PHASE TRANSITION IN THE ALTERNATING GRADIENT ACCELERATOR

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Phase stability in an accelerator is produced if a change in transit time accompagnies a variation in momentum. If the length along the orbit is s per acceleration and t and v are the time of transit and particle velocity, then differentiating $t = s/v$ gives the phase stability criterion

$$\Delta t/t = \Delta s/s - \Delta v/v.$$

Both s and v are expressed as functions of the momentum. If $\Delta t/t$ is negative stability obtains with positive dV/dt (V is the accelerating voltage) the inverse for positive values, and no phase stability if $\Delta t/t$ is zero. The second term is independent of the type of machine, and is $-(1-\beta^2)\,\Delta p/p = -(E_0{}^2/E^2)\,\Delta p/p$. The linear accelerator has $\Delta t/t$ negative because $\Delta s/s$ is a function of geometry, not momentum, and the phase stability is on the rising slope of the accelerating voltage. Blachman and Courant[1] pointed out the existence of a phase transition in the constant gradient machine. For the constant gradient machine with N straight sections of length L and orbit radius R

$$\frac{\Delta t}{t} = \left(\frac{1}{(1-n)(1+NL/2\pi R)} - \frac{E_0^2}{E^2}\right)\frac{\Delta p}{p}.$$

If $n = 0.6$ there can be a phase transition if $NL > 1.5\,(2\pi R)$, a condition which puts little restriction on practical design. The constant gradient accelerators are always operated with phase stability on the falling slope of the rf wave. The cyclotron and microtron are special cases with orbits of only a few revolutions, and the Thomas cyclotron has the useful property of transit time independent of momentum.

The alternating gradient synchrotron introduces the phase transition into the region of desirable design. This was noted by Christofilos and by Courant, Livingston and Snyder[2]. This paper expresses the variation in revolution period

$$\Delta t/t = (\alpha - E_0^2/E^2)\,\Delta p/p$$

with α a constant dependent on the geometry of the system, and of the order of 5/n.

K. Johnsen, and E. D. Courant, have found that a close approximation of the transition energy is given by $E_t = \nu E_0$ (E_t is the total energy at phase transition and ν is the number of radial betatron oscillations per revolution). This relationship indicates that electron alternating gradient synchrotrons can usually avoid the phase transition. Injection at 2 Mev kinetic energy will be above transition if $\nu < 5$, a convenient range for electron machines at 1-2 Bev. The large E_0 of the proton throws the transition energy into the middle of the operating range of most proton alternating gradient synchrotron designs.

The phase transition has been treated by E. D. Courant and H. S. Snyder[3], Kjell Johnsen[4] and R. Q. Twiss[5]. All three treatments use the same differential equation for small phase oscillations. In the nomenclature of Courant and Snyder θ is the phase angle of the particle with respect to the rf wave, θ_0 the stable phase, h the harmonic order, V the peak gap voltage, ω_0 the angular velocity of a particle with velocity c, E the total energy, and γ a function defined by $\Delta t/t = \gamma\Delta\,p/p$. If E_1 is the transition energy $\gamma = E_0^2\,(E^2 - E_1^2))/E^2E_1^2$. The equation of small phase motion

$$\frac{d}{dt}\left(\frac{E}{-\gamma}\frac{d\Phi}{dt}\right) + \frac{h\omega_0^2\,eV\cos\Phi_0}{2\pi}(\Phi - \Phi_0) = 0$$

can be solved readily if E and γ are slowly varying to give the phase oscillation frequency

$$\omega_\Phi = \left(\frac{-\gamma\, h\omega_0^2\, eV\cos\Phi_0}{2\pi E}\right)^{\frac{1}{2}}$$

and the amplitude variation

$$\Phi = \Phi_0\left[\left(\frac{-\gamma}{E}\right)^{1/4}\cos\left(\textstyle\int\omega_\Phi\,dt + \delta\right)\right]$$

The frequency goes to zero at $E = E_1$ ($\gamma = 0$) then rises to a relatively constant value. The phase oscillation amplitudes damp as $(|\gamma|/E)^{1/4}$ and the accompanying radial synchrotron oscillations damp as $\beta^{-1}(|\gamma|\,E^3)^{-1/4}$ or $\beta^{-1}\,(f^2 - 1/f)^{-1/4}$ where $f = E/E_1$. The infinity in the radial oscillations is not real because the approximation solution is not valid at the phase transition. In this region E/γ can be expressed as a linear function of time (or a power series) and the solution matched to the adiabatic solution at about a phase oscillation either side of the $\gamma = 0$ point. Typical curves for such a solution are given by Johnsen[6].

