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MATCHING UNEQUAL TRANSVERSE EMITTANCES FROM AN H⁻ ION-SOURCE INTO A RFQ

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By using more than two solenoids between an H^- ion-source, having different transverse emittances and a RFQ, it is possible to match a circular symmetric beam into the RFQ with the correct α and β values. Motion in the solenoids is analysed to show that this may be achieved by control of the coupling as well as the focusing and some examples of matching using more than two solenoids are given.

KEY WORDS: Ion source, radio-frequency devices, solenoid, matching

1 INTRODUCTION

A number of solutions have been proposed and tried for matching the output of an ion source to the input of a RFQ.[1–15] One of the most popular solutions uses only two solenoids.[9–15] The solenoids may be adjusted easily to provide the required match for α and β into the RFQ, but this solution assumes the emittances (phase space area/π) and the input beam parameters in the two transverse phase planes are equal. H^- Ion sources of the Magnetron and Penning type may have significantly different transverse emittances and matching into the RFQ is then more difficult since, at the input to the RFQ, the same values of α and β are required in the two phase planes together with the same beam size.[8] It is shown that by using more than two solenoids and controlling the coupling as well as the focusing it is possible to obtain a solution to the matching problem for unequal initial transverse emittances. Control of the coupling results in equal transverse emittances at the input to the RFQ and control of the focusing, equal values for both α and β in the two planes. Since to first order the coupling is linear there is no change to the hyper-volume of the four dimensional hyper-ellipsoid and the system could after subsequent linear transformation be 'uncoupled' again. The analysis assumes a space charge neutralised beam, but the effect of space charge is discussed and the principle for matching should still apply in the presence of linear space charge.

2 TRANSFORMATION OF A HYPER-ELLIPSOID IN A SOLENOID

The first order transformation of a four dimensional beam hyper-ellipsoid through a solenoid is initially studied to analyse the effect on coupling.

The hyper-ellipsoid may be represented in matrix form by:

$$[xx'yy'][\sigma]_0^{-1}\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix} = 1$$

where $[\sigma]_0$ gives the coefficients of the ellipsoid. This may be transformed by a linear matrix $[m]$ to another ellipsoid with coefficients given by $[\sigma]_1$, where:

$$[\sigma]_1 = [m][\sigma]_0[m]^T$$

Helm[16] obtains a description for the first order motion through a solenoid:

$$[s] = \begin{bmatrix} C^2 & \dfrac{SC}{K} & SC & \dfrac{S^2}{K} \\ -KSC & C^2 & -KS^2 & SC \\ -SC & \dfrac{-S^2}{K} & C^2 & \dfrac{SC}{K} \\ KS^2 & -SC & -KSC & C^2 \end{bmatrix}$$

where,

$$\begin{aligned} L &= \text{effective length of solenoid} \\ K &= \frac{B_0}{2B\rho} = \frac{\text{field inside solenoid}}{2 \times \text{momentum / electronic charge}} \\ C &= \cos(KL) \\ S &= \sin(KL) \end{aligned}$$

The matrix [s] may be shown to be a combination of linear focusing at the ends of the solenoid together with an axial rotation within the solenoid field.

$$[s] = \begin{bmatrix} C & 0 & S & 0 \\ 0 & C & 0 & S \\ -S & 0 & C & 0 \\ 0 & -S & 0 & C \end{bmatrix}\begin{bmatrix} C & S/K & 0 & 0 \\ -KS & C & 0 & 0 \\ 0 & 0 & C & S/K \\ 0 & 0 & -KS & C \end{bmatrix}$$

The rotation within the solenoid produces coupling that may be used to control the emittance.

In terms of the initial values of the Courant Snyder parameters and the initial emittances the transformation of the hyper-ellipsoid through the rotation matrix gives:

$$[\sigma]_1=\begin{bmatrix} C & 0 & S & 0 \\ 0 & C & 0 & S \\ -S & 0 & C & 0 \\ 0 & -S & 0 & C \end{bmatrix}\begin{bmatrix} \varepsilon_x\beta_x & -\varepsilon_x\alpha_x & 0 & 0 \\ -\varepsilon_x\alpha_x & \varepsilon_x\gamma_x & 0 & 0 \\ 0 & 0 & \varepsilon_y\beta_y & -\varepsilon_y\alpha_y \\ 0 & 0 & -\varepsilon_y\alpha_y & \varepsilon_y\gamma_y \end{bmatrix}_0\begin{bmatrix} C & 0 & -S & 0 \\ 0 & C & 0 & -S \\ S & 0 & C & 0 \\ 0 & S & 0 & C \end{bmatrix}$$

