Origin of the even-odd effect in the yields from high-energy reactions

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Abstract
The analysis of experimental production cross-sections of the light products of several nuclear reactions at high energy, measured at the FRS, GSI Darmstadt, revealed a very strong and complex even-odd staggering. The origin of this effect is related to the condensation process of heated nuclear matter while cooling down in the last evaporation step. The characteristics of the staggering correlate strongly with the lowest particle separation energy of the final experimentally observed nuclei. The study confirms the important role of the deexcitation process in fragmentation reactions, and indicates that sequential decay strongly influences the yields of light fragments, which are often used to extract information on the nature of nuclear reactions at high energies.

1 Introduction

In the last decades, a lot of effort has been invested to determine fundamental properties of hot nuclear matter. From the experimental side, one of the most outstanding finding was establishing the caloric curve [1], which gives evidence of the liquid-gas phase transition in nuclear matter. The experimental determination of the caloric curve was achieved by the study of multifragmentation products in high-energy nuclear reactions. It is exactly from the characteristics of the intermediate-mass fragments (IMF) produced in such reactions that the two fundamental variables – energy and temperature – are extracted. While the evaluation of the heat content of the hot nucleus (expressed as excitation energy per nucleon *E*/A*) is considered to be under control [2], larger problems are encountered in the measurement on the nuclear temperature *T* [3]. Most nuclear thermometers rely on the application of thermodynamic relations in the moment of "freeze-out", i.e. when, after the collision stage, the surviving piece of nuclear matter has evolved to a coexistence stage where the formed liquid pre-fragments and the gas of nucleons are assumed to be in thermal and chemical equilibrium. Here the term "pre-fragment" is used to underline that the hot composite fragments at freeze-out normally do not coincide with the experimentally-observed final cold fragments, due to the loss of mass by evaporation during the de-excitation process.

The most frequently used thermometers extract *T* from the measured yields of IMFs, assuming that the population at freeze-out follows a Boltzmann distribution. The probability of forming a pre-fragment (*A,Z*) in a heat bath with temperature *T* is proportional to:

$$Y(A,Z) \propto \sum_i g_i \mathrm{e}^{-E_i/T} \cdot \mathrm{e}^{-\mu/T} \qquad (1)$$

where the sum extends over all possible energy states; g_i is the degeneracy. The second factor comes from the condition of chemical equilibrium. The chemical potential μ depends on the binding energy *B(A,Z)* of the pre-fragment. The Boltzmann factor $\mathrm{e}^{-E/T}$ acts as a weighting factor, such that the lower is the energy of the state, the higher is the probability that the pre-fragment will be in that state. For this reason, it is often assumed that the population of pre-fragments at freeze-out is well represented the ground-state population, where the probability of one individual nucleus is given by Eq. (1), where *E* is its binding energy *B(A,Z)*. If this assumption is correct, the structural effects of the population at freeze-out should reflect those in the binding energies, e.g. those due to the pairing interaction. On the other hand, a heated nuclear system is expected to enter the region of thermal instabilities at quite high temperature, above ~ 5 MeV. If the heat-bath temperature is high, the excitation energy acquired by the pre-fragments is also high, and could be above the critical pairing energy even for very light pre-fragments. In this case, any structural effect related to pairing should be gone.

In this contribution, we will analyse the even-odd structure observed in the yields of light nuclei produced in high-energy nucleus-nucleus collision. We will discuss how the observed staggering can help in understanding the situation at freeze-out.

2 Experimental results

In 2003, the analysis of experimental production cross-sections of relatively light nuclei (Z>6) produced in the reaction ^{238}U+Ti at 1 *A* GeV revealed a very strong and complex even-odd staggering [4]. The experiment was performed using the high-resolution magnetic spectrometer FRagment Separator (FRS), at GSI. The produced nuclei were fully identified in mass and atomic number over an extended area of the chart of the nuclides, and kinematically separated to disentangle the different contributing reaction-mechanisms (namely, fragmentation and fission). In the meantime, a large amount of new experiments on nuclear reactions at high energy were performed at the FRS, and also at lower energies with the A1900 spectrometer at MSU, Michigan State, USA, and at Fermi energies at MARS, Texas University, USA. Recently, we analyzed the published results from many of these experiments, and found that the cross sections of the produced nuclei also appear to be modulated by the same complex and very strong even-odd structure, previously observed in 2003. The new systematic analysis of the even-odd effect over a large range of nuclear reactions and nuclear systems confirms the expected behaviors for the relatively light residual nuclei produced in rather violent collisions: The structural effects cannot be attributed to the surviving of nuclear structure of the colliding nuclei, like in low-energy fission; the structures appear as the result of the condensation process of heated nuclear matter while cooling down in the evaporation process [4].

