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THE DECOHERENCE AND RECOHERENCE OF THE BETATRON OSCILLATION SIGNAL AND AN APPLICATION

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The understanding of the strongly synchrotron frequency modulated collective betatron oscillation signal is achieved by both an analytical result and computer simulations. Those theoretical results agree well with the measurement data. One of the applications is the measurement of the value of the chromaticity times energy spread.

1. INTRODUCTION

It is observed in many accelerators and storage rings that after a horizontal kicker is fired to excite a single-bunch beam, the envelope of the betatron oscillation from a horizontal pickup is strongly modulated by the synchrotron frequency; see [Fig. 1(a), (b)]. This can be understood qualitatively as being due to the combined effects of the synchrotron motion and the machine chromaticity.[1]

The phenomenon of decoherence and later recoherence of collective oscillations also occurs in plasma waves,[2] where it is called an echo. If we apply an external disturbance to the plasma, the macroscopic field produced will decay through Landau damping (decoherence), but the disturbance in the microscopic motions remains. If we apply a second disturbance, the microscopic motions may later recohere, and a macroscopic field (the echo) may reappear in the plasma many Landau-damping periods after the application of the second disturbance. In the case of synchrobetatron oscillations in an accelerator, the phase adjustments that cause recoherence are due not to a second external disturbance, but to the effects of synchrotron oscillations on the betatron frequencies.

We study this phenomenon by pulsing an injection kicker magnet to a 400-MeV single-bunch stored beam in Aladdin, which is a 1-GeV electron storage ring serving as a light source. After the kicker pulse dies out, the beam will execute a coherent free betatron oscillation. A beam position monitor electrode which can sense the average position of the bunch is used to pick up the beam signal at each turn. After the detailed study of the phenomenon, we will find that the value of the product of the chromaticity and the energy spread can be measured from these signals. This provides a new method for the measurement of the

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chromaticity or the energy spread, as soon as we get one of the two values by traditional methods.[3,4]

2. THEORY

To study the theory of this phenomenon, we assume that every electron executes a free betatron oscillation with tune ν_i. Due to the original synchrotron oscillation, each electron has an energy difference ΔE_i, with respect to the synchronous electron. If the horizontal chromaticity ξ is not zero, then electrons with different energies will execute free betatron oscillations with different ν_i:

$$\nu_i = \nu_0 + \xi \frac{(\Delta P)_i}{P} = \nu_0 + \xi \frac{(\Delta \hat{P})_i}{P} \cos(\nu_s \omega t + \psi_i), \tag{1}$$

where P is the momentum of the synchronous electron, $(\Delta \hat{P})_i$ is the momentum oscillation amplitude of the electron i, ν_s is the synchrotron oscillation tune, ω is the angular revolution frequency, and ψ_i is the phase of the momentum oscillation of the electron i.

After some number of betatron oscillation periods, the signal from the pickup becomes smaller because electrons are no longer in phase and the displacement of the center of charge, $\bar{D}$, becomes smaller:

$$\bar{D}(t) = \frac{1}{N} \sum_{i=1}^{N} A \cos\left(\int_0^t \nu_i(t')\omega \, dt' + \phi\right), \tag{2}$$

where N is the total number of electrons. Notice here that the initial phase ϕ is the same for all electrons since they are given the same transverse impulse at the same time.

After one-half of a synchrotron oscillation period, the betatron oscillations appear very incoherent. But, if nonlinear effects are negligible, after one synchrotron period all the electrons will be in phase again and the displacement of the center of charge will be almost the same as just after the kicker was fired (except for a very small damping).