If ideal frequency tracking is assumed, and if Φ_0 is shifted $\pi - 2\Phi_0$ exactly at transition, the phase oscillations damp down to a very small value and then rise to a moderate fraction of their injection value; radial oscillations first diminish then rise to an amplitude less than the injection amplitude at the transition energy, then damp to a small value at high energy. However errors in shifting the phase will spread the bunch in azimuth and radius. Non linearities of the field and of the magnet end effects spread the time at which the various particles reach the neutral phase stability. Approximation calculations indicate that the time tolerances for the various processes are of the order of a millisecond, and that passage through the phase transition should be possible with little or no particle loss.

Programming, or computing, the frequency in the manner of the present proton synchrotrons at the neighborhood of the phase transition seems impracticably difficult. Design of a proton alternating gradient synchrotron for phase transition either below or above the operating region requires ν values either undesirably low or too high for reasonable alignment tolerances. V. V. Vladimirskii and E. K. Tarasov [7] have proposed a field configuration which can eliminate the transition energy. Part of the magnets have reversed field (but not gradient) and operation is maintained rather near an integral resonance. This scheme requires a larger circumference and more aperture, in addition to the disadvantage of working near an integral stop-band, but it also provides reduced momentum compaction—useful for attempts at multi-turn injection, and ejection of the beam.

The passage through phase transition by phase shifting the rf has been suggested by numerous authors. E. von Bodenstedt [8] has suggested that tolerances of phase shifting can be relieved by shifting the phase back and forth several times in the transition region. He has constructed an ingenious torsion pendulum analogue with variable moment of inertia and has carried it through phase transitions.

A radiofrequency system with the necessary frequency and phase precision can be evolved by using the bunched structure of the beam itself as the source of the accelerating waveform. The induction electrode signal is amplified, given the necessary phase shift and applied to the accelerating gap at constant voltage level. Individual particles have phase stability, but the center of gravity of the bunch does not have the phase stability provided by the usual independent rf system because the phase of the accelerating voltage maintains a constant relationship to the phase of the bunch. Long-time stability is inserted by observing the radial position of the beam with difference electrodes and using the resulting signal to actuate a phase shifter in the amplifier chain which then corrects any radial deviation. A narrow band system was built in 1953 to test the feedback scheme on the Cosmotron. About half way through the accelerating period the regular Cosmotron master oscillator was switched off and the feedback circuit switched in. The feedback system demonstrated in operation the stability expected, and was critically damped when a step function in radial beam position was inserted.

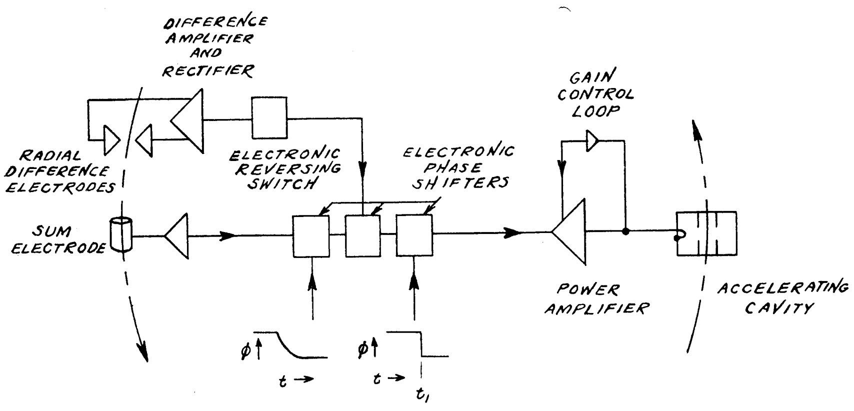
A feedback rf system was designed and constructed for the Brookhaven "Electron Analogue" accelerator. The Analogue is an alternating gradient synchrotron electron machine built to explore the characteristics of strong-focusing systems. It is made with accurately machined electrodes providing electrostatic fields, and is carefully degaussed to eliminate magnetic disturbances. Relevant parameters of this device are :

