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QUANTUM ELECTRODYNAMICS TESTS IN MUONIC SYSTEMS

E. Zavattini
CERN, Geneva, Switzerland

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Abstract

This paper is a discussion on some experimental tests of the vacuum-polarization contribution in quantum electrodynamics (QED).

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The possibility of testing -- under particularly favourable conditions -- effects, typical of the QED theory , due to the electronic vacuum polarization, by studying the energy levels of muonic atoms, was first suggested by Koslov, Fitch and Rainwater[1] soon after Fitch and Rainwater had discovered the muonic X-rays[2].

Before commenting on some experimental results obtained so far in muonic atom spectroscopy, let me, for clarification, speak briefly of some important new experimental results lately obtained in the field of experimental QED[3].

One considers, generally, an experimental result as a contribution to the experimental QED if this result can be explained theoretically only by including electromagnetic radiative corrections; these are calculated using the standard QED theory which today is a well-defined mathematical procedure by which one is able to compute electromagnetic processes to any order, provided a certain number of constants are given, in particular the fine structure constant α.

Of course, as is advisable, the value of α adopted in this case is the one obtained from experiments outside the experimental QED field. The most recent value of α obtained without QED (α_{WQED}) comes from the measurements of the proton gyromagnetic ratio at low field and the a.c. Josephson effect: this value is[4]

$$\alpha^{-1}_{\mathrm{WQED}} = 137.035987\ (29)\ . \qquad (1)$$

First, let me mention an important result obtained by Van Dyck, Ekstrom and Dehemelt of the University of Washington, who give a new experimental value for the g-factor anomaly of the electron, $a_e = (g_e-2)/2$, which is a purely QED quantity (it represents the deviation of the g_e factor of the electron from its value predicted by the Dirac equation).

The experimental result is[5]

$$a_e = (1159652410 \pm 200) \times 10^{-12}\ . \qquad (2)$$

This value has been obtained by trapping in a magnetic bottle a single electron!

Considering the experimental errors and the uncertainties with which the theoretical value can be computed, the value (2) is in quite good agreement with the value predicted by the QED theory.

It is interesting to know that the uncertainty $\delta a_{e\alpha}$ in the theoretical value due only to the uncertainty of the value of the fine structure α [Eq. (1)] is $\delta a_{e\alpha} = 250 \times 10^{-12}$. Among the various contributions to the value of ae the electronic vacuum polarization terms represent about one part in 10^4 of a_e; this implies that, with this experiment, given the value (1) one can experimentally check these contributions, assuming everything else known, at most to 0.25%, a limit already reached now with Eq. (2). It is clear that if a more precise experimental value for a_e is obtained in the near future, as seems possible[5], then from this, assuming the QED theory one will be able to deduce a more precise value of α.

Another important experimental result is that recently obtained at CERN by Bailey et al.[6] on the g-factor anomaly of the muon a_μ, which is also a purely QED quantity.

The new experimental value is

$$a_\mu = (g_\mu-2)/2 = (1165922000 \pm 9000) \times 10^{-12}\ . \qquad (3)$$

For this experiment, the situation concerning the contributions of the vacuum polarization terms to a_μ is quite different from that for the electron anomaly a_e; in fact, since in this case the average momentum transfer to the muon is $m_\mu c$, these contributions are now relatively large, so that the uncertainty in the value of α_{WQED} here does not impose any limitation. However just because of this high momentum transfer, contributions to a_μ coming from the strong interactions effects begin to have importance, as can be seen from Table 1 [7]. The contribution to a_μ coming from strong interaction effects has been computed using the results from experiments performed at the electron intersecting storage rings[7].

Table 1

The g-factor anomaly of the muon a_μ

$a_\mu \times 10^{12}$		Some of the contributions to a_μ in units of 10^{12}		
Exp. value	Theor. value	Elec. vacuum polarization	Muonic vacuum polarization	Strong interaction
1165922000 ± 9000	1165920600 ± 12000	≈ 6000000	≈ 86000	66700 ± 9400

One sees here that the limitations imposed for a test of the QED theory come mainly from the uncertainty in the strong interaction contributions: this implies that, at present, the vacuum polarization terms can be considered to be experimentally checked, assuming everything else is known, to a level similar to that obtained by the value (2) of a_e but at a momentum transfer two hundred times bigger.

One new important fact, that I wish to emphasize, is that to find agreement between experiment and theory it is necessary to take into account, in this case, also the muonic vacuum polarization, which therefore can be considered tested to a level of at most 25%.

Let us now discuss some of the latest contributions to experimental QED coming from measurements performed on nucleon-lepton bound systems.