To show this, we substitute Eq. (1) into Eq. (2) and expand it. We get:

$$\begin{aligned}\bar{D}(t) = \frac{A}{N} \sum_{i=1}^{N} \Big\{ &\cos(\nu_0 \omega t + \phi) \cos\left[\frac{\xi(\Delta \hat{P})_i}{\nu_s P}(\sin(\nu_s \omega t + \psi_i) - \sin \psi_i)\right] \\ &- \sin(\nu_0 \omega t + \phi) \sin\left[\frac{\xi(\Delta \hat{P})_i}{\nu_s P}(\sin(\nu_s \omega t + \psi_i) - \sin \psi_i)\right]\Big\}.\end{aligned} \tag{3}$$

In an electron storage ring, due to the quantum excitation effect, $(\Delta \hat{P})_i$ is Gaussian-distributed with mean value 0, and ψ_i is independently uniformly distributed between $-\pi$ and π. Because the sine function is an odd function, we can drop the last term in the above equation. It then becomes:

$$\begin{aligned}\bar{D}(t) &= \frac{A}{N} \sum_{i=1}^{N} \left\{ \cos(\nu_o \omega t + \phi) \cos\left[\frac{\xi(\Delta \hat{P})_i}{\nu_s P}(\sin(\nu_s \omega t + \psi_i) - \sin \psi_i)\right]\right\} \\ &= A \cos(\nu_0 \omega t + \phi)\mathscr{E}(t),\end{aligned} \tag{4}$$

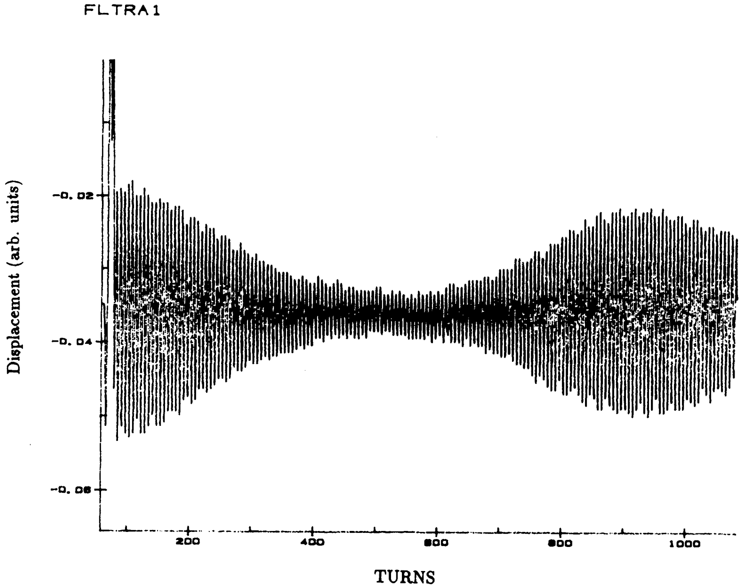

Fig. 1(a)

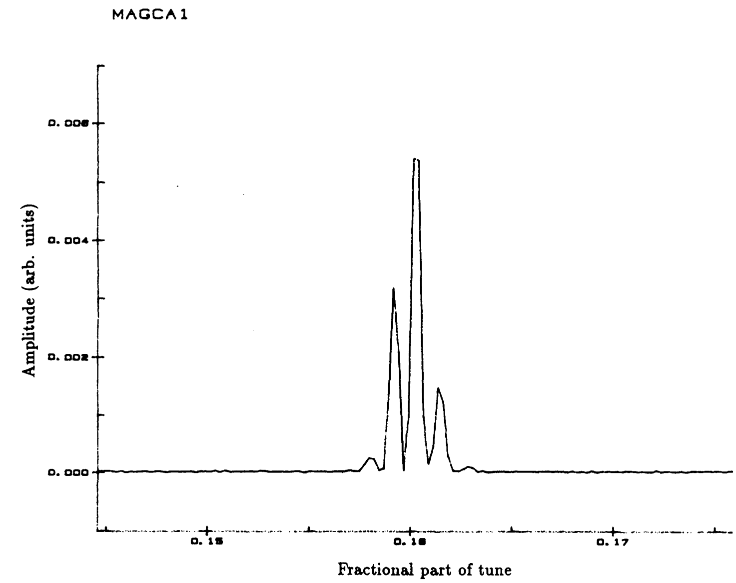

Fig. 1(b)

FIGURE 1 The measurement result for the case of non-zero chromaticity: (a) turn-by-turn signal; (b) the FFT of (a).

where

$$\mathscr{E}(t)=\frac{1}{N}\sum_{i=1}^{N}\cos\left[\frac{\xi(\Delta\hat{P})_i}{\nu_s P}(\sin(\nu_s\omega t+\psi_i)-\sin\psi_i)\right] \tag{5}$$

is the "envelope function", which is a slow modulation of the betatron oscillation $A\cos(\nu_0\omega t+\phi)$.