The final emittances may be obtained from the coefficients of the $[\sigma]_1$ matrix:

$$\varepsilon_{x_1}^2 = \left(\sigma_{11}\sigma_{22} - \sigma_{21}^2\right)_1$$

$$\varepsilon_{y_1}^2 = \left(\sigma_{33}\sigma_{44} - \sigma_{43}^2\right)_1$$

Substituting for the $[\sigma]_1$ coefficients in terms of the transformed $[\sigma]_0$ one obtains:

$$\varepsilon_{x_1}^2 = \varepsilon_{x_0}^2 C^4 + \varepsilon_{y_0}^2 S^4 + \varepsilon_{x0}\varepsilon_{y0} C^2 S^2 \left(\gamma_{y0}\beta_{x0} + \gamma_{x0}\beta_{y0} - 2\alpha_{x_0}\alpha_{y0}\right)$$

$$\varepsilon_{y_1}^2 = \varepsilon_{x_0}^2 S^4 + \varepsilon_{y_0}^2 C^4 + \varepsilon_{x0}\varepsilon_{y0} C^2 S^2 \left(\gamma_{y0}\beta_{x0} + \gamma_{x0}\beta_{y0} - 2\alpha_{x_0}\alpha_{y0}\right)$$

which gives the final emittances in terms of the initial emittances, the rotation angle and the values of the input parameters α, β, and γ in the two planes. The inconstancy of 2-dimensional emittances in a coupled system is also shown in a more general treatment of multi-dimensional beam emittance by Buon.[17]

3 CONTROL OF COUPLING

From the previous section it may be seen that by controlling the amount of rotation within a series of solenoids it is possible to have control over the coupling and final emittances in the transverse phase planes.

For the condition of equal initial emittances, $\varepsilon_{x_0} = \varepsilon_{y_0} = \varepsilon$, the final emittances are equal for all angles of rotation,

$$\varepsilon_{x_1}^2 = \varepsilon_{y_1}^2 = \varepsilon^2\left[S^4 + C^4 + C^2 S^2 \left(\gamma_{y_0}\beta_{x_0} + \gamma_{x_0}\beta_{y_0} - 2\alpha_{x_0}\alpha_{y_0}\right)\right],$$

but the value depends on the initial conditions and the rotation angle. If in addition $\alpha_{x_0} = \alpha_{y_0} = \alpha$ and $\beta_{x_0} = \beta_{y_0} = \beta$ and since $\beta\gamma - \alpha^2 = 1$ then, for all values of rotation

$$\varepsilon_{x_1} = \varepsilon_{y_1} = \varepsilon$$

These are the initial conditions to obtain a match with only two solenoids.

For the condition of unequal initial emittances, consider a rotation of $(n + \frac{1}{2})\pi$, $(n = 0, 1, 2 \ldots)$, then:- $C^2 = 0$, $S^2 = 1$,

$$\varepsilon_{x_1} = \varepsilon_{y_0} \; ; \quad \varepsilon_{y_1} = \varepsilon_{x_0}$$

and the emittances are interchanged between the two planes.

For a rotation of $n\pi$, $(n = 0, 1, 2 \ldots)$, then:- $C^2 = 1$, and $S^2 = 0$,

$$\varepsilon_{x_1} = \varepsilon_{x_0} \; ; \quad \varepsilon_{y_1} = \varepsilon_{y_0}$$

and the emittances are unchanged.

For the case where the rotation is $(2n + 1)\frac{\pi}{4}$, $(n = 0, 1, 2 \ldots)$, then:- $C^2 = \frac{1}{2}$, and $S^2 = \frac{1}{2}$,

$$\varepsilon_{x_1}^2 = \frac{1}{4}\left(\varepsilon_{x0}^2 + \varepsilon_{y0}^2 + \varepsilon_{x0}\varepsilon_{y0}\left(\gamma_{y0}\beta_{x0} + \gamma_{x0}\beta_{y0} - 2\alpha_{x_0}\alpha_{y_0}\right)\right)$$

$$\varepsilon_{y_1}^2 = \frac{1}{4}\left(\varepsilon_{x0}^2 + \varepsilon_{y0}^2 + \varepsilon_{x0}\varepsilon_{y0}\left(\gamma_{y0}\beta_{x0} + \gamma_{x0}\beta_{y0} - 2\alpha_{x_0}\alpha_{y_0}\right)\right)$$

and the bracketed terms are identical so the final emittances are equal, irrespective of the initial emittances and the initial conditions. The actual value of the final emittance in the two planes depends on the initial emittances and the initial values of α, β and γ. The final emittances are equal and a minimum if:

$$\gamma_{y0}\beta_{x0} + \gamma_{x0}\beta_{y0} - 2\alpha_{x_0}\alpha_{y_0} = 0$$

