Determination of the Pion and Kaon Structure Functions

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Quark structure functions have been extracted from low-p_T inclusive hadron production data for the pion and kaon with use of the recombination model. $n^\pi = 1.0 \pm 0.1$ and $n^K = 2.5 \pm 0.6$ is obtained, where n is the leading $(1-x)$ power of the nonstrange–valence-quark distribution. Both the pion and kaon nonstrange–sea-quark functions have $n \simeq 3.5$.

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The success of the quark/parton picture as applied to deep-inelastic leptoproduction and large-p_T hadronic-scattering experiments has prompted attempts to extend the picture to low-p_T hadronic processes.[1] The first objective of such a program is to model low-p_T scattering in terms of structure functions determined from leptoproduction experiments.[2] If this is accomplished, then it should be possible to use low-p_T data to derive structure functions for those hadrons which cannot be used as targets for lepton probes. The quark recombination model of Das and Hwa[3] has been used successfully to describe low-p_T proton fragmentation.[4] Here we use our data from a high-energy inclusive production experiment[5] to study the structure of pions and kaons in the context of the recombination model. This experiment was carried out with the Fermilab single-arm spectrometer[6]; the data reduction procedures are described elsewhere.[7]

The recombination model for the inclusive reaction $a + p \to c + X$ assumes that a quark and an antiquark from the fragmenting projectile, a, combine to form the outgoing meson, c. The invariant cross section for this process as a function of Feynman x is then[3]

$$x\frac{d\sigma^{a\to c}}{dx} = \sigma_{\text{inel}}{}^a \int_0^x\int_0^x F^a(x_1, x_2) \times R^c(x_1, x_2; x)\frac{dx_1}{x_1}\frac{dx_2}{x_2}, \quad (1)$$

where $R^c(x_1, x_2; x)$ is the recombination function, namely the probability that a quark q_1 at x_1 and an antiquark $\bar{q}_2$ at x_2 recombine as the valence quarks of meson c at x. $F^a(x_1, x_2)$ is the joint structure function for finding q_1 and $\bar{q}_2$ within projectile a, and is assumed to take the form[3]

$$F^a(x_1, x_2) = F_{q_1}{}^a(x_1) F_{\bar{q}_2}{}^a(x_2) \rho^a(x_1, x_2). \quad (2)$$

Here $F_q{}^a$ is the structure function for q within a, and p^a is a phase-space factor.

When projectile a is a proton, it has been demonstrated that the recombination model fits the inclusive meson spectrum well.[4] These fits with valence-quark structure functions derived from leptoproduction as input have determined the sea-quark structure. Repeating these fits, but allowing the data to determine the valence functions as well, we have also obtained results consistent with those from leptoproduction. A surprising result of the proton fits is that the valence and sea quarks together appear to carry all the momentum of the fragmenting proton, in contrast to leptoproduction results where neutral gluons carry approximately half of the momentum.

In this paper we study the nonleading reactions with incident and outgoing pions and kaons. For this analysis, the pion and kaon structure functions, F_q^a, are parametrized as follows. The nonstrange-valence-quark distribution is assumed to be[8]

$$V_u^a(x) = b_u^a x^{1/2}(1-x)^{n_u^a}, \tag{3}$$

where the superscript a denotes π or K. Charge-conjugation invariance ensures that $V_u^{a^+} = V_{\bar{u}}^{a^-} \equiv V_u^a$. The sea quarks are given by

$$S_q^a(x) = b_q^a(1-x)^{n_q^a}, \tag{4}$$

where the subscript q is either d for nonstrange sea quarks or s for strange ones. We assume light-quark universality, namely $S_d^a = S_{\bar{d}}^a = S_u^a = S_{\bar{u}}^a$, giving a total of six functions with two parameters each to be determined from the data. The functions are paired together according to (2) for six separate reactions as shown in Table I.

As suggested by Hwa and Roberts,[9] ρ^a is assumed to be unity for incident mesons. R^c is related to the valence structure function of the outgoing meson:

$$R^c(x_1, x_2; x) = \alpha^c\left(\frac{x_1}{x}\right)^{n_2^c}\left(\frac{x_2}{x}\right)^{n_1^c}\delta\left(\frac{x_1}{x}+\frac{x_2}{x}-1\right), \tag{5}$$

where the n_i^c are the leading $(1-x)$ powers of the valence structure functions as given in (3). For R^π we have $n_1^\pi = n_2^\pi = n_u^\pi$, while for R^K the power n_u^K refers only to the nonstrange valence quark. It is assumed that the $(1-x)$ power for the strange valence quark of the kaon is given by $(m_u/m_s)n_u^K$, where m_u/m_s is the ratio of the u- and s-quark effective masses[10] and is taken to be $\frac{2}{3}$. Finally, the constant α^c is obtained from the normalization condition[10]:

$$\int_0^1\int_0^1 R^c(x_1, x_2; 1)\,dx_1\,dx_2 = 1. \tag{6}$$

TABLE I. Structure function pairs used in the fits.