Now we look at Eq. (5); whenever $\nu_s\omega t=2n\pi$, $\mathscr{E}$ reaches its maximum value, and whenever $\nu_s\omega t=(2n+1)\pi$, $\mathscr{E}$ reaches its minimum value. This is exactly the reason for the synchrotron frequency modulation we see in Fig. 1. The values of $\mathscr{E}_{\max}$ and $\mathscr{E}_{\min}$ are

$$\mathscr{E}_{\max}=\frac{1}{N}\sum_{i=1}^{N}\cos\left[\frac{\xi(\Delta\hat{P})_i}{\nu_s P}(\sin(2n\pi+\psi_i)-\sin\psi_i)\right]=1 \tag{6}$$

$$\begin{aligned}\mathscr{E}_{\min}&=\frac{1}{N}\sum_{i=1}^{N}\cos\left[\frac{\xi(\Delta\hat{P})_i}{\nu_s P}(\sin((2n+1)\pi+\psi_i)-\sin\psi_i)\right]\\&=\frac{1}{N}\sum_{i=1}^{N}\cos\left[\frac{\xi(\Delta\hat{P})_i}{\nu_s P}(-2\sin\psi_i)\right].\end{aligned} \tag{7}$$

By replacing summation with integration and $(\Delta\hat{P})_i/P$ with $(\Delta\hat{E})_i/E$, we get:

$$\mathscr{E}_{\min}=\frac{1}{2\pi}\int_{-\pi}^{\pi}d\psi\int_{-\infty}^{\infty}dx\cos\left(-2\frac{\xi}{\nu_s}\frac{\sigma_\epsilon}{E}x\sin\psi\right)\frac{1}{\sqrt{2\pi}}e^{-x^2/2} \tag{8}$$

$$\equiv F\left(\frac{\xi}{\nu_s}\frac{\sigma_\epsilon}{E}\right). \tag{9}$$

From the above discussion we can also see the disappearance of the decoherence phenomenon when the chromaticity is set to zero. That is the case in Fig. 2. The universal function F is only dependent on the value of $(\xi/\nu_s)(\sigma_\epsilon/E)$ and the distribution of the charged particles. Therefore if we are dealing with a proton beam, we should replace the Gaussian distribution in Eq. (8) by the parabolic distribution usually assumed for a proton beam. Fig. 5 is a plot of the universal function F for the electron beam case.

The ratio of $\bar{D}_{\max}$ and $\bar{D}_{\min}$ is:

$$\begin{aligned}\frac{\bar{D}_{\min}}{\bar{D}_{\max}}&=\frac{A\cos(\nu_0\omega t_{\min}+\phi)\mathscr{E}_{\min}}{A\cos(\nu_0\omega t_{\max}+\phi)\mathscr{E}_{\max}}\\&=\frac{\cos(\nu_0\omega t_{\min}+\phi)}{\cos(\nu_0\omega t_{\max}+\phi)}F\left(\frac{\xi}{\nu_s}\frac{\sigma_\epsilon}{E}\right),\end{aligned} \tag{10}$$

