PHYSICAL REVIEW D **78,** 034004 (2008)

Propagation of field disturbances in Yang-Mills theory

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(Received 11 April 2008; revised manuscript received 2 July 2008; published 7 August 2008)

The propagation of field disturbances is examined in the context of the effective Yang-Mills Lagrangian, which is intended to be applied to QCD systems. It is shown that birefringence phenomena can occur in such systems provided some restrictive conditions, as causality, are fulfilled. Possible applications to phenomenology are addressed.

DOI: 10.1103/PhysRevD.78.034004 PACS numbers: 12.38.Mh, 24.70.+s, 42.25.Lc

I. INTRODUCTION

Small disturbances on nonlinear fields propagate with velocity depending on the polarization states. In general there will be two polarization modes, leading to the existence of two waves propagating with different velocities. This phenomenon is known in the literature as birefringence. In the Maxwell theory (i.e., Abelian gauge field) it can be found when light propagates inside certain material media [1]. It can also appear in the context of nonlinear spin-one fields [2–4], as it occurs in the quantum electrodynamics (QED). The effective Lagrangian for QED was derived long ago [5] for slowly varying but arbitrary strong electromagnetic fields. Its nonlinearities lead to effects like birefringence and photon splitting [2]. Some other investigations on this issue can be found in [6–9].

For the case of non-Abelian gauge theories, the quantum fluctuations lead to a vacuum state which does not coincide with the vacuum coming from the perturbation theories. The structure of the vacuum state was discussed for several models in [10–15]. The one-loop effective action for Yang-Mills theory was presented and discussed in, e.g., [10,11]. For the asymptotically free theory in the regime of large mean fields the effective action is controlled by perturbation theory. In this context the issue of event horizon formation in the physical vacuum associated with color confinement was considered in [16].

The mathematical formalism (see Sec. II for details) to deal with the propagation of small disturbances in nonlinear spin-one fields [2,4] depends on the Lagrangian as a general function of the field invariant. Thus, it can be used to examine the wave propagation in systems governed by a Yang-Mills effective Lagrangian. Particularly, it is worthwhile to analyze if effects like birefringence can occur in this context. Quantum chromodynamics (QCD) could be taken as a specific application, since it presents strong nonlinear properties.

In this manuscript the one-parameter effective Lagrangian presented in [11] is used as a "working model" when discussing Yang-Mills fields. Because of the possibility of two polarization modes presenting different velocities, as derived from our theoretical framework, it is shown that the birefringence phenomena can occur provided that causal conditions are fulfilled. In the QCD case, we discuss how to observe the birefringence phenomena associated with the propagation of small disturbances of the gluon field. Though the gluon is not directly observable due to confinement, a bulk of deconfined hot (and/or dense) quark-gluon matter is expected to exist in the ultrarelativistic heavy ion interactions, as well as in the early phase of the universe [17]. In those cases when the gluon propagates in the quark-gluon matter, it is argued that the birefringence of gluon field leads to local polarization of gluons. The polarization correlation is suggested to be measured in gold-gold collision on the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory and lead-lead collision on the Large Hadron Collider (LHC) at CERN.

The paper is organized as follows. In Sec. II the light cone conditions associated with the propagation of small disturbances in one-parameter spin-one theories are reviewed. In Sec. III the effective Lagrangian for a Yang-Mills field [11] is presented, and the procedure of taking volumetric spatial average is defined. Then we discuss the nontrivial behavior of the phase velocity. The conditions on the causal propagation are stated and some limiting cases from the effective Lagrangian are examined. Sections IV and V are dedicated to the possible applications to phenomenology and the effective geometry issue, respectively. Finally, some final remarks are presented in the conclusion.

The present investigation is considered under the regime of the eikonal approximation, as addressed in [4]. Latin indices run in the range (1, 2, 3) and Greek indices run in the range (0, 1, 2, 3). The Minkowski spacetime is used,

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employing a Cartesian coordinate system. The background metric is denoted by $\eta_{\mu\nu}$, which is defined by $\mathrm{diag}(+1, -1, -1, -1)$. Heaviside nonrationalized units are used and $c = 1 = \hbar$. The completely antisymmetric tensor $\epsilon^{\alpha\beta\mu\nu}$ is defined such that $\epsilon^{0123} = 1$.

