INSTITUTE OF NUCLEAR PHYSICS
SSSR ACADEMY OF SCIENCES, NOVOSIBI
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SYNCHROBETATRON RESONANCES AT ZERO VALUE OF CHROMATICITY

N.A. Vinokurov, V.N. Korchuganov, G.N. Kuliapanov,
V.N. Litvinenko, E.A. Perevedentsev, A.N. Skrinskij

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Abstract

The behaviour of the beam in storage rings and accelerators is significantly influenced by synchrobetatron resonances. They are generally considered to be caused by the non-compensated chromaticity (i.e. by the momentum dependence of the betatron frequency). In this approach, the synchrobetatron resonances vanish if the chromaticity goes to zero.

However, in this paper it is shown that in the presence of transverse (i.e. normal to the equilibrium orbit) electric or magnetic fields oscillating with an integer multiple of the revolution frequency ($\omega = q \cdot \omega_0$), synchrobetatron resonances $\nu_{x,z} + n\nu_s = m$ do arise at zero chromaticity. The resonance strength is proportional to $J_n(2\pi\ell/\lambda)$, where J_n is the n-th order Bessel function, ℓ is the bunch length, λ is the wavelength of the HF field. Thus these resonances are severe in high energy storage rings (SPEAR II, DORIS, VEPP-4, PEP) in which ν_s is large ($\simeq 0.1$) and $\ell/\lambda \simeq 1$.

The results obtained give an explanation of the chromaticity-independent synchrobetatron sidebands of the integer resonances, observed in SPEAR II with the orbit distortion in the RF cavities taken into account. A simple method for compensation of such resonances is suggested. The tolerable orbit distortions in the RF cavities of VEPP-4 are estimated.

In many accelerators and storage rings, synchrobetatron resonances (SBR) have been observed experimentally. They are known to significantly influence beam behaviour, its size and lifetime (see for example references 1) to 4). The main reason for these resonances is the energy dependence of betatron tune, i.e. chromaticity. However, the chromaticity can be compensated by sextupole corrections. Therefore, we will concentrate on other reasons resulting in the occurrence of SBR.

The most detailed study will be devoted to the case where there are transverse time-dependent electric or magnetic fields on the equilibrium orbit, which may occur if the orbit is distorted in the accelerating RF cavities. As it is shown below, the strength of the $\nu_{x,z} + n\nu_s = m$ SBR is then proportional to $J_n(2\pi\ell/\lambda)$, where J_n is the n-th order Bessel function, ℓ is the longitudinal oscillation amplitude, λ is the RF wavelength. Thus these resonances are severe for high energy electron storage rings such as SPEAR II, DORIS, VEPP-4, PEP and PETRA, where ν_s is rather high ($\simeq 0.1$) and $\ell/\lambda \simeq 0.5$. The estimates on orbit distortion tolerance in VEPP-4 are presented.

The results obtained may give an explanation of the SBR at zero chromaticity as those observed in SPEAR II.

A simple method to cure the resonances due to this mechanism is suggested.

1. Introductory notes on chromaticity-dependent SBR

Originally, the treatment of SBR accounted only for the chromaticity (i.e. dependence of the betatron tune on the energy of the particle)[5]. In the presence of the chromaticity the betatron oscillations appear to be frequency-modulated at the synchrotron frequency due to particle energy oscillations in a storage ring. The frequency modulation (FM) results in the appearance of side-bands in the betatron oscillation spectrum at frequencies $\nu = \nu_{x,z} + n\nu_s$. The amplitude of the n-th side-band (and also the strength of the SBR at $\nu_{x,z} + n\nu_s = m$) is proportional to $J_n(\xi_{x,z})$, where $\xi_{x,z} = 2\pi\, \Delta\nu_{x,z}/\nu_s$, $\Delta\nu_{x,z} = \partial\nu_{x,z}/\partial E \cdot \varepsilon$; $\Delta\nu_{x,z}$ is the FM amplitude, ε is the amplitude of the energy oscillations.

It must be noted that if the chromaticity is not compensated the SBR side-bands appear around all 'machine' resonances $k_x\nu_x + k_z\nu_z = m$, because here the betatron frequencies are modulated and the strengths of the SBR satellites in this approximation are determined by the chromaticity value and must therefore vanish if the chromaticity is compensated.