where

$$t_{\min}=\frac{(2n+1)\pi}{\nu_s\omega}$$

and

$$t_{\max}=\frac{2n\pi}{\nu_s\omega}.$$

If we look at only the envelope of the oscillation signal, we can drop the fast

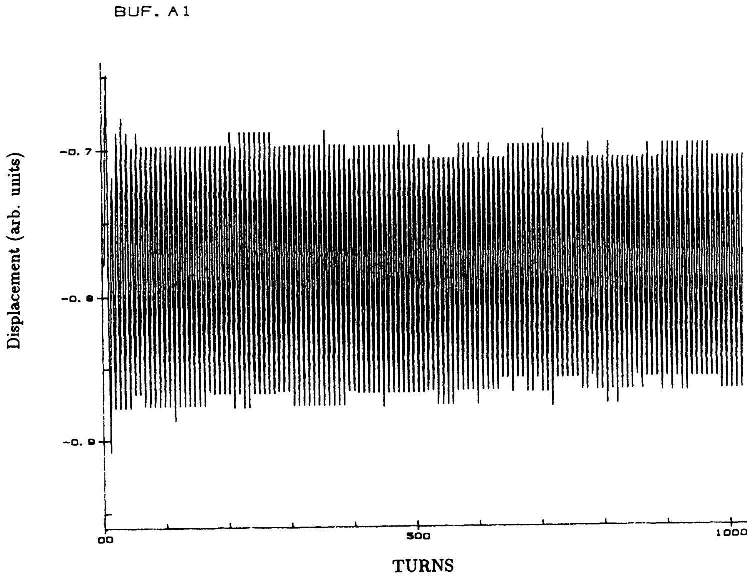

Fig. 2(a)

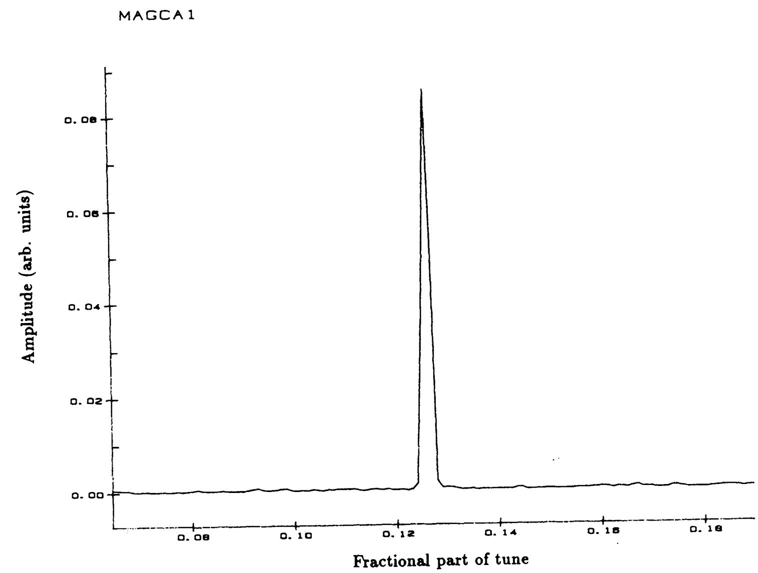

Fig. 2(b)

FIGURE 2 The measurement result for the case of zero chromaticity: (a) turn-by-turn signal; (b) the FFT of (a).

betatron oscillation part. Therefore we get

$$\frac{\tilde{\tilde{D}}_{\min}}{\tilde{\tilde{D}}_{\max}}=F\left(\frac{\epsilon}{\nu_s}\frac{\sigma_\epsilon}{E}\right). \tag{11}$$

In the above equation $\tilde{\tilde{D}}$ means we only look at the envelope of the oscillation signal. In this case, we can measure the energy spread, σ_ϵ, by another method[3] first, and by measuring the value of $\tilde{\tilde{D}}_{\min}/\tilde{\tilde{D}}_{\max}$ we can get the value of chromaticity or vice versa. One remark before ending this section is that if there exists some non-negligible tune spread due to nonlinear fields, then after the decoherence the echo amplitude will be reduced and will affect the accuracy of $\tilde{\tilde{D}}_{\min}/\tilde{\tilde{D}}_{\max}$. This can be easily checked by observing that the second maximum oscillation amplitude is as large as the first.

3. SIMULATION RESULT

Figure 3 and Fig. 4 are the results of computer simulations using Eq. (1) and Eq. (2) and the fast Fourier analysis spectrum of the results with both the cases of $\xi=0$ and $\xi\neq 0$. In those simulations $(\Delta\hat{P})_i/P$ is Gaussian distributed with mean value 0 and standard deviation σ_ϵ/E, and ψ_i is uniformly distributed between $-\pi$ and π. As we described in the previous section, the Gaussian distribution is only valid for the high-energy electron case and is due to the quantum excitation effect.