II. LIGHT CONE CONDITIONS

A. Field equations for one-parameter spin-one theories

The strength tensor field $F^{(a)}_{\mu\nu}$ and the gauge field $A^{(a)}_{\mu}$ are related by

$$F^{(a)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu + C^{abc} A^{(b)}_\mu A^{(c)}_\nu, \tag{1}$$

where C^{abc} represents the structure constant for a compact Lie group G. This tensor field can be conveniently defined in terms of the (non-Abelian) electric $E^{(a)}_\mu$ and magnetic $H^{(a)}_\mu$ fields as

$$F^{(a)}_{\mu\nu} = V_\mu E^{(a)}_\nu - V_\nu E^{(a)}_\mu - \epsilon_{\mu\nu}{}^{\alpha\beta} V_\alpha H^{(a)}_\beta, \tag{2}$$

where V_μ represents the four-velocity of an observer at rest with respect to the laboratory. In Cartesian coordinates it is given by $V_\mu = \delta^0_\mu$. In order to alleviate the notation, the "color" indices in the upper brackets will be omitted in what follows.

Let us assume the gauge invariant Lagrangian density as a general function of the Lorentz invariant $F \doteq F^{\mu\nu}F_{\mu\nu}$ as $L = L(F)$. From the minimal action principle we get the equation of motion

$$(L_F F^{\mu\nu})_{,\nu} = 0, \tag{3}$$

where a comma denotes partial derivatives with respect to the Cartesian coordinates. L_F represents the derivative of L with respect to the invariant F. L_{FF} is the second derivative. Using the relation $F_{,\nu} = 2F^{\alpha\beta}F_{\alpha\beta,\nu}$ in Eq. (3) we obtain

$$2L_{FF}F^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta,\nu} + L_F F^{\mu\nu}{}_{,\nu} = 0. \tag{4}$$

The field strength $F_{\mu\nu}$ must satisfy the identity

$$F_{\alpha\beta,\lambda} + F_{\beta\lambda,\alpha} + F_{\lambda\alpha,\beta} = 0. \tag{5}$$

Let us now derive the expression for the light cone conditions for this class of theories.

B. The propagation of the field disturbances

In this section we analyze the propagation of waves associated with the discontinuities of the field [18]. Let us consider a surface of discontinuity Σ defined by $\mathcal{Z}(x^\mu) = 0$. Whenever Σ is a global surface, it divides the spacetime in two distinct regions U^- and U^+ ($\mathcal{Z} < 0$ and $\mathcal{Z} > 0$, respectively). Given an arbitrary function of the coordinates, $f(x^\mu)$, we define its discontinuity on Σ as

$$[f(x^\alpha)]_\Sigma \doteq \lim_{\{P^\pm\}\to P} [f(P^+) - f(P^-)] \tag{6}$$

where P^+, P^- and P belong to U^+, U^-, and Σ, respectively. Applying the conditions [18] for the tensor field $F_{\mu\nu}$ and its derivative, we set

$$[F_{\alpha\beta}]_\Sigma = 0 \tag{7a}$$

$$[F_{\alpha\beta,\lambda}]_\Sigma = f_{\alpha\beta}k_\lambda \tag{7b}$$

where $f_{\alpha\beta}$ represents the discontinuities of field on the surface Σ and $k_\mu \doteq (\omega, \vec{k})$ represents the components of the wave 4-vector. The discontinuity of Eqs. (4) and (5) yields, respectively,

$$f_{\beta\lambda}k^\lambda = -\frac{2}{L_F} L_{FF} F_\beta{}^\mu F^{\nu\rho} f_{\nu\rho} k_\mu, \tag{8}$$

$$f_{\alpha\beta}k_\lambda + f_{\beta\lambda}k_\alpha + f_{\lambda\alpha}k_\beta = 0. \tag{9}$$

