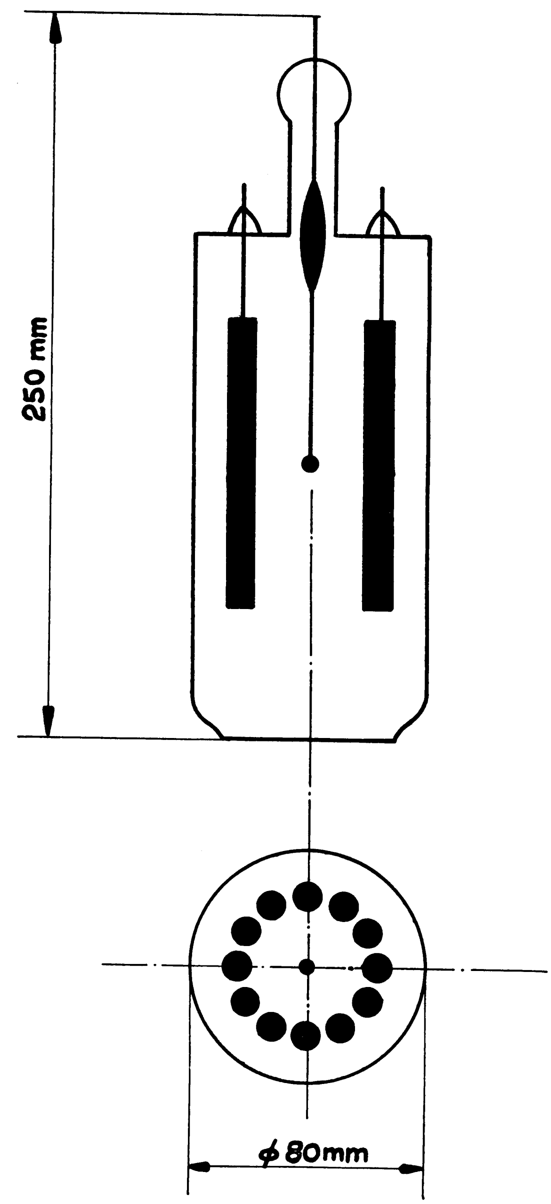

Fig. 1. Mechanical betatron analogue.