In Table I, we compare $\{\xi(\sigma_\epsilon/E)\}_{\text{theory}}$, which we got by measuring $\tilde{\tilde{D}}_{\min}/\tilde{\tilde{D}}_{\max}$ from the simulation result, with the product value of ξ and σ_ϵ/E (which we put into the simulation) for several different values of chromaticity and energy spread. The last column is the percentage error which is calculated as

$$\frac{\left\{\xi\dfrac{\sigma_\epsilon}{E}\right\}_{\text{theory}}-(\xi)\times\left(\dfrac{\sigma_\epsilon}{E}\right)}{(\xi)\times\left(\dfrac{\sigma_\epsilon}{E}\right)}.$$

In the above example, since $\tilde{\tilde{D}}_{\min}/\tilde{\tilde{D}}_{\max}$ is measured from the simulation result, the accuracy can be very high. Therefore, the major error is due to the fact that we only use a finite number of particles to simulate the Gaussian distribution. For the results in Table I, we used 10,000 particles in the simulation. If we use, for example, 5000 particles, the 1.0% error in the third row will go up to 3.9%. In a real measurement using the proposed method, the major error will be due to the inaccuracy of the measurement of $\tilde{\tilde{D}}_{\min}/\tilde{\tilde{D}}_{\max}$. This has been discussed in a paper reporting measurements of energy spread by the proposed method.[5] In that paper the authors compare the energy spread measured by the bunch length measurement method and the proposed method. Since the beam current in that measurement was well below the microwave instability, the energy spread should be a constant for different chromaticities and for different currents. The authors compare the constancy of the energy spread for different current and different

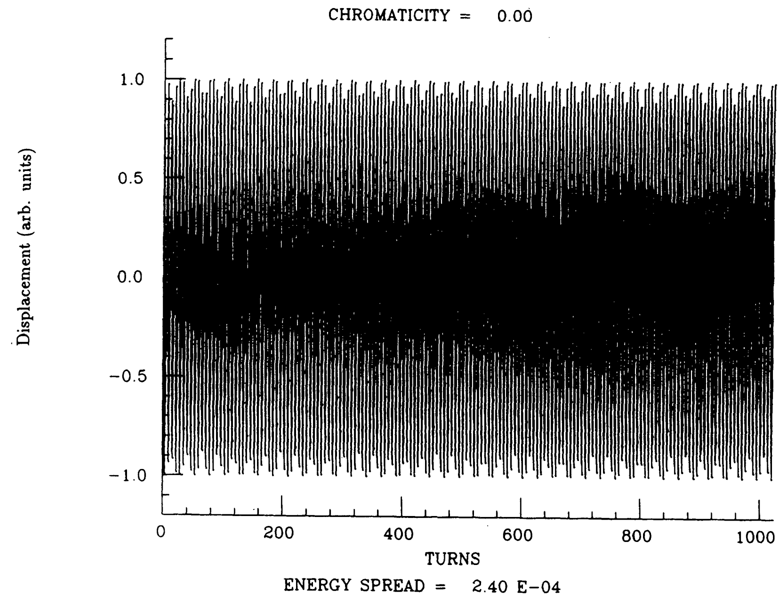

Fig. 3(a)