In this contribution, we want to focus on the reaction ^{56}Fe+Ti at 1 *A* GeV [5], where it was possible to extend the measurement of production cross sections down to lithium isotopes. The experiment was performed at the FRS, at GSI, Darmstadt. The residual nuclei were fully identified in mass and atomic number and their production cross sections were measured. As in the previous experiments, the longitudinal velocity of the fragments was measured with great precision exploiting the high resolution of the magnetic spectrometer. This provided a kind of multiplicity filter and allowed disentangling fragmentation/multifragmentation products from binary-decay products. In the following, we refer exclusively to fragmentation/multifragmentation products. More details on the experimental technique and on the data analysis can be found in ref. [5]. In Fig.1, the data are presented according to the neutron excess *N-Z*. The production cross sections of the observed fragments, grouped according to this filter, reveal a complex structure. All even-mass nuclei (left panel) present a visible even-odd effect, which is particularly strong for *N*=*Z* nuclei. Odd-mass nuclei (right panel) show a reversed even-odd effect with enhanced production of odd-*Z* nuclei. This enhancement is stronger for nuclei with larger values of *N-Z*. However, for nuclei with *N-Z*=1 the reversed even-odd effect vanishes out at about *Z*=16, and an enhanced production of even-*Z* nuclei can again be observed for *Z* > 16. Contrary to other neutron-rich chains, the neutron-deficient chain *N*=*Z*-1 shows a "typical" even-odd effect, i.e. an enhanced production of even-*Z* nuclei. Finally, all the observed structural effects seem to vanish out as the mass of the fragment increases.

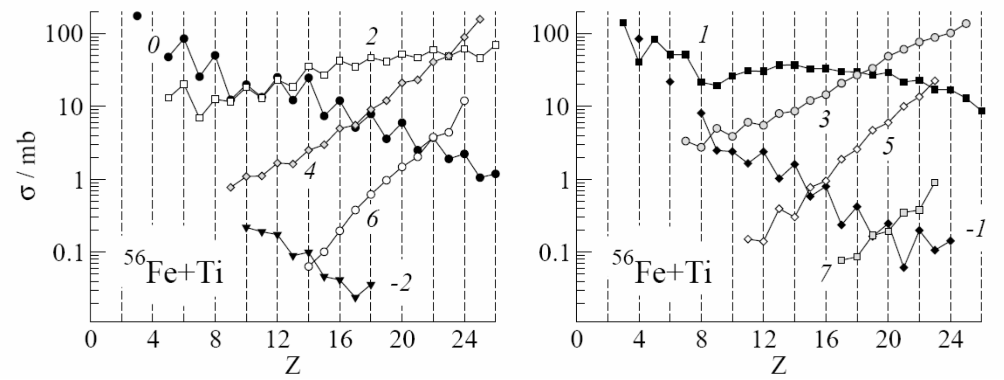

Fig. 1: Measured fragmentation cross sections of the residues from the reaction ^{56}Fe + Ti, 1 *A* GeV [5]. The data are grouped in sequences of nuclei with given value of *N-Z*. The sequences are labelled with the *N-Z* number.

3 Investigation of the population of fragments at freeze-out

By analysing the longitudinal velocity of the residual nuclei, P. Napolitani *et al.* [5] showed that the lightest fragments of Fig. 1 are produced in multifragmentation events. Here, we assume that the multifragmentation process happens as a consequence of a thermal instability and that at freeze-out thermal and chemical equilibrium are reached. This assumption is probably reasonable for nucleus-nucleus reaction at 1 GeV, since dynamical effects should not play a major role.