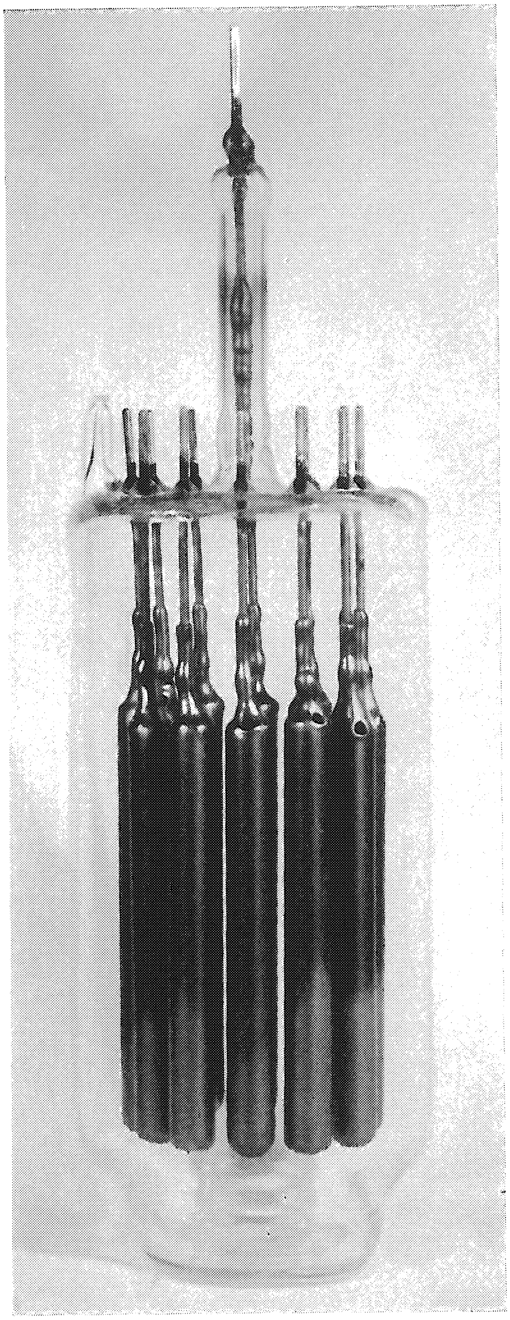

Fig. 2. Photograph of the tube

$$F\varphi = -\frac{q}{r}\frac{\partial V}{\partial \varphi} = q \sum \frac{nV_n(t)}{R}\left(\frac{r}{R}\right)^{n-1} \sin n\varphi$$

q being the charge on the ball.

Fig. 2 shows a photograph of the tube.

Satisfactory models have been obtained with a pendulum having about 5 cm. length, 0.1 mm. thickness, and a ball volume of 1 mm³. The fundamental frequency of oscillation is about 10 c/s and the voltages applied need not exceed a few kilovolts.

By applying a constant 6- or 8-pole field for having a quadratic or cubic non-linearity, the oscillation frequency is made dependent on the amplitude. Fig. 3 shows the frequency shift which must be linear with the square of the amplitude, obtained for a cubic non-linearity, and unidimensional motion.

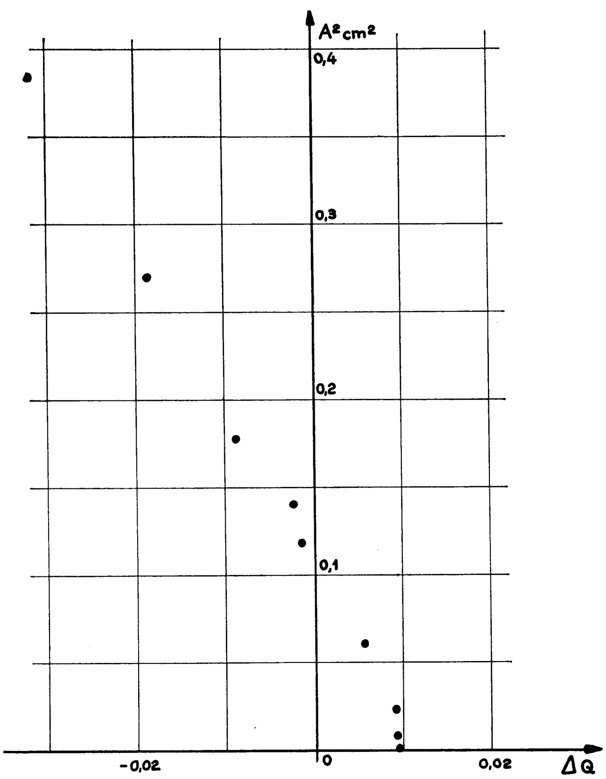

Fig. 3. Amplitude squared against frequency shift with cubic non linearity.

Fig. 4 shows a 1st-order resonance curve in the presence of a cubic non-linearity, giving the maximum amplitudes reached when exciting the pendulum around its own natural frequency by a dipole field, starting from the equilibrium position. The shift of the frequency for maximum response and the sharp assymmetry of the curve are typical.