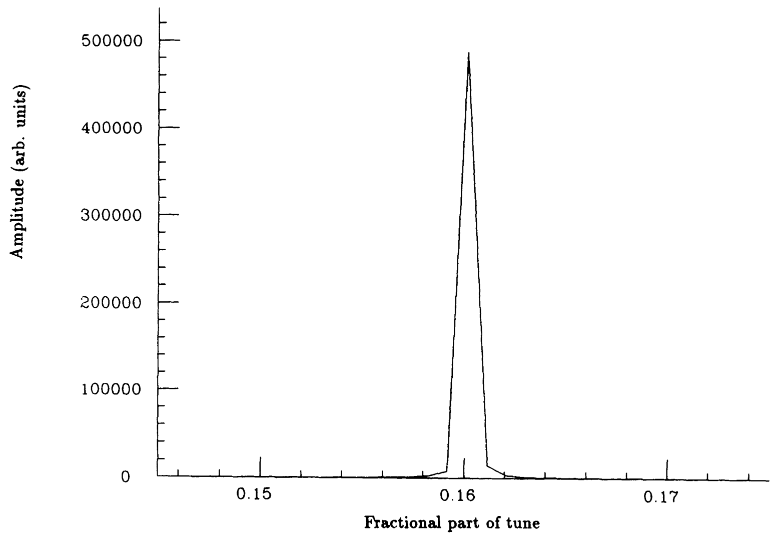

Fig. 3(b)

FIGURE 3 The simulation result for the case of zero chromaticity: (a) turn-by-turn signal; (b) the FFT of (a).

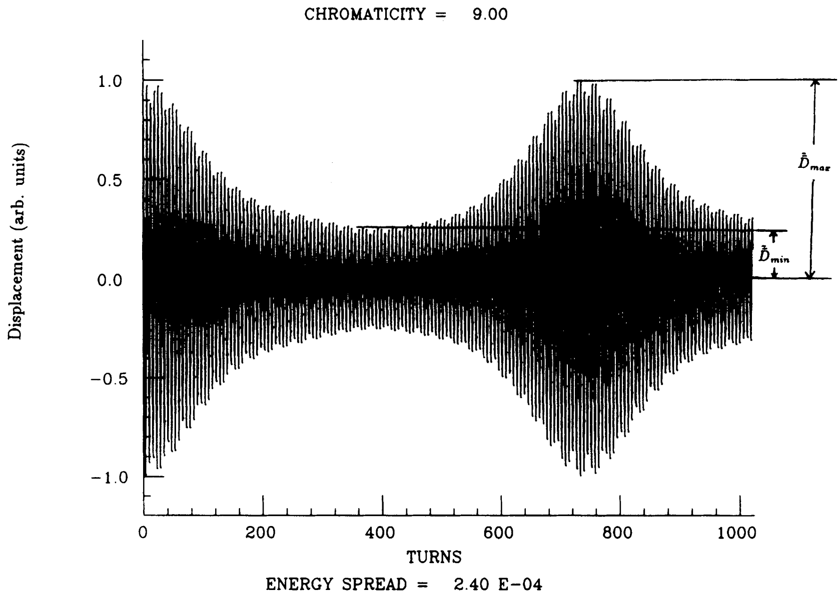

Fig. 4(a)

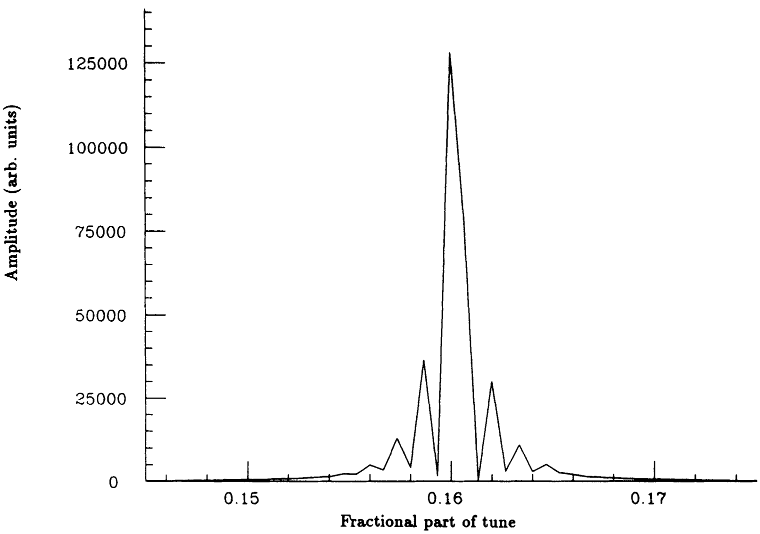

Fig. 4(b)

FIGURE 4 The simulation result for the case of non-zero chromaticity: (a) turn-by-turn signal; (b) the FFT of (a).