It is not clear that such a condition can be obtained, but for a low value of the final emittance it is clear that the initial values of α_{x_0} and α_{y_0} should be of the same sign. If $\alpha_{x_0} = \alpha_{y_0} = \alpha$ and $\beta_{x_0} = \beta_{y_0} = \beta$ and since $\beta\gamma - \alpha^2 = 1$ then for these restricted conditions, the final emittance is equal to the average value of the initial emittances,

$$\varepsilon_{x_1} = \varepsilon_{y_1} = \frac{\varepsilon_{x_0} + \varepsilon_{y_0}}{2}$$

The change in the two transverse emittances as a function of the rotation angle may be seen from the results shown in Figure 1, for the examples listed in Table 1 using four solenoids to match to a RFQ. In all the cases studied using more than two solenoids, the matching condition was obtained for a total rotation angle of 0.75π. It may be seen from Figure 1 that for this condition the emittances are equal. They are also equal for a rotation angle of 0.25π, but emittances are interchanged for a rotation angle of 0.5π. Since to first order the coupling described above is linear, there is no change to the hyper-volume of the four dimensional hyper-ellipsoid and the system could after subsequent linear transformation be 'uncoupled' again.

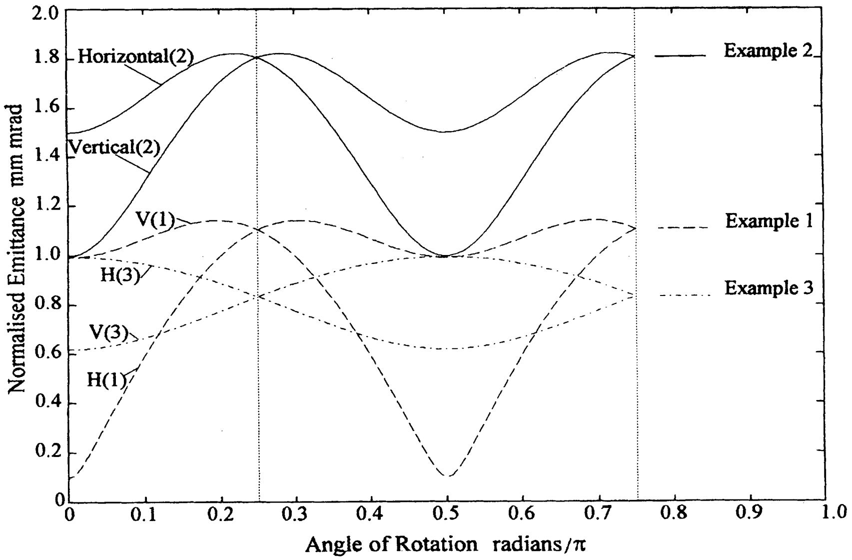

FIGURE 1: Plot showing change of emittance in the two transverse phase-planes as a function of rotation angle for three examples of matching using four solenoids.

The analysis presented is to first order and assumes a space-charge neutralised beam. The effect of space charge to first order is to defocus the beam, which can be counteracted by suitable adjustments of the solenoid fields. Therefore, the general principle for matching should still apply in the presence of linear space charge.

4 MATCHING USING MORE THAN TWO SOLENOIDS

In principle only one extra solenoid is needed to control the coupling since the matching requirements into the RFQ are axisymmetric. It was found that in the examples considered below, it was not possible to find solutions using three solenoids equally distributed along the beam transfer line, but this may be connected with the algorithm used for convergence in the computer code. By using four solenoids symmetrically placed in pairs towards each end of the drift space it was possible to find solutions in all cases. Using the results from the four solenoid solution and coupling the last two solenoids together to form one solenoid it has then been possible to obtain a three solenoid solution, see Table 2. In the examples studied, the use of more than two solenoids had the added advantage that the first solenoid in the matching system, the one close to the ion source, had a very low field requirement, which should make magnetic shielding between the source and solenoid easier.

The results for a number of examples using more than two solenoids to match between an ion source and a RFQ are given in Table 1. The computer program TRANSPORT,[19] modified to calculate emittances and the Courant-Snyder parameters, was used to calculate

TABLE 1:

Parameter	Example 1 ref. 18 Penning source		Example 2 ref. 14 Magnetron source		Example 3 ref. 15 Magnetron source	
	Initial	Final	Initial	Final	Initial	Final
α_x	−13.9212	0.569	−6.327	0.569	−1.4045	0.569
β_xm	9.6677	0.0206	1.057	0.0206	0.349	0.0206
$(\varepsilon_x)_n$ μm rad	0.1	1.106	1.5	1.81	1.0	0.83
$\hat{x}$ mm	12.5	1.92	16.0	2.45	7.5	1.67
$\hat{x}'$ mrad	18.0	107.03	97.0	137.03	37.0	93.13
α_y	−3.1136	0.569	−4.147	0.569	−2.589	0.569
β_y m	0.9733	0.0206	1.046	0.0206	0.562	0.0206
$(\varepsilon_y)_n$ μm rad	1.0	1.106	1.0	1.81	0.62	0.83
$\hat{y}$ mm	12.5	1.92	13.0	2.45	7.5	1.67
$\hat{y}'$ mrad	42.0	107.03	53.0	137.03	37.0	93.13

