

## NUMERICAL STUDIES OF THE JUNGLE GYM SLOW-WAVE ACCELERATING STRUCTURE†

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Many structures of interest to accelerator physicists have been inaccessible to analysis because there was no three-dimensional electromagnetic field-solving code. This report discusses the analysis of one such structure, the Jungle Gym, using the recently developed MAFIA codes. The Cornell version of the Jungle Gym has been modeled and the dispersion curves for the  $TM_{010}$  and  $TM_{020}$  modes calculated and plotted. The shunt impedance ( $r$ ) and  $r/Q$  for the  $\pi$ -mode were also calculated and compared with experimental data. The calculations and experimental results were in good agreement.

### 1. INTRODUCTION

Jungle Gym is a bar-loaded traveling-wave structure for use in accelerators. This slow-wave structure, used in the Cornell electron synchrotron, was known to be easy to cool and to have a reasonable shunt impedance. It was also very compact and mechanically rigid. The three-dimensional MAFIA<sup>1</sup> codes were used to simulate this structure.

### 2. THE MODEL

A unit cell begins and ends with half a pair of bars and with another pair of bars rotated 90° separating these end bars. The number of cells to model depends on the phase shift per cell desired. This requirement is discussed in more detail in the next section. A three-cell model is plotted by the MAFIA mesh generator in Fig. 1. Only one-quarter of the actual transverse physical structure needs to be generated because of its two-fold symmetry. Table I shows the dimensions of a unit cell.

Although circular tapered bars were used in the actual structure, octagonal cylindrical bars were used for the model. An estimated average radius of 1.466 cm was used for the bars, and the *cavity* radius was changed until the  $\pi$ -mode was at 714.3 MHz; i.e., the phase velocity equaled the particle velocity (see Appendix A). The resulting cavity radius was 12.136 cm, a difference of less than 4% from the actual value of 12.625 cm. This small error may result from the coarseness of the mesh used.

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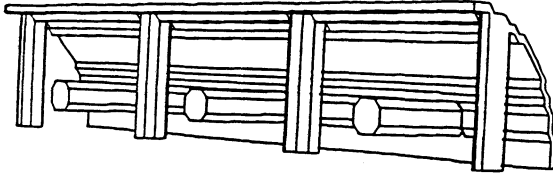


FIGURE 1 A three-cell Jungle Gym model.

### 3. BOUNDARY CONDITIONS AND THE DISPERSION CURVE

Both Dirichlet (tangential electric field equals zero) and Neumann (tangential magnetic field equals zero) boundary conditions were used in solving for the structure. By placing Neumann boundary conditions on the sides where the cuts were taken in the nonpropagating ( $x$  and  $y$ ) directions to model one-quarter of the cavity, the lowest-order TE modes are eliminated.

In the propagating ( $z$ ) direction, to find the desired phase shifts per cell,  $\phi$ 's, for the lowest mode, one chooses the number of cells ( $n$ ) and the boundary conditions according to the following formulas:

Desired $\phi$ 's	$k$	Boundary Condition
$k\pi/n$	$0, 1, \dots, 2n - 1$	Dirichlet–Dirichlet
$k\pi/n$	$1, 2, \dots, 2n$	Neumann–Neumann
$k\pi/(2n)$	$1, 3, \dots, 2n - 1$	Dirichlet–Neumann

For example, in a three-cell model, the first two sets of boundary conditions (Dirichlet–Dirichlet or Neumann–Neumann) gave the modes with phase shifts being multiples of  $\pi/3$  whereas the last set (Dirichlet–Neumann) gave modes with phase shifts being multiples of  $\pi/6$ . These phase shifts, along with their corresponding frequencies, are shown in Table II for the lowest band.

The smooth curves formed from the plot of resonant frequency versus phase shift per cell in Fig. 2 show that the boundary conditions used appear to be correct in modeling the slow-wave rf structure. Modes with phase shifts being multiples of  $\pi/12$  were also found using six cells, and these fit perfectly on the

TABLE I  
Dimensions of the Jungle Gym Model

Radius of bars	1.466 cm
Length of bars (& radius of cavity)	12.136 cm
Spacing between bars	10.493 cm
Longitudinal length per cell	20.865 cm

