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IS FEEDBACK ABLE TO COUNTERACT THE TRANSVERSE MODE COUPLING INSTABILITY?

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ABSTRACT The current in many large electron storage rings is limited by the transverse mode coupling instability.
It has been proposed to increase the threshold of this instability by using a feedback system.
The purpose of this paper is to present and discuss an appropriate treatment of the transverse mode coupling instability in the presence of a feedback system in the framework of the Vlasov equation. It is shown that a feedback system is only effective as a cure for transverse mode coupling instability under extremely restricted conditions.

INTRODUCTION

The current in large electron storage rings is often limited by the transverse mode coupling instability [1,2].
The instability occurs when the frequencies of the fundamental or dipole mode (frequency ω_β for zero current) and the first head tail mode (frequency $\omega_\beta - \omega_s$) become degenerate. It is known that the threshold current of this instability can be increased when the frequency separation of the two modes is enlarged.
In reference 3 it was proposed to enlarge the frequency separation with help of a feedback system which only causes a frequency shift of the dipole mode (therefore named dipole feedback) without causing any damping. Such a system is termed reactive whereas a system causing damping is called resistive.
It had been found out that resistive feedback is not able to counteract the mode coupling instability [3].
The investigation of reactive feedback using the two particle model promised an increase of the threshold current by a factor of two to four [4]. This result was confirmed by studying the problem in the framework of the Vlasov equation [3].
These promising results, however, could not be confirmed by applying a few par-

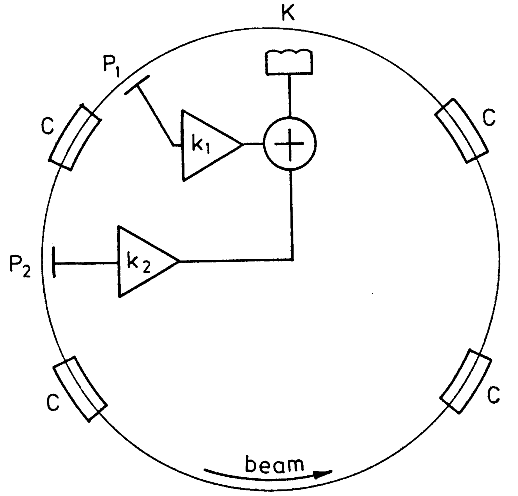

Figure 1: Schematic presentation of an accelerator with localised cavities (C) pickup stations (Pi) and kicker (K); the gain of the amplifiers is denoted by k_i

ticle simulation technique [5]. On the contrary it was found that the feedback in general reduces the threshold current.

In contrast to the studies using the two particle model and the Vlasov equation the simulation takes into account the location of the elements causing the wakefields and the components of the feedback system.

It was therefore conjectured that the different results are due to the different assumptions concerning the location of the elements.

The purpose of this paper is to discuss the problem in the framework of the Vlasov equation taking into account the location of all elements. A condition is given which must be imposed on the feedback system to ensure that it is purely reactive. In general this requirement cannot be satisfied and this means that an actual feadback contains reactive and resistive parts. As a consequence the feedback is only effective for small values of the synchrotron tune.

THE FORMALISM

The Vlasov equation with piecewise constant impedance

Figure 1 shows a schematic description of an accelerator with localised cavities (C) and a feedback system consisting of pickup stations (P_i) and a kicker magnet (K).

To examine such an accelerator in the framework of the Vlasov equation one needs a formalism to treat elements with piecewise constant impedances.

The starting point is the linearized Vlasov equation. This equation is transformed into an infinite set of ordinary differential equations by applying the usual mathematical methods [6,7]:

$$\frac{d\vec{a}(\theta)}{d\theta} = \mathcal{A}(\theta)\vec{a}(\theta) \ . \tag{1}$$

The components of $\vec{a}$ are the amplitudes of the head tail modes — the internal oscillation modes of the bunch.

The matrix $\mathcal{A}$ depends on the angular position θ since the impedance is not uniformly distributed around the ring. But it is reasonable to assume the impedance to be piecewise constant. In this case equation (1) can be integrated for each section of constant impedance yielding a transfer matrix for each section

$$\mathcal{A}(\theta) = \mathcal{A}_i \text{ for } \theta_i \leq \theta \leq \theta_{i+1} \tag{2}$$

$$\vec{a}(\theta_{i+1}) = T_j\,\vec{a}(\theta_i) \ \text{ with } T_j = \exp(\mathcal{A}_j\,(\theta_{i+1} - \theta_i)) \ . \tag{3}$$

The single turn matrix T_0 can be computed by matrix multiplication. The eigenvalues of T_0 determine the stability. The transfer matrices T_j satisfy the following two important similarity relations [7] :

$$J\,T^{-1}\,J^{-1} = T^T \tag{4}$$

$$K^{-1}\,T\,K = T^* \ . \tag{5}$$

Since the single turn matrix is the product of transfer matrices it also satisfies the similarity relations. These two relations fix the eigenvalue spectrum in such a way that an instability can occure only if two eigenvalues ($\lambda = \rho \exp{(i\phi)}$) have the same phase ϕ which means that two eigenmodes have the same frequency [7]. This is just the condition for the occurence of an instability mentioned in the introduction.

Transfer matrix of the feedback system

The feedback system is build up of two elements: the pickup electrodes and the kicker magnet. Present kicker magnets can only affect the dipole moment of a bunch since the bandwidth of such divices is limited. Because of that restriction it is assumed that the pickup stations are also restricted to measuring only the dipole moment.