Let me first talk about some new results, recently obtained at the CERN Synchro-cyclotron by a CERN-Pisa University Collaboration, on the value S_2 of the energy level difference $S_2 = 2S_{1/2} - 2P_{3/2}$ in the muonic ion $(\mu^{-4}\text{He})^+$: Fig. 1 shows the diagram of the first lower levels of such a system and Table 2 gives the theoretical predictions for S_1 and S_2 according to the analysis of Rinker$^{8)}$.

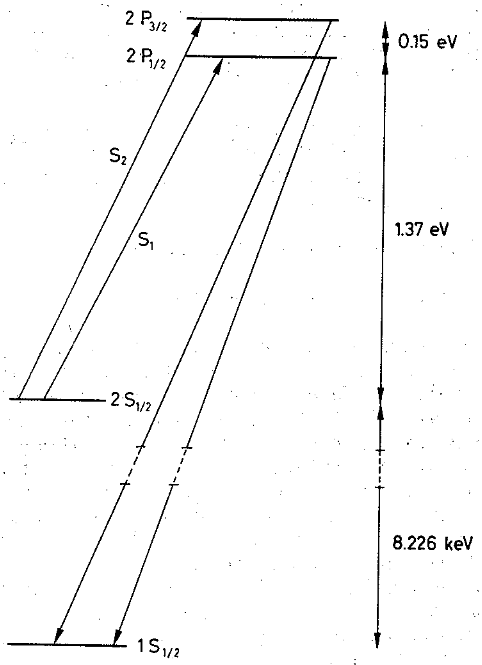

Fig. 1 Schema of the lowest energy levels of the $(\mu^4\text{He})^+$ muonic ion.

In a quite recent analysis, Borie$^{10)}$ confirmed the values found by Rinker. She also calculated the contributions due to the two-photon term and to the hadron vacuum polarization term; these contributions were found to be, respectively, -0.44 and 0.05 MeV (they are not included in Table 2).

The new value for S_2 experimentally found is$^{11)}$

$$S_2^{\text{exp}} = 1527.5 \pm 0.3 \text{ meV} . \qquad (4)$$

This value has been obtained by irradiating $(\mu^4\text{He})^+$ systems, previously prepared in the 2S level, with infrared radiation pulses (20 nsec wide) generated by a suitable dye laser. The value (4) corresponds to that wavelength λ_2 for which the muonic ion $(\mu^{-4}\text{He})^+_{2S}$ has a maximum probability to perform the $2S_{1/2} \to 2P_{3/2}$ transition; the transition is identified by detecting the subsequent prompt 2P → 1S decay by looking at the characteristic 8.2 muonic X-ray emitted essentially in coincidence with the light pulses.

Recently, with new data on electron scattering from ^{4}He, Sick et al.$^{9)}$ have obtained for the ^{4}He r.m.s. charge radius the new value

$$\langle r^2 \rangle^{1/2} = (1.674 \pm 0.012) \text{ fm} . \qquad (5)$$

Using values (4) and (5) we see that the difference D between the theoretical value (taken from Table 2) and the experimental one is

$$D = 0.2 \pm 4.2 \text{ meV} . \qquad (6)$$

The uncertainty on D is mainly due to the uncertainty on the value (5) and defines for D a 95% confidence level. Equation (6) shows that the electronic vacuum polarization contribution to the S_2 difference is experimentally checked here to about 0.25%. The limit obtained here is very near to the one that it is possible to obtain from the measured value of a_e and a_μ; in conditions, however, where the measured quantity is essentially all due to the vacuum polarization effect$^{12)}$. The average momentum transfer in this case is $\alpha(m_\mu c)/2$.

In order to have a direct proof that the experimental value (4) really represents S_2 (and not S_1 as might be, due to some crazy situation), before quitting the experimental floor, the CERN-Pisa Collaboration has also looked for the transition S_1 [of course assuming now the value (4) for S_2 and taking for the $S_2 - S_1$ difference the fine structure term as given by Table 2]. As expected they observed a peak in the right region and, as a preliminary result, they quote the value$^{13)}$

$$S_1^{\text{exp}} = (1381.3 \pm 1) \text{ meV} . \qquad (7)$$

Table 2

Contribution$^{a)}$ to the energy difference between the 2S and 2P levels in the $(\mu^{-4}\text{He})^+$ system (Z = 2). Taken from Ref. 8. Energies in meV; $\langle r^2 \rangle$ charge r.m.s. radius of helium in fm^2.