Hereafter, we want to show that the analysis of the even-odd staggering of these fragments can gives us very specific information on the nature of the population of fragments at freeze-out. We will consider two possible scenarios at freeze-out.

In the first scenario, the IMFs are essentially produced in their ground state; the yields are governed by Boltzmann statistics under certain conditions. In this case, it is not the phase space provided by the IMFs, which determines the yield, because each fragment offers only one state. It is the population probability of just the ground state of each IMF as part of all the possible states of the total system, which is decisive. In other words, the probability for the production of one or the other IMF is given by the phase space of the rest, which is left over when the IMF is produced. It is high if the energy left for the rest is high; it is low if the energy for the rest is low. This implies that the essential parameter is the binding energy of the respective IMF: $Y(A,Z) \propto \mathrm{e}^{-B(A,Z)/T}$.

In the second scenario, the intermediate-mass pre-fragments at freeze-out are essentially produced with excitation energies above the critical pairing energy. In this case, the population of pre-fragments at freeze-out cannot show any even-odd staggering, because there is no pairing interaction above the critical energy. The population of pre-fragments will change during the de-excitation process, but it will remain always smooth as long as the excitation energy remains above the critical energy. Only when the excitation energy drops eventually below the pairing critical energy and the de-excitation enters in the last evaporation step structural effects are restored [4] and the even-odd staggering in the final population of IMFs becomes visible.

3.1 Even-odd staggering in the binding energy

The even-odd staggering in the binding energy was deeply investigated since several decades. We want just to recall here the main characteristics, as explained in ref. [6]. Based on the idea that the interaction energy between two fermions will be greater when the two interacting densities have identical (congruent) nodal structures, compared to the case of two uncorrelated densities, Myers and

Swiatecki calculated with simple combinatory the number of pairs with identical spatial wave-functions. As a result, they got that extra binding associated with the presence of congruent pairs is expressed by:

$$-\frac{3}{2}A + |N - Z| + \delta \qquad (2)$$

with δ= 0 for even-even nuclei, δ= ½ for odd-even or even-odd nuclei, δ= 1 for odd-odd nuclei, and δ= 2 for N=Z=odd nuclei. The interaction energy between a pair of nucleons interacting by short-range forces is inversely proportional to the volume – itself approximately proportional to A – in which the nucleons are confined. It follows that the extra binding associated with the presence of congruent pairs is:

$$-\frac{3}{2} + |I| + \frac{\delta}{A} \quad \text{with} \quad I = \frac{N - Z}{A} \qquad (3)$$

The quantity $C = -3/2 + |I|$ is the "congruence energy" and δ/A is the pairing correction. The congruence energy is often parameterized as $C = -a + b \cdot |I|$ (a ~ 7 MeV and b ~ 42 MeV), where $b \cdot |I|$ is the Wigner term. Thus, the staggering in binding energy can come only from the staggering of δ. In Fig. 2, the experimental staggering in the binding energy is presented for four sequences of nuclei with constant neutron-excess: N=Z-1, N=Z, N=Z+1, N=Z+2. The binding energy staggering is given by the difference between the tabulated experimental binding energies from Audi and Wapstra [7] and the liquid drop binding energies, calculated according to Myers and Swiatecki [8]. Please note that the long-range fluctuation is related to shell effects (not considered in the liquid-drop formula). The staggering in the experimental binding energy is consistent with the prescription of Myers and Swiatecki. The N=Z and N=Z+2 chains are made of even-even and odd-odd nuclei alternating; the extra binding energy associated with pairing alternates between δ=0 and δ=2 or δ=1, producing a strong even-odd staggering. The N=Z-1 and N=Z+1 chains are made of even-odd and odd-even nuclei alternating; the extra binding energy associated with pairing is given by δ= ½ for both type of nuclei, producing no even-odd staggering.

Thus, we can conclude that the staggering in the binding energies cannot be responsible for the observed even-odd staggering in the final yields. Consequently, the first scenario described above seems not to be realistic.