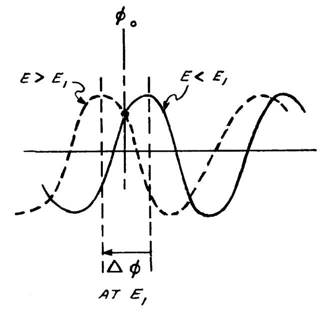
Radius	22 - $^1/_2$ ft
Lattice Structure	$^1/_2$F 0 $^1/_2$F $^1/_2$D 0 $^1/_2$D
Number of periods, N	40
Length of $^1/_2$F, $^1/_2$D, or 0	7 in.
n	225
ν	$\sim 6\text{-}^1/_2$
mode, μ	$\sim \pi/3$
Injection energy	1 Mev
Transition energy, E	$\sim$ 2.75 Mev
Maximum energy	10 Mev
Rise time	5 msec
Aperture	0.8 $\times$ 0.8 in.
Radiofrequency, f	$\sim$ 7 mc/s
$\Delta f/f$	0.055

Sixteen quadrupole lenses, half between $^1/_2$F $^1/_2$F bending lenses and half between $^1/_2$D $^1/_2$D, will change ν by ± 1 in any direction in the stability diagram. Sixteen sextupoles and sixteen octopoles similarly located provide either quadratic or cubic non-linearities when excited.

Electrons are injected through an electrostatic pulsed-off inflector, normally timed to leave about one-half revolution of particles in the orbit. The pickup electrode signal then has a large component at the rf fundamental, and this is amplified and used to accelerate the beam. A simplified block diagram (p. 105) shows the signal passing through three electronic phase shifters. The first is programmed with an exponential and approximately compensates the variation in phase shift with frequency produced in the cables and circuits. The second is controlled by a signal proportional to radial beam position. The third rapidly advances the rf phase when triggered by a timer. A gain control loop with heavy feedback keeps the gap voltage constant over large variations in beam intensity.

In operation the injected beam is accelerated at approximately constant radius near the center of the aperture. The programmed phase shifter, and a delay line for setting the proper starting condition, are adjusted to give minimum demand on the radial control. At the critical energy the





rf phase is rapidly advanced, and simultaneously the sense of the radial control is reversed by an electronic switch. (This is necessary because the slope of the accelerating voltage at the stable phase reverses sign.) Acceleration then proceeds at the new stable phase angle.

When acceleration was first attempted on the Analogue, the radial control circuit had poor transient response and destroyed the beam. While the circuit was being rebuilt the phase was controlled with the programmed phase shifter and acceleration through the transition was accomplished by very careful adjustment. Although the beam jittered from pulse to pulse, it could be carried beyond 5 Mev before being lost under best conditions. Timing of the phase jump could be varied $\pm$ 100 μsec, corresponding to about $\pm$ 0.2 Mev, before the beam was lost at transition. As might be supposed, operation was very critical and initial phase settings to $\pm 2^{\circ}$ were necessary.

An improved radial control provided stable operation with essentially no beam loss at phase transition, or after it, to an energy of about 6 Mev where the guide field voltage was cut off. A gap voltage of some 300 volts peak was adequate for the energy rise rate of about 200 volts per turn. The Analogue is quite close to $\nu_z = 6\text{-}^1/_2$ with no quadrupole voltage and better operation was obtained with sawtooth voltages on the quadrupoles giving $\nu_r \sim 6.7$ and $\nu_z \sim 6.6$. Operation was also somewhat improved with excitation of one set of sextupoles. The timing of the phase jump, or the radial signal reversal, could be varied $\pm$ 100 μsec before operation became unstable. The phase jump was set to span 30 μsec. Phase oscillation frequency of the Analogue is about 15 kc/s at injection, falls to zero at about 1.1 m/sec, then rises to about 3 kc/s. Thus there are fewer than ten phase oscillations before, and after, the transition.

Performance of the Analogue deteriorated over a period of time, and the process was hastened by seismic disturbances from heavy equipment excavating the AGS ring. It was stripped, releveled, and readjusted. We hope that quantitative data on the passage through phase transition, including non-linear effects, can be obtained later.

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