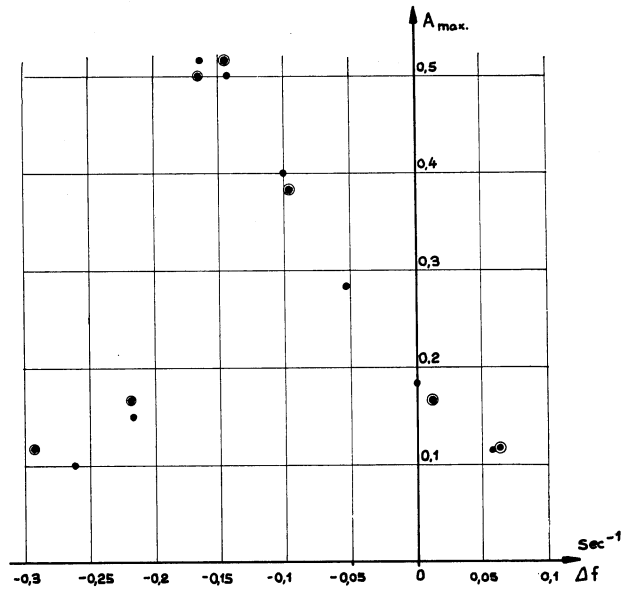

Fig. 4. 1st order resonance line with cubic non linearity.

If we apply to the tube time varying 6-, 8-, 10-pole fields of very nearly 3, 4, 5 times the natural oscillating frequency we observe sub-resonances. Fig. 5, 6, 7 show the observed sub-resonance lines of 3rd, 4th, 5th-order in the Q-diagram. (Ordinate and abscissa are proportional to the ratios of the applied perturbation frequency to the natural frequencies in the 2 principal axes of vibration of the rod, which can be made more or less different by applying a fixed electrical quadrupole).

All the lines calculated by Hagedorn show up exactly. In particular we observed only sum resonance-lines

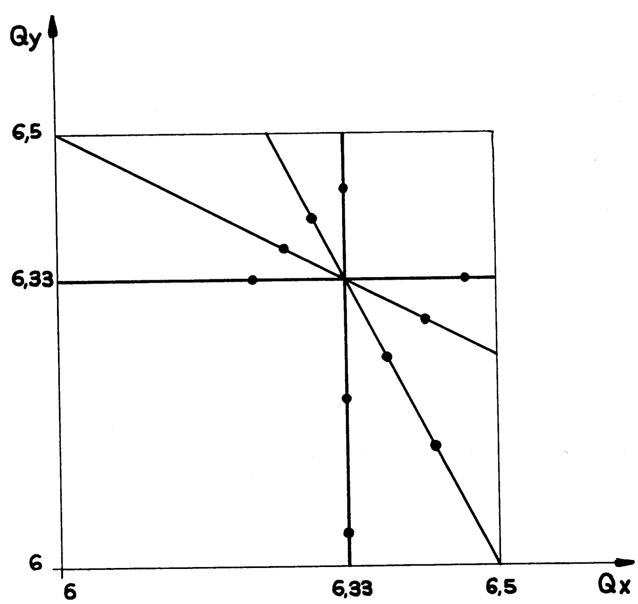

Fig. 5. 3rd order subresonance lines.

(negative slope in the diagram) and no difference resonance lines (positive slope), as predicted.

On vertical and horizontal lines, growth of amplitude takes place only along the corresponding coordinate axis; on the inclined lines, growth of amplitude is observed along both coordinates. The last mentioned lines were excited more easily (the amplitude building up more quickly with the same perturbation). All this is in accordance with theory.