TABLE II  
Phase Shift per Cell from a Three-Cell Model

Frequency (MHz)	Phase Shift per Cell	Boundary Conditions
617.9	$2\pi$	Neumann–Neumann
620.8	$11\pi/6$	Dirichlet–Neumann
629.4	$5\pi/3$	Dirichlet–Dirichlet or Neumann–Neumann
643.5	$3\pi/2$	Dirichlet–Neumann
662.7	$4\pi/3$	Dirichlet–Dirichlet or Neumann–Neumann
686.5	$7\pi/6$	Dirichlet–Neumann
714.3	$\pi$	Dirichlet–Dirichlet or Neumann–Neumann
745.8	$5\pi/6$	Dirichlet–Neumann
780.4	$2\pi/3$	Dirichlet–Dirichlet or Neumann–Neumann
817.3	$\pi/2$	Dirichlet–Neumann
855.6	$\pi/3$	Dirichlet–Dirichlet or Neumann–Neumann
891.9	$\pi/6$	Dirichlet–Neumann
911.5	0	Dirichlet–Dirichlet

curves in this figure. The lowest band corresponds to the  $TM_{010}$ , whereas the next band of modes corresponds to the  $TM_{020}$  mode. The dashed curve is the  $TM_{010}$  mode found by Tigner and is given for comparison.<sup>2</sup>

According to Tigner, the upper frequency of the lowest band is given approximately by the  $TM_{01}$  cut-off of an empty pipe of the same diameter.<sup>1</sup> This cutoff frequency for a radius of 12.136 cm is 946.1 MHz, which is approximately the calculated frequency of 911.5 MHz.

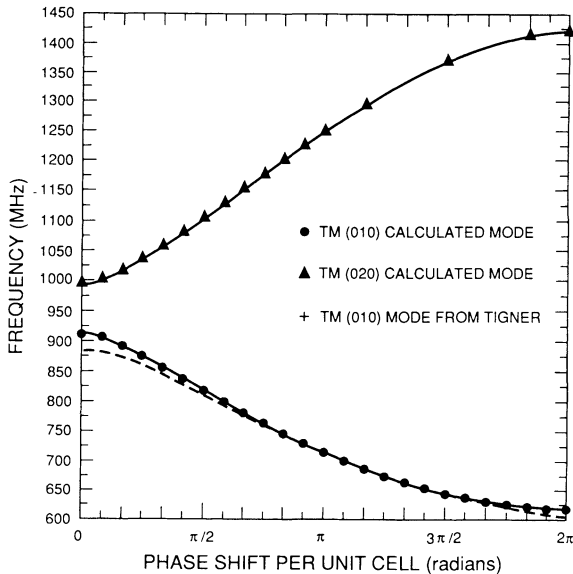


FIGURE 2 The Jungle Gym dispersion curve.

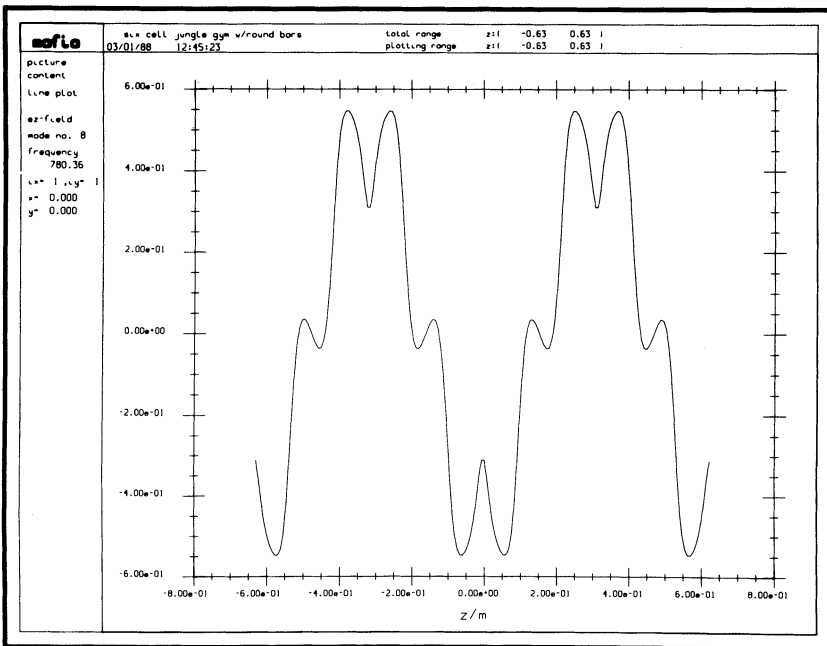
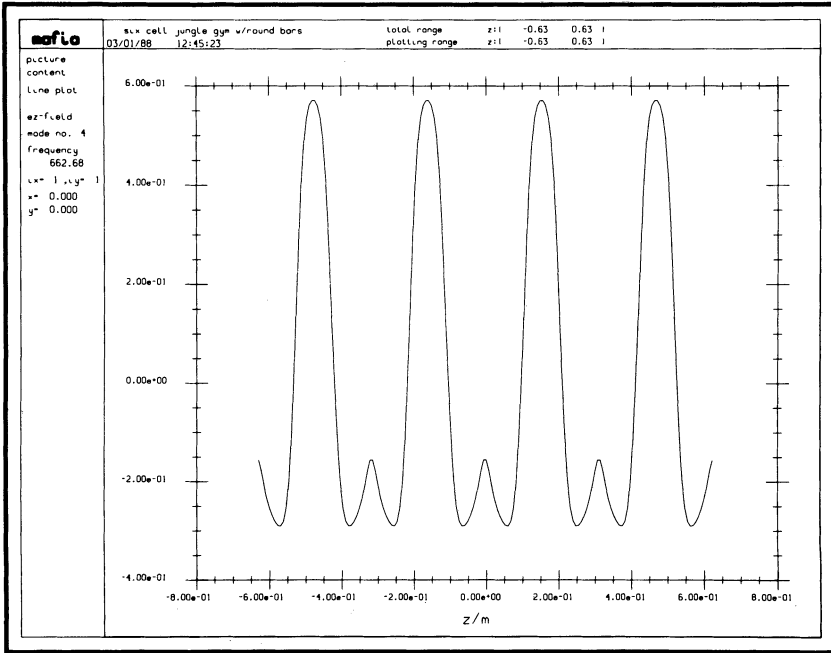