To $a+p\to c+k$	F_q^a	$F_{\bar{q}}^a$
$\pi^\pm \to K^\pm$	$V_u^\pi + S_d^\pi$	S_s^π
$\pi^\pm \to K^\mp$	S_s^π	S_d^π
$\pi^\pm \to \pi^\mp$	S_d^π	S_d^π
$K^\pm \to \pi^\pm$	$V_u^K + S_d^K$	S_d^K
$K^\pm \to \pi^\mp$	S_d^K	S_d^K
$K^\pm \to K^\mp$	S_s^K	S_d^K

With the assumption of scaling and charge-conjugation invariance, the data for the six reactions listed in Table I from both 100 and 175 GeV/c incident momenta and with both positive and negative beam have been simultaneously fitted with the formulas outlined in (1)–(5). The p_T range of the data is limited to three x sweeps centered at $p_T = 0.3$, 0.5, 0.75 GeV/c, with $0.3 \leqslant x \leqslant 0.7$. The upper limit on x is imposed to minimize the effects of forward resonance production and triple Regge contributions.[5] The reaction $\pi^\pm \to \pi^\mp$ includes appreciable resonance production at large x and so is further limited to $x \leqslant 0.5$. There are still resonance contributions below these limits which are not accounted for in the model.[11]

Since we fit differential and not p_T-integrated cross sections, the six normalization parameters b_q^a in (3) and (4) are in principle p_T dependent. However, the values of b_d^a and b_s^a are determined relative to b_u^a by averaging over p_T. The parameters b_u^a are then obtained if we assume one valence u quark per meson:

$$\int_0^1 V_u^a(x)\,dx/x = 1. \tag{7}$$

We also evaluate the structure-function first moments

$$\bar{x}_u^a = \int_0^1 x V_u^a(x)\,dx/x,$$

$$\bar{x}_{d,s}^a = \int_0^1 x S_{d,s}^a(x)\,dx/x, \tag{8}$$

which give the average momentum fraction carried by each quark.

The results of the simultaneous fit to all our data are given in Table II. A comparison of the model with a fraction of our data is shown in Fig. 1 to illustrate the quality of the fit (χ^2/degree of freedom $= 251/206$). Although the data are plotted with statistical error bars, a 10% systematic normalization uncertainty has been added in quadrature for the fits. Integrating the p_T dependence of the data determines the normalization parameters, σ_{inel}^a, in (1). The values obtained are $\sigma_{\text{inel}}^\pi = 20 \pm 5$ mb and $\sigma_{\text{inel}}^K = 13 \pm 4$ mb, which are in reasonable agreement with measured inelastic total cross sections.

The fitted structure functions are displayed in Fig. 2. As expected, the valence functions fall

TABLE II. Results of the recombination-model fit to all the data.

Function	n	b	$\tilde{x}$
V_u^{π}	1.0 ± 0.1	0.75 ± 0.01	0.20 ± 0.01
S_d^{π}	3.5 ± 0.2	0.82 ± 0.08	0.18 ± 0.02
S_s^{π}	2.2 ± 0.2	0.07 ± 0.01	0.02 ± 0.01
V_u^{K}	2.5 ± 0.6	1.02 ± 0.04	0.12 ± 0.02
S_d^{K}	3.6 ± 0.3	0.71 ± 0.21	0.15 ± 0.02
S_s^{K}	7.5 ± 2.6	0.47 ± 0.27	0.05 ± 0.02
${F_u}_{\text{eff}}^{\pi}$	1.2 ± 0.2	0.71 ± 0.07	0.32 ± 0.02
${F_u}_{\text{eff}}^{K}$	2.6 ± 0.3	1.02 ± 0.10	0.28 ± 0.02

less steeply with x than the sea functions. Our pion valence function compares well with the results of an experiment studying pion-produced muon pairs,[12] which are included in Fig. 2. The value, $n_u^{\pi}\simeq 1$, is also in agreement with other experimental results[13] and theoretical predictions.[2,14] In particular, quark-counting rules would predict $n = 2n_{\text{spec}} - 1 = 1$, where n_{spec} is the number of spectator, or left-over, quarks.[15] The indication that the kaon valence function has $n_u^K \simeq 2.5$, and is, therefore, steeper than the pion one, is a new result. The value $n_d \simeq 3.5$ for both the nonstrange sea functions is also in reasonable agreement with the counting rules, which predict either 5 or 3 depending on whether one counts valence and sea spectators, or only valence spectators.[16]