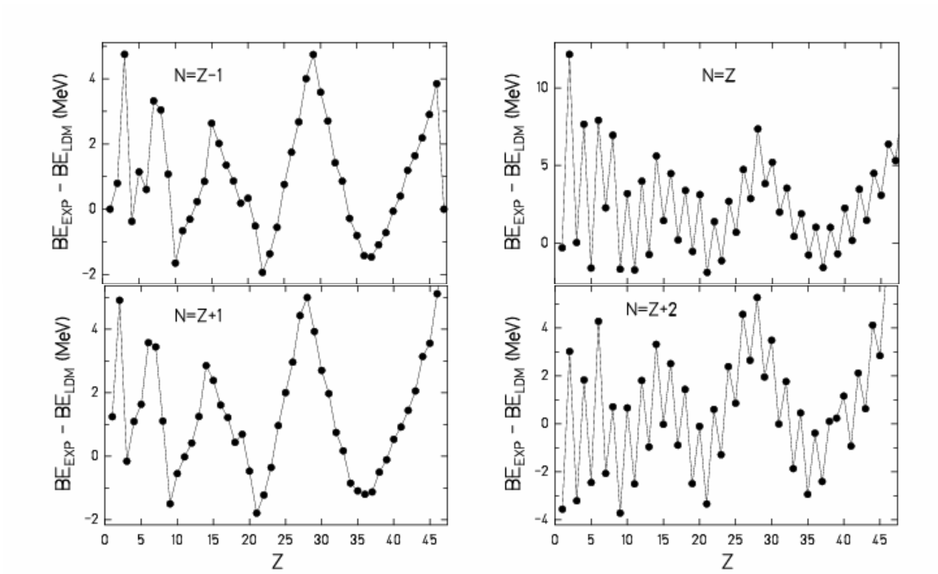

Fig. 2: Binding energy staggering given by the difference between the tabulated experimental binding energies from [7] and the liquid drop binding energies [8].

3.2 Even-odd effect in the remnants of an evaporation chain

We want to discuss now which kind of staggering we have to expect in the cold final products due to the influence of an evaporation cascade starting from hot pre-fragments. In this case, after their formation, the initial pre-fragments are definitely free of any even-odd structure, because there is no pairing above the critical energy. There are a large number of states available for the direct production. The number of available states from a certain total excitation energy of the decaying system is essentially determined by a mostly structureless level density above a fictive liquid-drop ground state. Some influence of shell effects is still possible. This produces an essentially smooth 2-dimensional distribution in N and Z on the chart of the nuclides with an excitation-energy distribution, which varies smoothly as a function of N and Z. This structureless cloud rains down in excitation energy and in nucleon number due to evaporation. This process is essentially deterministic, because in most cases one particle dominates and the variation of the kinetic energy of the emitted particle is small compared to the binding energy. In this way, a well-defined 3-dimensional subspace of the N, Z ,E^* initial production cloud ends up in a certain fragment. As a consequence, the final yields will be modulated by the range of excitation energy below the lowest particle threshold: There is a fine structure, proportional to the fluctuations of the lowest particle threshold. This is an old idea of J. Hüfner, C. Sander and G. Wolschin presented in ref. [9] (see in particular Fig. 1 from ref. [9]), where they wrote: "... the decay is strongly determined by threshold effects. The production cross sections are determined by how much excitation cross-section there is in a given interval between two thresholds, *not* by how the energy is precisely distributed over the available degrees of freedom". This idea is at the base of the de-excitation model of X. Campi and J. Hüfner [10], later modified and improved by J. J. Gaimard and K. H. Schmidt [11], where the evaporation stage is treated in a macroscopic way on the basis of a master equation which leads to a diffusion equation. There, it is written: "The mass yield curve $\sigma(A)$ is directly related to the pre-fragment distribution because of conservation of probability and energy, essentially".

In order to clarify better this concept, we consider Fig. 3, where the last evaporation step is depicted, for the 4 possible types of final fragments: odd-odd, even-odd, odd-even, even-even.