FIGURE 3 The  $E_z$  line plots for the six-cell structure with Dirichlet boundary conditions.

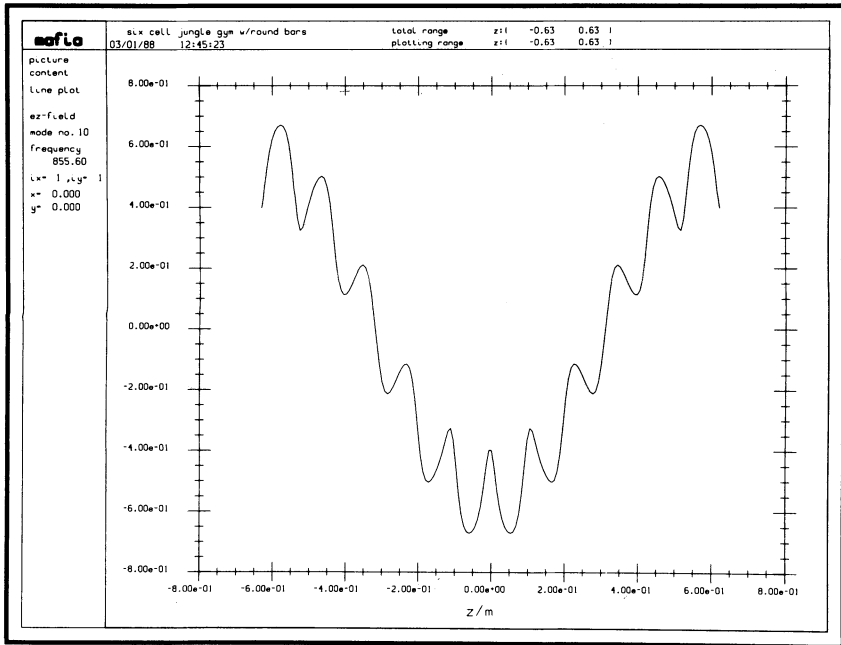
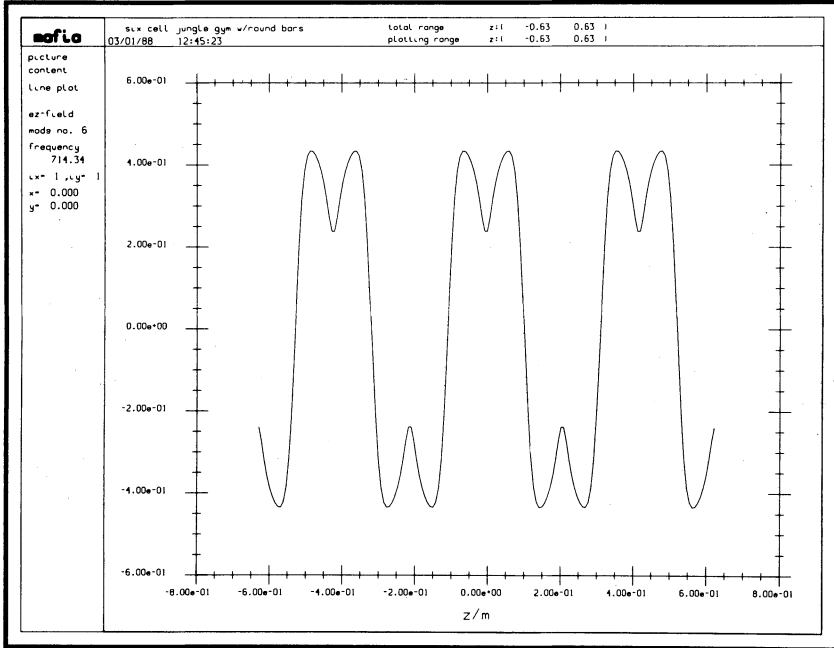