For the case where $f_{\alpha\beta}$ is the wave propagation tensor given by Eq. (7b), for which Eq. (9) applies, it follows that

$$f_{\alpha\beta} = \sigma(\epsilon_\alpha k_\beta - \epsilon_\beta k_\alpha), \tag{10}$$

where σ is the strength of the wavelet and ϵ_β represents the polarization vector. Working with Eqs. (8)–(10) we obtain the eigenvalue equation

$$Z^\mu{}_\nu \epsilon^\nu = 0, \tag{11}$$

where we defined

$$Z^\mu{}_\nu \doteq \delta^\mu{}_\nu + \frac{4}{L_F k^2} L_{FF} F^{\mu\alpha} F^{\nu\beta} k_\alpha k_\beta, \tag{12}$$

with $k^2 \doteq k^\mu k_\mu$. The eigenvectors of $Z^\mu{}_\nu$ represent the dynamically allowed polarization modes (e_+, e_-). The general solution for the eigenvalue equation is formally given by $\det|Z^\mu{}_\beta| = 0$, and results in the following light cone conditions [2,4,19]:

$$k_+^2 = \gamma F^{\lambda\mu} F^\nu{}_\lambda k^+_\mu k^+_\nu, \qquad \gamma \doteq \frac{4L_{FF}}{L_F} \tag{13}$$

$$k_-^2 = 0, \tag{14}$$

The $\pm$ signs are related with the two possible polarization modes associated with the wave propagation [2]. The existence of these two solutions shows that birefringence effects may appear, provided that $L_{FF}/L_F \neq 0$. In the formalism of geometrical optics it is usually said that generally there will be two rays inside the medium, the ordinary ray (o-ray) and the extraordinary ray (e-ray). The former does not depend on the direction of wave propagation and its velocity is equal to the light velocity in the classical vacuum of electrodynamics. The latter presents an explicit dependence on the direction of propagation. The light cone conditions for two-parameter Lagrangians can be obtained in the same lines. For further details, see Refs. [2,4].

III. WAVE PROPAGATION IN THE YANG-MILLS FIELD

A. The effective Lagrangian

The effective Lagrangian for QCD in terms of the parameter background field F can be presented [11] in the form

$$\mathcal{L}_{\text{eff}} = \frac{1}{4}\frac{F}{\bar{g}(t)^2}, \qquad t \doteq \log\frac{F}{\mu^4} \tag{15}$$

where the effective coupling $\bar{g}(t)$ is implicitly given by

$$t = \int_{g}^{\bar{g}(t)} dg \frac{1}{\beta(g)}, \tag{16}$$

with $\beta(g)$ the Callan-Symanzik β-function and g the gauge field coupling constant appearing in the basic QCD Lagrangian.

In fact, there are many invariants of the Yang-Mills field with the number dependent on the specific gauge group [20]. The ansatz used to derive this effective Lagrangian takes into consideration only the algebraic invariant F and imposes consistency with the trace anomaly for the energy-momentum tensor [21].

For the present proposes, the system described by Eq. (15) is assumed to satisfy the following requirements:

(1) the volumetric spatial average of the color field strength is independent of direction;

(2) it is equally probable that the products E^iE^j and H^iH^j (with $i \neq j$), at any time, take positive or negative values;

(3) there is no net flow of energy as measured by an observer at rest with respect to the system.

The above-mentioned volumetric spatial average of an arbitrary quantity X for a given instant of time τ is defined as

$$\bar{X} \doteq \lim_{V\to V_o} \frac{1}{V}\int X\sqrt{-g}d^3x^i, \tag{17}$$

with $V = \int\sqrt{-g}d^3x^i$, and V_o stands for the time dependent volume of the whole space. A similar average procedure has been already considered in the context of general relativity solutions [22–24].