Nevertheless, in SPEAR II, SBR sidebands at $\nu_{x,z} - n\nu_s = 5$ have been observed which are independent of the chromaticity in a wide range ($E \frac{\partial\nu}{\partial E} = 0 - 10$) but strongly dependent on orbit distortions. This fact gives rise to a study of SBR at zero chromaticity.

2. SBR sidebands due to transverse components of the accelerating RF fields

Let us consider the particle motion in a storage ring in the presence of transverse (i.e. normal to the equilibrium orbit) fields which oscillate at the frequency $\omega = \nu_d \omega_o$, where ω_o is the revolution frequency of the equilibrium particle. The revolution time T of a particle depends on its energy and therefore oscillates with respect to the equilibrium value T_o with the synchrotron frequency. Hence the driving force acting on the particle is phase-modulated at the synchrotron oscillation frequency. The phase modulation gives rise to sidebands at frequencies $\nu = \nu_d + n\nu_s$ in the spectrum of the driving force and therefore SBR arise.

Note that in contrast to the case of SBR with $\partial\nu_{x,z}/\partial E \neq 0$ where synchrotron oscillations enrich the spectrum of betatron oscillation, here the spectrum of the driving force is enriched.

Let us write down the linearized equation of betatron oscillations taking into account radiation damping

$$\frac{d^2z}{ds^2} + 2u_z(s)\,\frac{dz}{ds} + \left[g_z(s) + u'_z(s)\right] z = f_z(s)e^{i\omega t} \tag{1}$$

where s is the azimuth, $g_z(s)$ is the rigidity, $u_z(s) = \frac{1}{2E_o}\frac{dE_{rad}}{ds}$ is the radiation damping rate and

$$f_z(s) = \frac{e}{E_o}\left[E_z(s) - H_x(s)\right], \tag{2}$$

here E_z, H_x are the field components, E_o is the equilibrium energy, e is the particle's charge. Here and below we assume the motion to be ultrarelativistic : $\gamma_o \equiv E_o/mc^2 >> 1$.

Because of the synchrotron oscillations:

$$\phi \equiv \omega_o t = \theta + \phi_o \sin \nu_s\theta, \tag{3}$$

where $\theta \equiv S/\bar{R}$ and ϕ_o is the amplitude of the phase oscillations, we obtain :

$$\omega t = \nu_d \theta + \kappa \sin \nu_s \theta \qquad (4)$$

$$\kappa \equiv \nu_d \phi_o ,$$

i.e. phase modulation of the driving force.

Let us consider the vicinity of the resonance

$$\pm \nu_z + \nu_d + n\nu_s + k = \Delta/2\pi \quad (\Delta << 1),$$

where Δ is the detuning. Leaving out non-resonant terms and transforming to 'slow' variables (betatron amplitude and phase) we obtain an equation for the stationary amplitude :

$$A_z(s) = w_z(s) \frac{|C_k|}{(\gamma^2_z + \Delta^2)^{1/2}} J_n(\kappa) \qquad (5)$$

$$C_k = \int_o^{2\pi} f_z(s)\ w_z(s)\ \exp\left[i(\pm\chi_z(s) - k\frac{s}{R})\right] ds \qquad (6)$$

where $\omega_z = \sqrt{\beta_z}$ is the modulus and χ_z is the phase of the Floquet function, and γ_z is the radiation damping decrement per turn.

It is convenient to express γ_z in terms of the radiation energy loss per turn

$$\gamma_z = \frac{eU_o \cos \phi_s}{2E_o} \qquad (7)$$

Here U_o is the peak RF voltage and ϕ_s is the equilibrium phase. If the driving force is concentrated in δ-function at the azimuth s_o, equ. (5) yields

$$A_z(s) = w_z(s)\ w_z(s_o) \frac{(\bar{E}_z - \bar{H}_x)d}{U_o \cos \phi_s} \frac{J_n(x)}{(1+\delta^2)^{1/2}} \qquad (8)$$

where d is effective length of the domain in which the fields are concentrated, and $\delta \equiv \Delta/\gamma_z$.