FIGURE 3 (contd.)

#### 4. PHASE SHIFTS PER CELL

The  $E_z$  line plots generated by the MAFIA postprocessor verified the phase shifts per cell. Figure 3 shows some of these plots for the six-cell structure with Dirichlet boundary conditions. These plots clearly indicate the phase shift of the  $E_z$  field per cell. For example, in the lower left plot, there are two complete field patterns or  $2(2\pi) = 4\pi$  in six cells; therefore, this is the  $(4\pi/6) = (2\pi/3)$  mode.

The values of these phase shifts per cell were also verified by having the postprocessor do electric-field arrow plots at selected  $x$  or  $y$  planes. Figure 4 shows the arrow plots of different modes at the  $x = 0$  plane in the length of the structure with Dirichlet boundary conditions on the ends. The darkened octagons represent the ends of the loading bars for the one-quarter Jungle Gym structure, whereas  $\phi$  indicates the phase shift per cell.

Looking at one end of the structure and comparing it to the other end, we see that for the odd multiples of  $\pi/3$ , the fields are completely reversed; for the even multiples of  $\pi/3$ , the fields are pointing in the same direction, as expected.

#### 5. SHUNT-IMPEDANCE CALCULATIONS

The Jungle Gym studied was designed to operate with particle velocities at the speed of light, essentially, and with a phase shift per cell of  $\pi$  radians. The

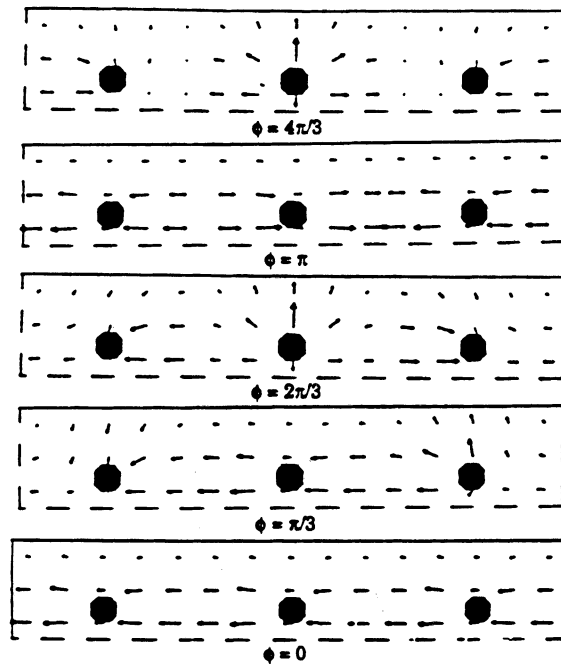


FIGURE 4 Electric-field arrow plots from the MAFIA postprocessor.

MAFIA postprocessor computes the force integrals needed to calculate the shunt impedance.

The formula used for shunt impedance per unit length for a traveling wave (see derivation in Appendix B) is

$$r_{\text{TW}} = \frac{2 \left\{ \left[ \int E_z \cos(kz) dz \right]^2 + \left[ \int E_z \sin(kz) dz \right]^2 \right\}}{(P_{\text{loss}})_{\text{SW}} \Delta z}, \quad (1)$$

where

$\Delta z$  is the distance along the structure over which the integration takes place,

$V$  is the maximum potential difference,

$(P_{\text{loss}})_{\text{SW}}$  is the power loss through the walls, as calculated for the standing wave

$E_z$  is the electric field along the  $z$ -direction,

$k$  is  $\omega/v$ , where  $\omega = 2\pi$  (frequency), and

$v$  is the speed of the particle.

The power loss and the two integrals on the right side of the equation are computed by the postprocessor. The  $Q$  is also given by the postprocessor, so that the  $r/Q$  values could be calculated.