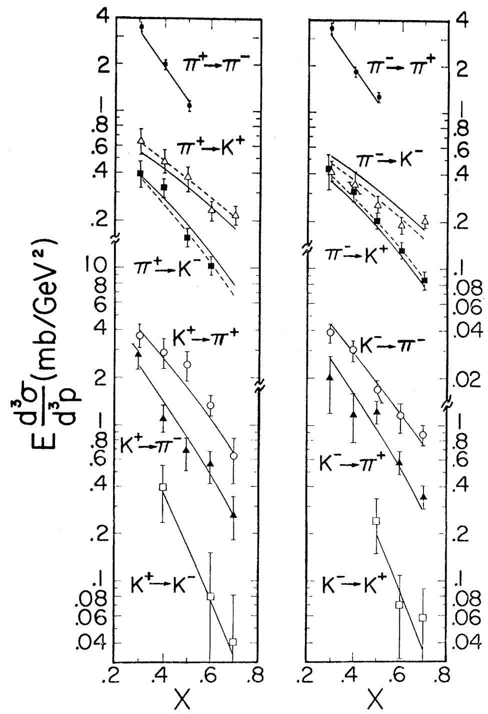

FIG. 1. Invariant cross sections for the reactions $a+p\rightarrow c+X$ at $p_T = 0.3$ GeV/c and $p_{\text{beam}} = 100$ GeV/c. The solid curves are the recombination-model fit to all the data. The dashed curves represent separate fits to the positive- and negative-beam data where they differ from the composite fit.

As in the case of the recombination-model analysis of proton-induced reactions,[4] the sea quarks appear to carry a large fraction of the incident meson's momentum. The total momentum fraction carried by the quarks in the pion is

$$\tilde{x}_{\text{tot}}^{\pi} = 2\tilde{x}_u^{\pi} + 4\tilde{x}_d^{\pi} + 2\tilde{x}_s^{\pi}, \tag{9}$$

where $\tilde{x}_u^{\pi}$ refers only to valence quarks. A similar relation holds for kaons which takes into account the somewhat larger value of x for the strange valence quark. From the results in Table II, $\tilde{x}_{\text{tot}}^{\pi} = 1.2\pm 0.1$ and $\tilde{x}_{\text{tot}}^{K} = 1.0\pm 0.1$. The saturation of the sum rule (9) by the quarks thus suggests that a "turbulent" sea accounts for the momentum usually carried by gluons. However, in addition to the fitted errors for $\tilde{x}_q^{a}$ shown in Table II, there is an overall normalization uncertainty resulting from the use of the condition (7) together with the function V_u^{a} given in (3). Although our data are insensitive to the small-x behavior of V_u^{a}, the assumed factor of $\sqrt{x}$ has a strong influence on the integral in (7). For example, if V_u^{a} were instead parametrized as $x(1-x)^n$, the values of n and the relative values of $\tilde{x}$ would not be significantly affected, but all values of $\tilde{x}$ would be approximately 40% lower.

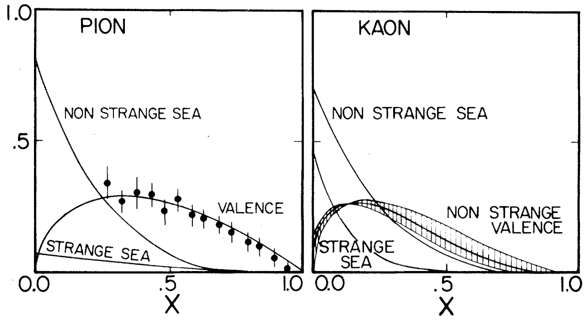

FIG. 2. Pion and kaon structure functions obtained from the recombination-model fit. The data points for the pion valence function are from Ref. 12. A 1-standard-deviation error band is shown for the kaon valence function.

We have also tried many variations of these fits. For example, extending the x range of the fit does not significantly change the results. Fitting the positive- and negative-beam data independently gives results similar to those in Table II, except that the normalization of V_u^π is about 3 standard deviations higher (lower) for the π^+ (π^-) data. The dashed curves in Fig. 1 show the positive- and negative-beam fits to the $\pi \rightarrow K$ channels; the difference between these curves and the composite fit gives rise to the V_u^π difference. Finally, we have reparametrized F_u^a as a single function,

$$F_{u_{\rm eff}}{}^a(x) = b_{u_{\rm eff}}{}^a(1-x)^{n_{u_{\rm eff}}{}^a}, \qquad (10)$$

instead of $V_u^a + S_d^a$. The resulting sea-quark distributions are unchanged, and the fitted parameters of $F_{u_{\rm eff}}{}^a$ are included in Table II.

In conclusion, within the framework of the recombination model, we have extracted information on pion and kaon structure functions from our low-p_T inclusive scattering data. The pion valence function is in agreement with previous determinations, while the kaon nonstrange-valence-quark distribution behaves like $(1-x)^{2.5}$. The nonstrange sea quarks have a $(1-x)^{3.5}$ behavior for both the pion and kaon, and appear to carry most of the momentum of the fragmenting meson.

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