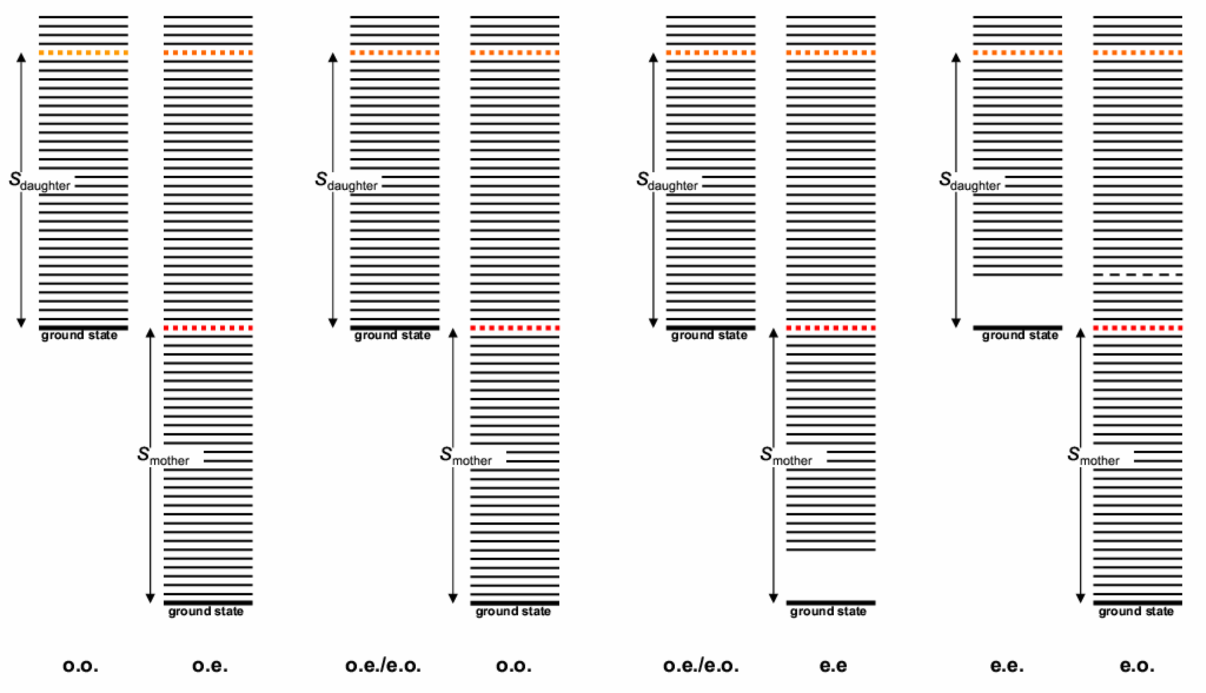

Fig. 3: Artistic view of the last evaporation step. See text for details.

Under the assumption that we are considering the last evaporation step, then the mother nucleus has excitation energy E^* between the lower (red) and upper (orange) dotted lines. If the energy of the mother nucleus would be above the upper (orange) dotted line, then it would decay in the daughter nucleus, but it would not be the last evaporation step: in turn, the daughter nucleus would decay since its E^* would be above its separation energy. If the energy of the mother nucleus would be below the lower (red) dotted line, then it would not decay into the daughter nucleus; it would deexcite by gamma emission. In reality, the mother nucleus can have E^* a little bit above the upper (orange) dotted line and still decay in the daughter nucleus, because some energy would go in the kinetic energy of the emitted particle, but this quantity is small and in first approximation we can neglect it and assume that the two dotted lines define the "energy range" occupied by the mother nucleus. The lower (red) line is defined by the separation energy of the mother nucleus, which coincides – by definition – to the ground state of the daughter nucleus. The upper (orange) dotted line is defined by the separation energy of the daughter nucleus. In conclusion: if the energy of the mother nucleus is between the ground state and particle separation energy of the daughter nucleus, then it will decay into the daughter nucleus. The same reasoning is valid also for the case of an even-even final fragment, whose first excited state is well separate from the ground state. If the energy of the mother nucleus is above its particle separation energy (lower (red) dotted line) and below the dashed line, it will decay into the ground state of the daughter nucleus; this does not make any difference in terms of final yield. Here we must point out that things go differently for heavy nuclei, where gamma emission becomes a competitive channel, as discussed in ref. [4].