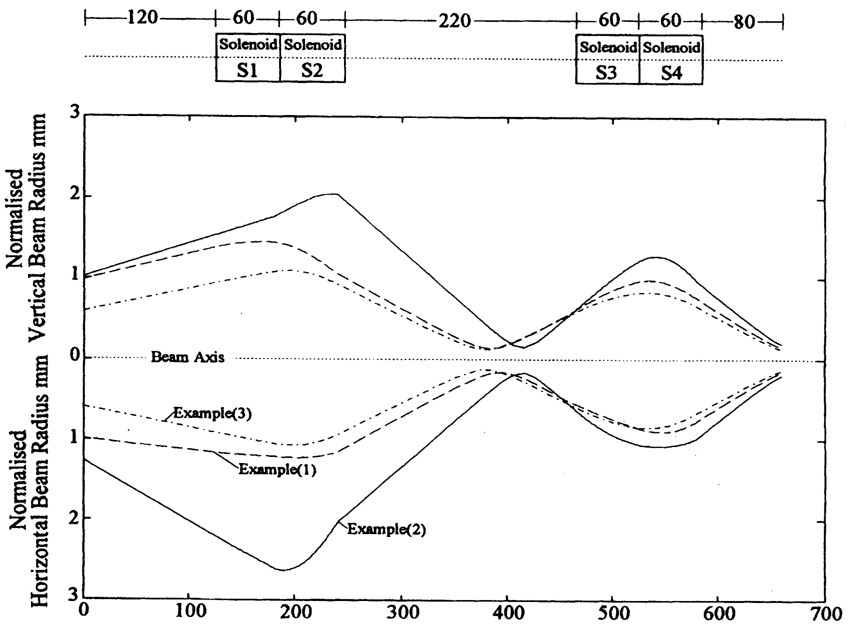

FIGURE 2: Beam profiles for three examples using four solenoids to match to a RFQ, starting with unequal initial emittances.

TABLE 2:

Solenoid Field	Example 1		Example 2		Example 3	
T	3 Sol	4 Sol	3-Sol	4 Sol	3 Sol	4 Sol
S1	0.127311	0.223131	0.045124	0.070949	0.101947	0.115951
S2	0.438215	0.378430	0.460159	0.443298	0.479374	0.463242
S3	0.478945	0.327720	0.509070	0.393348	0.471047	0.397220
S4	0.478945	0.594120	0.509070	0.615820	0.471047	0.547004

the matched solutions. The output parameters for α and β, the drift lengths and the total length of the solenoids are the same for all three examples and are taken from reference 15. The input parameters are taken or estimated from the references shown in Table 1, but for reference 15 the effect of unequal emittances is explored assuming the emittances are in a similar ratio to reference 14. The beam profiles through the system using four solenoids to produce a match, are plotted in Figure 2 for each of the examples listed in Table 1. The solenoid fields required to give the matched solutions are given in Table 2. The solutions with 'three' solenoids also produce a rotation of 0.75π to give a match and have small differences in beam profiles in comparison with the four solenoid solutions. However, the differences in profiles are within 5% of the four solenoid solutions and occur within the solenoids and the drift space between the pairs of solenoids.

The examples show matching to a RFQ for a wide range of different input parameters. From the results it may be seen that for the input conditions of Examples 1 and 2, the final emittances are equal at a value only slightly larger than the maximum initial emittance. For Example 3 the final emittances are equal at a value that lies between the initial horizontal and vertical emittances. On substituting the values for the initial conditions it is found that Example 3 is closest to meeting the condition for minimum emittance with:

$$\gamma_{y0}\beta_{x0} + \gamma_{x0}\beta_{y0} - 2\alpha_{x0}\alpha_{y0} = 2.298$$

whereas, Examples 1 and 2 have values of 39.149 and 6.517 respectively. This is reflected in the final emittance of Example 3 being close to the average value of the initial emittances and only a factor 1.4 larger than the minimum compared with factors of 2.2 and 2.0 respectively for Examples 1 and 2.

5 CONCLUSION

By using more than two solenoids to control coupling as well as focusing, it is possible to obtain a good match between an ion source and a RFQ even with different initial emittance values in the two transverse phase planes.

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