In terms of the color fields, these requirements imply that

$$\bar{E}_i = 0, \qquad \bar{H}_i = 0, \qquad \overline{E_iH_j - H_iE_j} = 0, \tag{18}$$

$$\overline{E_iE_j} = -\tfrac{1}{3}E^2\eta_{ij}, \tag{19}$$

$$\overline{H_iH_j} = -\tfrac{1}{3}H^2\eta_{ij}, \tag{20}$$

where we have defined $E^2 \doteq -E^iE_i$ and $H^2 \doteq -H^iH_i$. The above average procedure consists of an idealization to deal with systems like a quark-gluon plasma. Nevertheless, it has adequate elements for our discussions. From the point of view of a statistical ensemble, we can assume that the field average over the whole bulk is vanishing compared to the fluctuation, so it is isotropic as a whole, while anisotropic for each local area.

B. Application to the effective Yang-Mills Lagrangian

For the special Lagrangian presented in Eq. (15), the factor γ in Eq. (13) is given by

$$\gamma = \frac{-4}{E^2(Z^2-1)}G(\bar{g}), \tag{21}$$

where we have defined the quantities

$$Z^2 \doteq \frac{H^2}{E^2}, \tag{22}$$

$$G(\bar{g}) \doteq \frac{\bar{g}\,\dot{\bar{g}} - 3\dot{\bar{g}}^2 + \bar{g}\,\ddot{\bar{g}}}{\bar{g}^2 - 2\bar{g}\,\dot{\bar{g}}}, \tag{23}$$

with $\dot{\bar{g}} \doteq \partial\bar{g}/\partial t$.

The phase velocity for the wave perturbation can be obtained from the dispersion relation as $v_e^2 = \omega/|\vec{k}|$, where the index e stands for the e-ray. The o-ray propagates with the light velocity, as determined by Eq. (14). Now using the previous results, we obtain

$$v_e^2 = 1 - \frac{8}{3}\frac{(Z^2+1)G(\bar{g})}{(Z^2-1)+4G(\bar{g})}. \tag{24}$$

In order to guarantee causality, the physical solutions must satisfy the requirement $0 \leq v_e \leq 1$, which implies that

$$0 \leq \frac{8}{3}\frac{(Z^2+1)G(\bar{g})}{(Z^2-1)+4G(\bar{g})} \leq 1. \tag{25}$$

From the analysis of the energy density for the effective action associated with this problem, it can be inferred that the case $E^2 > H^2$ leads to a metastability of the vacuum. The interpretation for this behavior is that if a region in the system develops a large E field, it will decay quickly to a configuration where $H^2 > E^2$ [11]. Therefore, it is adopted here that $H^2 > E^2$ (which means $Z^2 > 1$ and $F > 0$) and the condition stated by Eq. (25) yields

$$0 \leq G(\bar{g}) \leq \frac{3(Z^2-1)}{4(2Z^2-1)}. \tag{26}$$

Now we are going to examine two cases in which the explicit form of the effective coupling was presented in the literature. The first one is obtained when the regime of small coupling is taken into consideration, and the second one was previously proposed in [25] as a suggestion for the case of a large coupling constant.

For the case of small coupling the beta function can be expanded as [25–28]

$$\beta(g) = -\tfrac{1}{2}b_0g^3 + b_1g^5 + \cdots \tag{27}$$

where b_0 and b_1 are constants. Now, taking the limit of large mean fields ($F \to \infty$) we obtain from Eq. (16) that

$$\frac{1}{\bar{g}(t)} = b_0 t - 2\frac{b_1}{b_0}\log t + \cdots \tag{28}$$

Introducing these results in the effective Lagrangian we obtain [11,29]

$$\mathcal{L}_{\rm eff} = \frac{1}{4} b_0 F \log\frac{F}{\mu^4}. \tag{29}$$

For this case the function $G(\bar{g})$ results in

$$G(\bar{g}) = -\frac{1}{2}\frac{1}{t+1}. \tag{30}$$

Since $t > 1$ at the large mean field regime, we conclude that $G(\bar{g}) < 0$, and there will be no propagation associated with the e-ray. The o-ray travels with the unperturbed velocity $v_- = 1$ and does not depend on the direction of propagation.