The important conclusion is that it is not the number of levels of the daughter nucleus between its ground-state and its separation energy which determines the probability to decay into the daughter nucleus; it is not even the number of levels of the mother nucleus in the energy range between the ground-state and separation energy of the daughter nucleus which determines the probability to decay into the daughter nucleus. The phase-space, which is related to the level density, is not anymore the relevant quantity in the last evaporation step, which can be considered quasi deterministic. It is the separation energy, which gives the range of excitation energy, which "catches" the evaporation flux in particle-stable states, in the sense of Campi and Hüfner. The relevant quantity for the staggering in the final yields is the "particle threshold", which we called "energy range", represented by the lowest value of the particle separation energy of the daughter nucleus.

In Fig. 4, we present on the chart of the nuclides the lowest particle separation energy for light nuclides. The values of proton and neutron separation energies, S_p and S_n respectively, were taken from the compilation of Audi and Wapstra [7]. As stated before, it is the lowest particle separation energy which determines the final flux of phase space, and therefore, the staggering in the final yields.

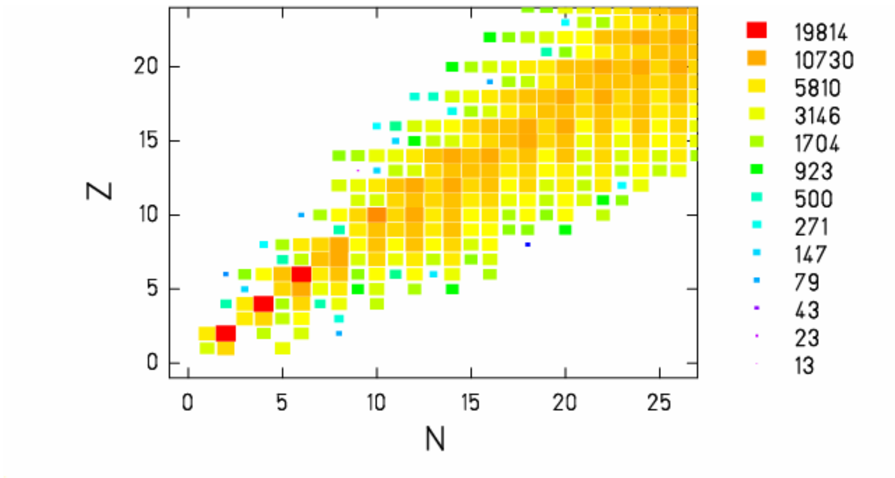

Fig.4: Lowest particle separation energy for each nuclide (keV). Values are taken from Ref. [7].

In reality, the proton separation energy cannot be compared directly to the neutron separation energy because of the Coulomb barrier. S_n and S_p are approximately equal along the stability line. However, when we consider an evaporation process, the emitted particle has to overcome the Coulomb barrier. This is why the evaporation corridor [12] is defined by the "attractor line" [13] and not by the stability line. A more precise evaluation of the "energy range" should take into account the Coulomb barrier. However, to calculate this reduction of S_p is not trivial because for proton emission tunneling through the barrier is not at all negligible. On the other hand, the S_p is given by the mass difference of the two close nuclei, so at infinite distance, *not* above the Coulomb barrier. So, to reduce S_p by an amount equal to a coulomb barrier would also be wrong. Anyhow, Coulomb effects are small for light nuclei considered here, and might be neglected in the first step. In conclusion, we think that the lowest particle separation energy is a good representative of the "energy range", at least to understand qualitatively the staggering phenomenon.

4 Origin of the even-odd effect in the light fragments

We have now all the information to understand the origin of the even-odd effect in the yields presented in Fig.1. The lowest particle threshold (Fig. 4) results in a strong and complex even-odd staggering. Fig. 4 shows that along the chain N=Z, even-even nuclei show the highest energy range. Along the chain N=Z+1, it is the energy range of Z=odd nuclei which is enhanced. On the contrary, along the chain N=Z-1, it is the energy range of Z=even nuclei which is enhanced. These features are fully consistent with the staggering in the experimental yields. On the other hand, the results presented in Fig. 2 indicate that there is no staggering in the binding energy of even-mass nuclei, which is not reflecting the behavior observed in the yields of Fig. 1.