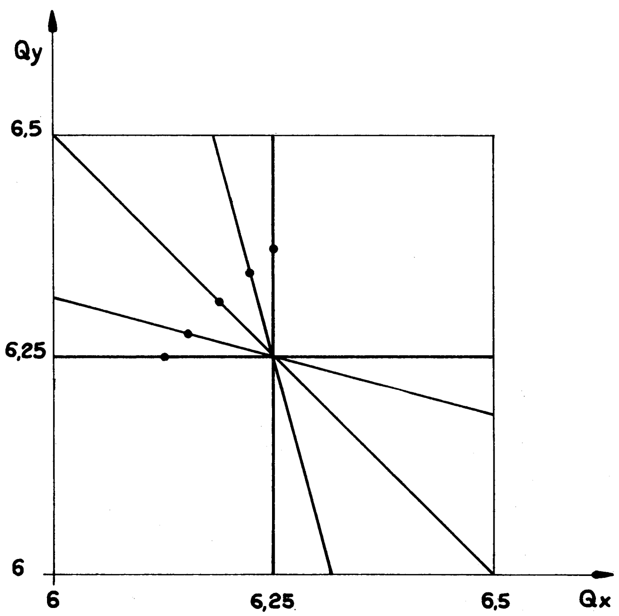

Fig. 6. 4th order subresonance lines.

The next project was to quench a sub-resonance with a constant non-linearity. Fig. 8 shows a 3rd order sub-resonance curve (excited by a 6-pole on the 3rd harmonic) with a constant cubic non-linearity (8-pole). The asymmetry of the sub-resonance curve and its sharp drop break are again typical.

Further work is to be done on the dynamic behaviour of oscillation while sweeping repeatedly to and fro through sub-resonance lines as is the case due to a synchrotron oscillation.

Work is also contemplated on the form of the closed orbit for various misalignments for magnets, which can be done with the same model.

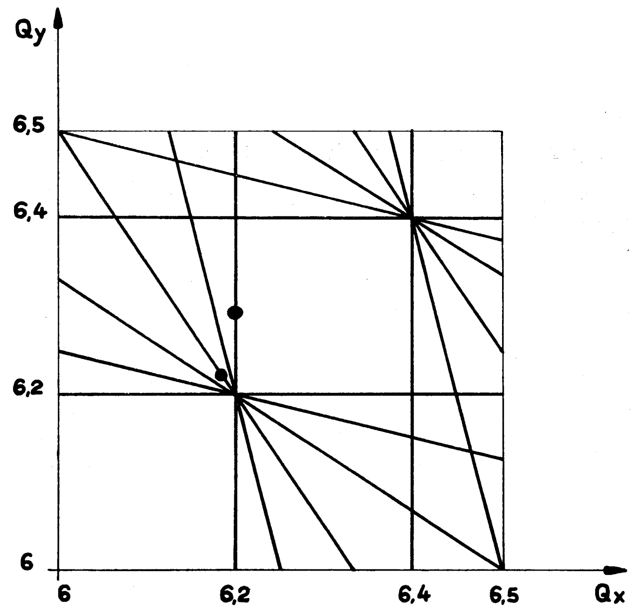

Fig. 7. 5th order subresonance lines.

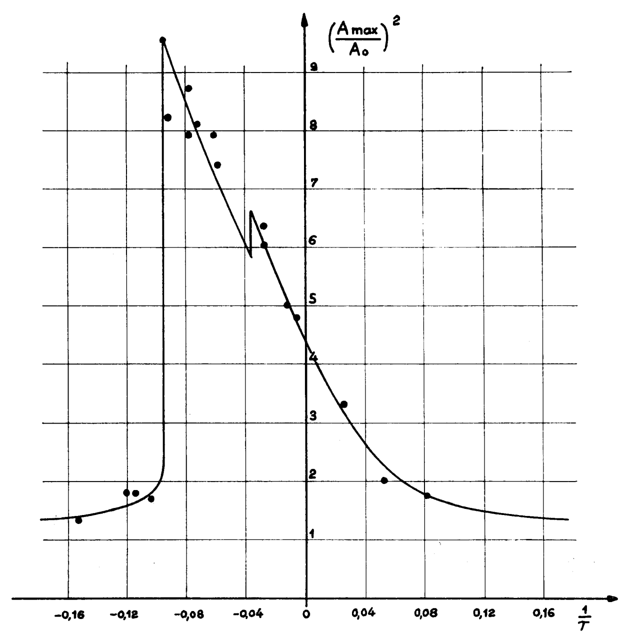

Fig. 8. 3rd order subresonance line with constant cubic non linearity.