Contributions		$S_1 = 2S_{1/2} \to 2P_{1/2}$	$S_2 = 2S_{1/2} \to 2P_{3/2}$
Electronic vacuum polarization	Uehling terms $\alpha(\alpha Z)$	1666.1	1666.1
	Källen-Sabry terms $\alpha^2(\alpha Z)$	11.6	11.6
Muon vacuum polarization	$\alpha(\alpha Z)$	-	0.3
Recoil		0.3	0.3
Nuclear polarizability		3.1	3.1
Fine structure		-	145.6
Finite size corrections		$-3.1-102.0 \langle r^2 \rangle$	$-3.1-102.0 \langle r^2 \rangle$
Vertex correction	$\alpha(\alpha Z)$ $\alpha(\alpha Z)^2$	-10.9 -0.2	-10.6 -0.2
Total in meV $^{b)}$		$1666.9-102.0 \langle r^2 \rangle$	$1813.1-102.0 \langle r^2 \rangle$
Total in meV $^{c)}$		1381.7 ± 4.2	1527.3 ± 4.2
Total in Å $^{d)}$		8973.5 ± 21.8	8118 ± 21.8

a) The contribution due to weak interaction is $\approx 2 \times 10^{-5}$ meV; J. Bernabeu et al., CERN TH-1853 (1974).

b) The theoretical uncertainty is ±1.1 meV and comes mostly from the uncertainty of the nuclear polarization contribution.

c) $\langle r^2 \rangle$ taken from Ref. 9; the error given here also takes into account the uncertainty with which $\langle r^2 \rangle$ is given.

d) The natural width of S_1 and S_2 is $\Gamma = 8$ Å.

The error in (7) is relatively large, because of statistics; the Clebsch-Gordan coefficient disfavours the $2S_{1/2} \rightarrow 2P_{1/2}$ transition. Moreover, given the apparatus as it was built, an absolute calibration line, sufficiently near to the value (7), could not be used.

Value (7) confirms (see Table 2), as was already assumed, that indeed value (4) refers to the $2S_{1/2} \rightarrow 2P_{3/2}$ transition.

Accurate measurements of the X-rays emitted in transitions between n states in high-Z muonic atoms also provide a very good test of QED corrections to the energy levels; in particular, as already mentioned, of those corrections due to the electronic vacuum polarization. In these cases the momentum transfer is relatively high, since it is $(m_\mu c)\alpha Z/n$: moreover, in general, for n sufficiently large, effects connected with the nuclear charge radius are negligible.

In the past, a number of measurements were in disagreement with each other and with the theory: now the situation is changed, partly due to the fact that more complete calculations are now available [see article by Mohr[3]] and partly because more accurate experiments have been performed. In particular, Dubler et al. at SIN and Hargrove et al. at SREL have provided very accurate measurements of muonic atom transition energies in Pb, Ba, Sr, Ca, etc., in the 150-450 keV energy region. All these measurements are done by measuring directly the energy of the X-ray emitted, by means of a solid-state detector. All I can say is that these authors state that both the experimental situation and the theoretical one are now enough under control that it is possible to establish a check of the electronic vacuum polarization corrections to the transition energies to a level of 0.2%. I suppose we will hear directly from these authors at this conference about their interesting results.

Before making some final remarks, let me, also for completeness, say what is the situation about the Lamb shift in the hydrogen atom[3]. The different contributions to the Lamb shift, as calculated by Mohr[3,14] are listed in Table 3: the theoretical value S_{th}, after assuming for the r.m.s. $\langle r^2 \rangle^{1/2}$ charge radius of the proton the value

$$\langle r^2 \rangle^{1/2} = 0.87 \pm 0.02 \text{ fm} , \qquad (8)$$

is

$$S_{th} = 1057.888 \pm 0.013 \text{ MHz} , \qquad (9)$$

where the error is the estimated uncertainty for the uncalculated terms.

Table 3

Contributions to the Lamb shift in hydrogen

Correction	Order (mc^2)	Value (MHz)
Self energy	$\alpha(Z\alpha)^4[\ln(Z\alpha)^{-2},1,Z\alpha,\ldots]$	1085.812
Vacuum polarization	$\alpha(Z\alpha)^4[1,Z\alpha,\ldots]$	-26.897
Fourth order	$\alpha^2(Z\alpha)^4$	0.101
Reduced mass	$\alpha(Z\alpha)^4 m/M[\ln(Z\alpha)^{-2},1]$	-1.636
Relativistic recoil	$(Z\alpha)^5 m/M[\ln(Z\alpha)^{-2},1]$	0.359
Nuclear size	$(Z\alpha)^4(R/\lambda)^2$	0.148
Total		1057.888

I wish to make two remarks here:

i) The value (8) for $\langle r^2 \rangle^{1/2}$ lately found by Borkowski et al.[15] is quite different, within the quoted errors, from the value 0.80 accepted earlier (and for quite a long time): the uncertainty induced in (9) by the uncertainty of (8) is 0.007.