To visualize it better, in Fig. 5, we present a comparison of the experimental production cross-section from the reaction ^{56}Fe+Ti at 1 *A* GeV, for two sequences of fragments: the N=Z chain, and the N=Z+1 chain. The experimental cross sections (expressed in mb) are compared with the biding energies staggering (expressed in MeV) and with the lowest particle separation energies (expressed in MeV).

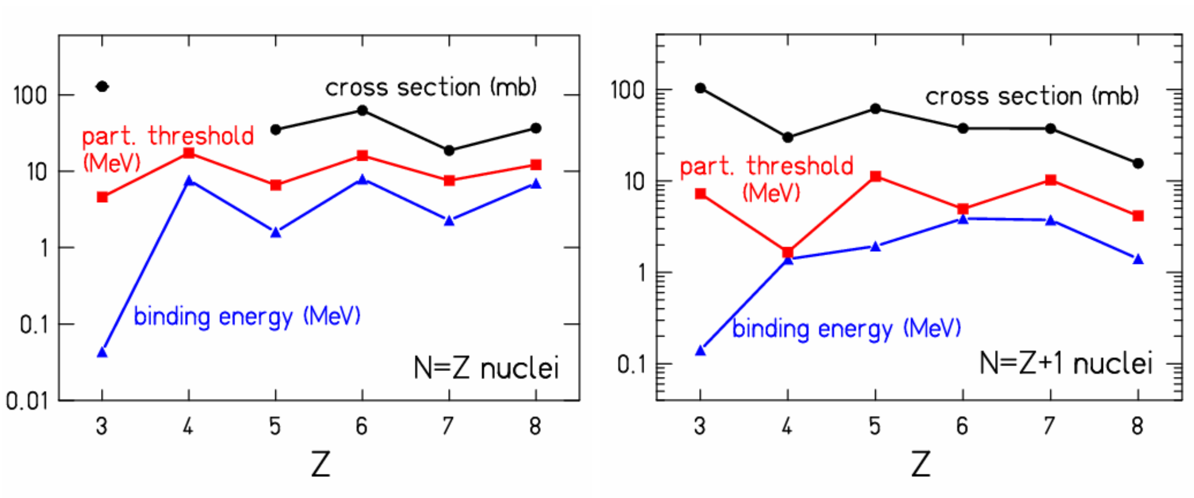

Fig.5: Results for the sequences of nuclei with N=Z (left) and N-Z=*1* (right).
Dots: Measured fragmentation cross sections of the residues from the reaction ^{56}Fe + Ti, 1 *A* GeV.
Squares: Lowest particle separation energy (particle threshold).
Triangles: Extra binding energy due to pairing.

The results indicate clearly that the assumption that IMFs are produced cold at freeze-out is wrong, even for very light IMFs such as lithium isotopes (Z=3). The use of Eq. (1) to determine the probability to form a given IMF at freeze-out is incorrect, because the population of fragments is not predominantly in the ground state but above the pairing critical energy. So, it is dangerous to extract fundamental properties of hot nuclear matter assuming that the population at freeze-out is well represented by the Boltzmann factor of Eq. (1). The situation is even more severe when these methods are extended to treat even heavier final fragments. It has already been recognised previously that the determination of the freeze-out temperature with the isotopic yield thermometer might be disturbed by the evaporation process or sequential decay. Several authors investigated this problem and proposed suitable corrections to this effect [14,15]. Our analysis opens a new view on this problem by demonstrating that the fine structure observed in the final yields can fully be explained by the evaporation process. That means that this specific feature does not show any noticeable influence of the Boltzmann factor, which is behind the isotopic thermometer method. Quantitative conclusions on the validity of the isotopic thermometer method are beyond the scope of this work.

5 Conclusions

We have studied the origin of the even-odd staggering observed in the production cross-section from high-energy reactions. The observed fine structure in the yields of the fragments agrees well with the fluctuations of the lowest particle threshold as a function of neutron and proton number. The structure is not consistent with the fluctuations of the binding energies. This finding proves that the primary fragments produced in multifragmentation before secondary decay are mostly produced in excited states. The direct production in their ground state seems to be weak. Implications on the application of the isotopic thermometer are expected.

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