In all electron storage rings there are accelerating structures that have longitudinal electric fields varying quickly in time. If the equilibrium orbit crosses the accelerating structure at an angle with respect to its electric axis, then a transverse component of the electric field arises which drives a SBR at $\nu_{x,z} + n\nu_s = m$, because in this case $\nu_d = q$, q is the harmonic number of the accelerating RF voltage. Now

$$\bar{E}_z = \frac{U_o}{d} \sin \alpha_z$$

where α_z is the projection of the angle between the orbit and the RF cavity axis on the plane of the vertical betatron oscillations.

Because of the relation $\phi_o = \ell/\bar{R}$, where ℓ is an amplitude of the longitudinal oscillations, rewriting $q/\bar{R} = 2\pi/\lambda$, where λ is the acceleration field wavelength, we obtain from equ. (8) :

$$A_z(s) = w_z(s)\, w_z(s_o) \frac{J_n(2\pi\ell/\lambda)}{(1 + \delta 2)^{\frac{1}{2}}} \frac{\sin \alpha_z}{\cos \phi_s} \quad (9)$$

Horizontal SBR at $\nu_x + n\nu_s = m$ may be considered in a similar way by replacing γ_z by $\gamma_x = G_x\gamma_z$ where G_x is the dimensionless damping partition number $(G_x + G_s = 3)$:

$$A_x(s) = w_x(s)\ w_x(s_o) \frac{J_n(2\pi\ell/\lambda)}{G_x\ (1 + \delta^2)^{\frac{1}{2}}} \frac{\sin \alpha_x}{\cos \phi_s} \quad (10)$$

The above consideration does not account for higher harmonics in synchrotron oscillations due to their nonlinearity. To take this into account, equ. (3) is rewritten in the form :

$$\phi = \theta + \phi_o \sin \nu_s\theta + \sum_{k=2}^{\infty} \phi_k \sin k\nu_s\theta \quad (11)$$

Hence the nonlinearity of the synchrotron oscillations can be included by introducing a numerical factor b_n into equs. (9) and (10). In the table below values of b_n are presented for several sidebands for the case where $\cos \phi_s << 1$, and the RF voltage is assumed to be sinusoidal (with $J_n(x) \simeq \frac{1}{n!} (\frac{x}{2})^n$ in equ. (11) as $x \lesssim 0.5$) :

$\lvert n \rvert$	1	2	3	4	5	6
b_n	1	1	9/8	3/2	75/32	135/32

3. SBR sidebands due to dispersion in cavities

It has been shown in ref. 7 that a non-zero dispersion function η (and/or its derivative η') in the accelerating structure can also give rise to SBR. Since the reason of excitation of SBR in this case and in the case considered above is the same (i.e. the particle's trajectory changes with respect to the instantaneous off-energy orbit), equs. (9) and (10) can be readily generalized for $\eta,\eta' \neq 0$ by substitution

$$\sin \alpha_{x,z} \rightarrow \left[(\sin \alpha_{x,z} - \eta'_{x,z} + \frac{\beta'_{x,z}}{2\beta_{x,z}} \eta_{x,z})^2 + \frac{\eta^2_{x,z}}{\beta^2_{x,z}} \right]^{1/2} \qquad (11)$$

Note that for a perfect ring geometry $\eta_z, \eta'_z \equiv 0$ and SBR at $\nu_z + n\nu_s = m$ can only be excited by the equilibrium orbit distortions in RF cavities ($\alpha_z \neq 0$).

4. Estimates of SBR in VEPP-4

In the storage ring VEPP-4 design, the non-zero dispersion η_x and η'_x in the cavities will result in a noticeable increase of the horizontal beam size at SBR $\nu_x - n\nu_s = 10$. Thus, for $n = 4$, the horizontal beam size will be increased by a factor of 2. A vertical angle $\alpha_z \approx 10^{-2}$ between the cavity axis and the orbit will increase the vertical beam size at SBR $\nu_z - 4\nu_s = 10$ by the same factor.

5. Cure for SBR at zero chromaticity

However, SBR excitation at zero chromaticity can be compensated. As one can see from equs. (5), (6), it is sufficient to use two RF correctors for each lateral degree of freedom (x and z).

As the simplest corrector deflection plates can be used which give transverse electric and magnetic fields. To cure the SBR excitation the plates should be fed by an RF voltage at an integer multiple of the revolution frequency and with appropriate amplitude and phase.

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