The nonperturbative expression for the beta function is still unknown. Nevertheless, a suggestion about the strong coupling form of the beta function was presented long ago [25], and is given by

$$\beta(g) = -\frac{a}{2} g, \tag{31}$$

with a a positive constant. If we assume this form for the beta function, we obtain from Eqs. (16) and (31)

$$\frac{1}{\bar{g}(t)^2} = \frac{1}{g^2}\left(\frac{F}{\mu^4}\right)^a. \tag{32}$$

In terms of the parameter t it follows that

$$\bar{g}(t)^2 = g^2 e^{-at}. \tag{33}$$

Introducing these results in the effective Lagrangian we obtain

$$\mathcal{L}_{\rm eff} = \frac{1}{4g^2} F\left(\frac{F}{\mu^4}\right)^a. \tag{34}$$

For this case the function $G(\bar{g})$ results in

$$G(\bar{g}) = -\tfrac{1}{2}a. \tag{35}$$

Thus, since $a > 0$, we conclude that $G(\bar{g}) < 0$, and again there will be no propagation associated with the e-ray. In the both cases, once the superluminal propagation is suppressed, the medium allows just one polarization mode to propagate. So it seems to behave like a polarizer.

IV. OBSERVABLE FOR BIREFRINGENCE

As shown in the previous section, birefringence effects can occur in the Yang-Mills fields. Nevertheless, due to the confinement phenomenon, a direct measurement of these effects on gluons propagating in an external color field seems to be improbable. However, deconfined quark-gluon matter, also known as quark-gluon plasma (QGP), is expected to be produced in the gold-gold interaction at RHIC or lead-lead interaction at LHC. This provides an opportunity to investigate the issue of gluon propagation in QGP systems.

The different velocities in which the field disturbances can propagate are associated with different polarization modes. It can be simply understood as a quantum measurement on the gluon, by which the gluon falls onto the eigenstate of the polarization mode. In the special cases where the e-ray gluon is forbidden by causality, the external field works like a polarizer, and only the o-ray gluon is allowed there. The polarization is assigned by the external field. When one gluon propagates in a QGP, it is "measured" time and time by the local field. Hence, the last polarization direction before its hadronization into hadrons is completely out of control. This will also destroy any global polarization information [30]. However, due to the complete polarization at any local area, the polarization correlation will be very strong. A crucial point is how to identify two particles at the same local area with the same polarization. In order to explore this point let us consider, for simplicity, a "gluon plasma." When the hard parton propagates in the medium, it works as a source of small disturbances on the external field, and could lead to the emission of Cherenkov radiation. The o-gluon can exist within the Cherenkov cone, but the e-gluon must be outside. If the gluon is not confined it can be measured that the polarizations for two gluons inside the cone are parallel. More specifically, $\hat{\epsilon}_1 \cdot \hat{\epsilon}_2 = 1$. The experimental results depend not only on the polarization transfer from the gluon to a certain kind of hadron in hadronization process, but also on the identification of the Cherenkov cone. So, one suggestion is to measure the polarization correlation of two particles of the same kind (2 vector mesons, 2 hyperons, etc.) with almost parallel momenta and within the same jet cone (when the jet can be identified, as expected in LHC). By studying the correlation dependent on the jet cone angle, one may have a way to measure the Cherenkov angle.

One experimental observable for the polarization correlation can be suggested. This can be extracted from the ideal case of two Λ particles with the same polarization $\vec{P}$, with $P = |\vec{P}|$ representing the polarization rate. The conventional way to measure $\vec{P}$ of a single Λ is by measuring the direction vector (denoted as $\hat{p}$) of the momenta of the daughter particles, e.g., proton or pion from the Λ decay, at the rest frame of Λ. Then the angular distribution,

$$\frac{dN}{d\cos\theta} \sim 1 + \alpha\hat{p}\cdot\vec{P} = 1 + \alpha P\cos\theta, \tag{36}$$

can give the information on the polarization. Here α is the hyperon decay parameter. From this equation we see that if the direction of $\vec{P}$ is random, the average of all $\Lambda's$ gives zero, then P is not able to be measured. However, for the two $\Lambda's$ with the same polarization, in the rest frame of *each* Λ, *respectively*, the direction vectors $\hat{p}_1$ and $\hat{p}_2$ can be measured. Then we calculate the expectation value $\langle\hat{p}_1 \cdot$