TABLE I

Comparison with Simulation

$\frac{\sigma_\epsilon}{E} \times 10^{-4}$	ξ	$(\xi)\left(\frac{\sigma_\epsilon}{E}\right) \times 10^{-4}$	$\frac{\bar{\bar{D}}_{min}}{\bar{\bar{D}}_{max}}$	$\left\{\xi \frac{\sigma_\epsilon}{E}\right\}_{theory} \times 10^{-4}$	% error
	9.0	21.60	0.263	21.86	1.2
2.4	6.0	14.40	0.442	14.72	2.2
	3.0	7.20	0.764	7.27	1.0
	9.0	64.80	0.083	65.60	1.2
7.2	6.0	43.20	0.129	42.44	−1.8
	3.0	21.60	0.274	21.0	−2.5

chromaticities. The result from the proposed method is about a factor of three better than that from the bunch length measurement method.

4. CONCLUSION

After analyzing the decoherence and the recoherence of the coherent transverse betatron oscillation, we have concluded with a method for the measurement of $\xi(\sigma_\epsilon/E)$. The advantages of this method are:

1. It is machine-independent. Even for a proton machine, we need only change the particle distribution function in Eq. (8) to the parabolic distribution usually assumed.

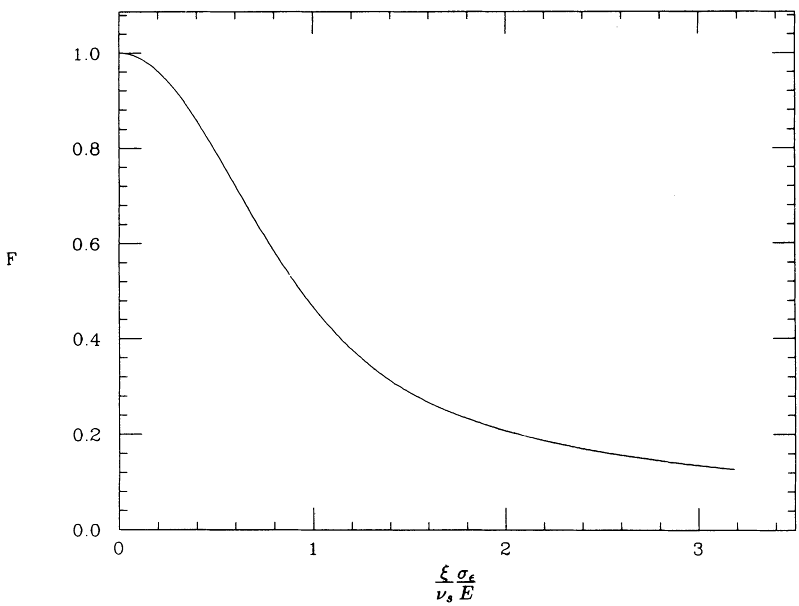

FIGURE 5 The universal function F as function of $\xi\sigma_\epsilon/\nu_s E$.

2. It is current-independent. The result depends on $\tilde{\tilde{D}}_{min}/\tilde{\tilde{D}}_{max}$ in which current cancels out.
3. It is independent of the kicker strength, provided we do not kick so hard as to get into the nonlinear region.
4. It can be used even at very small currents, in which case the bunch length measurement method for measuring energy spread is difficult.

However, the disadvantage of this method is that the measured value is the product of two unknowns. If we want to get one of the unknowns by this method, the result will depend on the accuracy of the other value which must be obtained by another method. The linearity of the signal pickup system is important. It is also clear from Fig. 5 that, for $(\xi\sigma_\epsilon/\nu_s E) > 2.0$, a small measurement error in $\tilde{\tilde{D}}_{min}/\tilde{\tilde{D}}_{max}$ will produce a large error in $\xi\sigma_\epsilon/\nu_s E$.

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