ii) There exists another theoretical estimate of the Lamb shift, due to Erickson, which gives, assuming (8) for R, the value[16].

$$S'_{th} = 1057.939 \pm 0.011 \text{ MHz} . \qquad (10)$$

The experimental value obtained by Lundeen and Pipkin[17] is

$$S_{exp} = 1057.893 \pm 0.020 \text{ MHz} . \qquad (11)$$

From the comparison of the value (11) with the theoretical ones, one can say that the electronic vacuum polarization correction to the 2S level of the hydrogen atom is checked (assuming everything else is known) to about 0.3% at least: i.e. one obtains, at present, a limit very similar to the ones obtained above. In this case the average momentum transfer is $\alpha(m_e c)/2$.

In conclusion, we can say that we see that electronic vacuum polarization contributions can be considered well checked to a level of about 0.2-0.3% and this for a very large variety of momentum transfer: it looks an interesting challenge for the muonic atom spectroscopy to go beyond these results.

A possible line of action is the one followed by a group of experimentalists at SIN; that is to look at some specific transition X-rays for the case of muonic atoms with Z around 12, for which the nuclear finite size contribution is negligible and, also, the electron screening effect (which seems to be one of the limitations for the case of high Z muonic atoms) is minimized[18]. This is one of the reasons for which a high resolution bent crystal spectrometer, now operating in the SIN μ channel, was built.

Another way to obtain a better check for the vacuum polarization contributions to the muonic atom energy level is to perform the same type of experiment as the CERN-Pisa Collaboration applied to the muonic helium system, on the neutral muonic atom μp: in this case, the correction due to the finite size is relatively less important and therefore the uncertainty on the proton r.m.s. charge radius gives a smaller uncertainty in the theoretical prediction for the 2S - 2P differences.

This is why a group of physicists at SIN are trying to see if it is possible, under some experimental conditions, to maintain μp systems in a 2S level, in a low-density gas target, for a sufficiently long time.

Table 4

Contribution to the 2S - 2P splitting for the μp muonic atom. The states are indicated as ${}^{2F+1}L_j$, where $F = j$ + nuclear spin I. Energies are given in units of α^2Ry = 0.13461 eV. The disappearance rate of the 2P state is 1.2×10^{11} sec^{-1}; in the given units the width of the 2P level is 0.0006 α^2Ry (which for λ causes a linewidth Γ of 20 Å).

Contributions		Transitions					
		${}^1S_{1/2} \rightarrow {}^3P_{1/2}$	${}^1S_{1/2} \rightarrow {}^3P_{3/2}$	${}^3S_{1/2} \rightarrow {}^1P_{1/2}$	${}^3S_{1/2} \rightarrow {}^3P_{1/2}$	${}^3S_{1/2} \rightarrow {}^3P_{3/2}$	${}^3S_{1/2} \rightarrow {}^5P_{3/2}$
Electronic vacuum polarization	$\alpha(\alpha Z)$	1.5225	1.5225	1.5225	1.5225	1.5225	1.5225
	$\alpha^2(\alpha Z)$	0.0112	0.0112	0.0112	0.0112	0.0112	0.0112
Fine structure		-	0.0625	-	-	0.0625	0.0625
Vertex correction $\alpha(\alpha Z)$		-0.0049	-0.0049	-0.0049	-0.0049	-0.0049	-0.0049
Hyperfine structure		0.1417	0.1135	-0.0845	-0.0280	-0.0563	-0.0337
Finite-size correction a) $= -0.03859 \langle r^2 \rangle$		-0.0292 ± 0.0013	-0.0292 ± 0.0013	-0.0292 ± 0.0013	-0.0292 ± 0.0013	-0.0292 ± 0.0013	-0.0292 ± 0.0013
Total (in α^2Ry)		1.6413	1.6756	1.4151	1.4716	1.5058	1.5284
Total in Å b)		56116 ± 51	54967 ± 51	65086 ± 51	62587 ± 51	61166 ± 51	60261 ± 51

a) Assuming for $\langle r^2 \rangle^{1/2}$ the value (7).

b) Linewidth Γ = 20 Å.

The last published values on the expected 2S - 2P energy differences for the muonic atom μp[19,20] were obtained using the old value for the r.m.s. charge radius of the proton R; in Table 4 are given the new values taking for R the quantity (8).

It is clear, however, that additional experimental information on the proton r.m.s. charge radius would be extremely welcome in order to eliminate any doubt on this very important quantity necessary to describe accurately the lower S levels of the proton-lepton bound systems ep and μp.

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