$\hat{p}_2\rangle$, which results in

$$P = 3\sqrt{\langle \hat{p}_1 \cdot \hat{p}_2 \rangle}/\alpha. \tag{37}$$

Because $\langle \hat{p}_1 \cdot \hat{p}_2 \rangle$ is a SO(3) scalar, it does not depend on the direction of polarization. Thus, it can be averaged for all the jets of all the (QGP) events in order to get the scalar value of averaged polarization from Eq. (37). In an experiment, the Λ pair of the same jet [34] are expected to have the same polarization with larger probability, as discussed above.

V. THE EFFECTIVE GEOMETRY ISSUE

The effective theory approach has long been considered as a possible way to understand confinement phenomena. One of the possible ways to investigate these phenomena consists in the construction of analogue models in which confinement would be related with an event horizon formation, as it occurs in the black hole physics. Such an interpretation was considered in [16,35] (see also the references therein).

The results discussed in this paper can be read in the context of the optical geometry. In this context some comments are in order. Equations (13) and (14) can be presented in the appealing form

$$g_{\pm}^{\mu\nu} k_\mu k_\nu = 0, \tag{38}$$

where we define the two symmetric contravariant tensors

$$g_{+}^{\mu\nu} = \eta^{\mu\nu} - \gamma F^{\lambda\mu} F^{\nu}{}_{\lambda}, \tag{39}$$

$$g_{-}^{\mu\nu} = \eta^{\mu\nu}. \tag{40}$$

The inverse symmetric tensor $g_{\mu\nu}$ is defined in such way that $g^{\mu\alpha} g_{\alpha\nu} = \delta^\mu_\nu$. Therefore, for each propagation vector k_ν, the corresponding tensor $g^{\mu\nu}$ plays the role of an effective metric tensor. Indeed, k_ν is a lightlike (or null) vector with respect to the associated metric tensor. It can be shown [36] that it also satisfies a geodesic equation in terms of the Christoffel symbols $\Gamma^\alpha{}_{\mu\nu}$ associated with this metric. In this way we can refer to k_ν as a geodesic null vector with respect to the effective metric tensor $g^{\mu\nu}$. This geometric interpretation could be used in order to produce an analogue model for confinement based on the possible formation of an event horizon. In this case, the requirement of causal propagation would assume a nontrivial role. Inside the hadron, where quark and gluons can interact and propagate, the velocity of the field disturbances would be expected to be smaller than 1. On the other hand, if the vacuum outside the hadron is described by the effective Lagrangian, the larger than 1 velocity could be interpreted as an indication of confinement, since no physical observable could propagate there.

VI. CONCLUSIONS

In this paper the propagation of field disturbances was investigated in the context of the effective Yang-Mills Lagrangian. The general dispersion relations for one-parameter Lagrangians, Eqs. (13) and (14), were derived employing the method presented in [2,4]. It was shown that birefringence phenomena can occur.

Let us remark on some points. First, it should be stressed that the method depends on the effective Lagrangian as $L(F)$, so the conclusions are quite general. Second, the assumption of causal propagation of the signals sets nontrivial constraint when exploring specific solutions. Finally, for the case of a deconfined quark-gluon system, which naturally provides an effective external field, the birefringence phenomena with gluons and its local polarization effects are expected to be observed at RHIC and LHC by measuring the strong local spin correlations of various hadrons from QGP. The measurements of the spin correlations suggested here, at the same time, can be useful in assigning the details of the effective fields in QGP. This information provides opportunities to develop the effective Lagrangian framework and hence the better understanding of QCD.

ACKNOWLEDGMENTS

This work is partially supported by the Brazilian research agencies CNPq and FAPEMIG (V. A. D. L.); and National Natural Science Foundation of China (NSFC) with Grant No. 10775090, and China Scholarship Council (CSC) (S. Y. L.).

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