The kicker exerts a force on the beam which is proportional to the dipole moment at each pickup $D(\theta_{Pi})$ measured on the same turn:

$$F(\theta_K) = \sum_{i=1}^{n} k_i\,D(\theta_{Pi}). \tag{6}$$

The kicker can be treated as a localised element and a transfer matrix can be derived in analogy to the case of elements with constant impedance. Assuming that the kicker is a "thin" element one finally gets for the change of the vector $\vec{a}$ [7]

$$\Delta\vec{a} = \vec{a}(\theta_K^+) - \vec{a}(\theta_K^-) = \vec{p}(\sum_{i=1}^{n} k_i D(\theta_{Pi})) \tag{7}$$

where $\vec{p}$ is a kind of projection operator ensuring that only the dipole moment is changed. The angle $\theta_K^{\pm}$ denotes the angular position just in front of or just behind the kicker respectively.
The connection between the dipole moment at the pickup station and the vector $\vec{a}$ at the kicker is given by the corresponding transfer matrix T_{PiK}:

$$D(\theta_{Pi}) = \vec{p}^T \cdot T_{PiK}^{-1}\,\vec{a}(\theta_K). \tag{8}$$

where p^T denotes the transpose of $\vec{p}$.
With the help of equation (8) the transfer matrix of the feedback system can be written in the following form:

$$\vec{a}(\theta_K^+) = T_{FB}\,\vec{a}(\theta_K^-) \tag{9}$$

$$T_{FB} = \mathbb{1} + \mathcal{K}\sum_{i=1}^{n} k_i T_{PiK}^{-1} \tag{10}$$

where the kick matrix is defined by

$$\mathcal{K} = \vec{p} \cdot \vec{p}^T \tag{11}$$

and $\mathbb{1}$ denotes the unit matrix.
The transfer matrix of the whole ring with feedback is given by

$$T = T_{FB} \cdot T_0 . \tag{12}$$

To investigate the stability, the eigenvalues of T must be evaluated.

RESULTS

In the introduction an intuitive definition of reactive feedback is presented. In order to adjust the kick parameters k_i, however, a formal definition is needed. Before giving this definition a simple example is discussed.
In the case that all modes are uncoupled the investigation of the feedback system reduces to the problem of studying the stability of pure transverse dipole oscillations in the presence of a feedback system. In this simple example the single turn matrix is a two by two matrix. It can easily be shown that there is now damping if

$$\det T = 1 . \tag{13}$$

This is just the condition used in reference 3 to define a reactive feedback system. If there is only one pickup it is well known that this condition can be satisfied by choosing a phase advance of an integer multiple of π between pickup and kicker.
Generally all modes are coupled so that the requirement (13) normally does not lead to a reactive system.
Therefore the definition of a reactive feedback has to be more restrictive.
A feedback is purely reactive if the the feedback matrix (10) satisfies the pair of

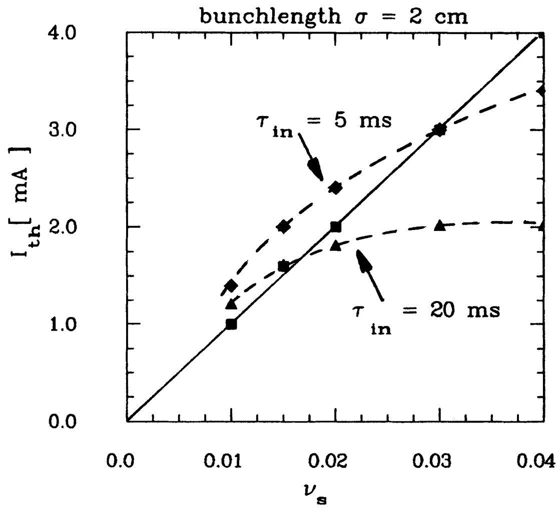

Figure 2: Threshold current as a function of the synchrotron tune for a PETRA like machine for different values of the internal damping constants τ_{in} (dashed curves); the solid curve describes the case without feedback

similarity relations (4) and (5). The relation (4) implies that the determinant of the feedback matrix is unity but in general the converse is not true so that the given requirement is stronger.
In the case that the similarity relations are satisfied, the feedback reacts like a special cavity which only produces a frequency shift.
If there is only one cavity the reqirements (4) and (5) can be fulfilled if the betatron phase advance between the cavity and the kicker magnet is just an integer multiple of π [7]. Thus the feedback effect can be accomplished. This means that the threshold current can be enhanced by a factor of two to four.
In general the number of cavities is bigger than one and it is not possible to achieve the required phase advance between each cavity and the kicker.
Therefore the feedback system is always a mixture of reactive and resistive parts where the resistive part leads to damped and antidamped modes due to the coupling.
But in every machine there are internal damping mechanisms such as Landau damping or radiation damping. If this damping is able to compensate the antidamping of the feedback system it is possible to increase the threshold current by applying feedback.
The stability of a realistic accelerator with lots of cavities and a localised feedback system can be investigated by nummerical evaluation of the eigenvalues of the single turn matrix (see eq. (12)). In figure 2 the threshold current computed as a function of the synchrotron tune is shown. One can see that there is a positive

feedback effect only in case of small synchrotron tunes. The positive effect of the feedback depends on the internal damping rate (which is difficult to determine for the higher order head tail modes) and on the strength of the coupling between the dipole and the first head tail mode.
These two ingredients must be well known before an accurate estimate of the synchrotron tune up to which the feedback is effective can be obtained.

CONCLUSION

In this paper it is shown that a pure reactive feedback system cannot be realized in general. The application of feedback has a a positive effect only if the synchrotron tune of the ring is small (around 0.02). Therefore feedback might be helpful in small synchrotron radiation sources and in proton storage rings which naturally have a small synchrotron tune.

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