The values of  $r$  and  $r/Q$  calculated for each of the phase shifts per cell between 0 and  $2\pi$  (Table III) came from the six-cell structure with a short on each end. This setup did not include the  $2\pi$ -mode. As shown in Table III, the shunt impedance calculated at the  $\pi$ -mode was 25.4 M $\Omega$ /m. To two significant digits, this is the same value measured by Tigner. The  $r/Q$  found was 1370  $\Omega$ /m, which is within 1.4% of Tigner's value of 1389  $\Omega$ /m.

TABLE III  
Shunt Impedance ( $r$ ) and  $r/Q$  from a Six-Cell Model

Frequency (MHz)	Phase Shift per Cell	$r$ (M $\Omega$ /m)	$r/Q$ ( $\Omega$ /m)
617.9	$2\pi$		
620.8	$11\pi/6$	0.00272	0.177
629.4	$5\pi/3$	0.0506	3.23
643.5	$3\pi/2$	0.152	9.42
662.7	$4\pi/3$	0.309	18.4
686.5	$7\pi/6$	8.05	459.
714.3	$\pi$	25.4	1370.
745.8	$5\pi/6$	15.4	786.
780.4	$2\pi/3$	1.90	90.2
817.3	$\pi/2$	0.181	7.94
855.6	$\pi/3$	1.61	63.6
891.9	$\pi/6$	1.93	64.2
911.5	0	0.349	9.56

## 6. CONCLUSION

It can be seen from this study on the Jungle Gym that by taking a simple standing-wave model for a traveling-wave structure and using the MAFIA codes with the correct boundary conditions, one obtains the desired traveling-wave properties. In particular, the shunt-impedance calculations for the  $\pi$ -mode and the dispersion curve for the  $TM_{010}$  mode compared well with experiment.

## ACKNOWLEDGEMENTS

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## REFERENCES

1. R. Klatt, F. Krawczyk, W.-R. Novender, C. Palm, T. Weiland, B. Steffen, T. Barts, M. J. Browman, R. Cooper, C. T. Mottershead, G. Rodenz, S. G. Wipf, "MAFIA—A Three-Dimensional Electromagnetic CAD System for Magnets, RF Structures, and Transient Wake-field Calculations," *1986 Linear Accelerator Conference Proc.*, Stanford Linear Accelerator Center Report SLAC-303, 276–278 (1986).
2. M. Tigner, Bar Loaded Waveguide for Accelerator Service, *IEEE Transactions on Nuclear Science*, NS-18, No. 3 (1971).

## APPENDIX A

### Derivation of the Frequency at the $\pi$ -Mode

The phase velocity is defined to be

$$v_p = \frac{\omega}{\beta},$$

where  $\omega (= 2\pi f)$  is the radian frequency of the wave, and  $\beta$  is the propagation constant equal to  $2\pi/\lambda$ . For the  $\pi$ -mode,  $\lambda = 2L$ , where  $L$  is the distance between cells. Because we would like the phase velocity to be the same as the particle velocity, which is essentially at the speed of light  $c$ , we set  $v_p = c$ .

The equation above then becomes (for the  $\pi$ -mode)

$$c = \frac{2\pi f}{(2\pi/\lambda)} = f\lambda = f(2L)$$

or

$$f = \frac{c}{2L}.$$

For the Jungle Gym modeled,  $L = 0.2098$  m; this gives  $f = 714.3$  MHz.



**APPENDIX B****Derivation of the Shunt-Impedance Formula**

In general, for a standing wave, the shunt impedance per unit length is

$$r_{\text{SW}} = \frac{R}{\Delta z} = \frac{|V|^2}{(P_{\text{loss}})_{\text{SW}} \Delta z} = \frac{\left| \int E_z e^{jkz} dz \right|^2}{(P_{\text{loss}})_{\text{SW}} \Delta z}$$

$$= \frac{\left[ \int E_z \cos(kz) dz \right]^2 + \left[ \int E_z \sin(kz) dz \right]^2}{(P_{\text{loss}})_{\text{SW}} \Delta z}.$$

For a traveling wave, in general,

$$(P_{\text{loss}})_{\text{TW}} = \frac{1}{2}(P_{\text{loss}})_{\text{SW}}.$$

Therefore,

$$r_{\text{TW}} = \frac{2 \left\{ \left[ \int E_z \cos(kz) dz \right]^2 + \left[ \int E_z \sin(kz) dz \right]^2 \right\}}{(P_{\text{loss}})_{\text{SW}} \Delta z}.$$