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BEHAVIOR OF SINGLE-ENDED VS. PUSH-PULL AMPLIFIERS FOR THE ACCELERATING SYSTEMS OF HIGH-CURRENT BEAM STORAGE RINGS*

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Push-pull and single-ended amplifiers, both on common-anode configuration, are examined under very heavy beamloading conditions. The topological difference in behavior is demonstrated, and it is shown why a single-ended amplifier is to be preferred. Two technical examples are analyzed and discussed. Because of the inherent non-linearity of the problem, state-variable analysis is being used.

I. INTRODUCTION

In many heavy-particle accelerators, and particularly in proton storage rings, acceleration is achieved by means of ferrite-loaded resonators. These are excited by high-power amplifiers that, because of the relative low frequencies being used, fall into the telephone-transmitter amplifier category.

Figure 1 shows the cavity coupling: schemes that could be used for a storage ring a coaxial resonator, excited by a common-anode push-pull amplifier (a), or by a single-ended cathode follower (b).

As in telephone circuits, a push-pull arrangement with grounded cathodes has been always preferred to the corresponding single-ended amplifier, based on several arguments that are summarized below:

(i) Neutralization in a push-pull is straightforward and troublefree during operation, while neutralizing a single-ended amplifier is often accomplished with some difficulty.

(ii) The linearity of a push-pull is well known, and its efficiency is much higher than that of a corresponding single-ended amplifier.

(iii) dc and rf voltages amount exactly to half the voltages to be used with corresponding single-ended, and it is well known that costs and failure chances grow rapidly with applied voltages.

Steady demand for better performances of rf systems for storage rings, and particularly the needs of the storage ring ISABELLE[1] led us to a critical review of possible circuit arrangements. Our conclusions are that for ISABELLE a single-ended circuit configuration should be preferred. The main arguments are:

As in all large accelerators, also in ISABELLE, the last amplifier output impedance is chosen to be very low (10 to 50 ohms, for instance), while at the same time the current accelerated is very large (peak values of several tens of amperes are a normal condition).

A suitable negative feedback can help meet the first requirement. For the large current requirement, the instantaneous power output of the last amplifier must be very large.

It is clear that the low output-impedance criterion is, per se, not tailored to push-pull amplifiers, because a push-pull amplifier has an output impedance at least twice than that of a corresponding single-ended amplifier. There are also other arguments, and stronger ones, to rule out the push-pull configuration.

The accelerated particle beam extracts a certain amount of energy from the cavity resonator in a very short time, thus reducing the excitation level of the resonator itself; this level must be restored by the amplifier. Because of the negative feedback needed, as seen above, to lower the output impedance, a signal depending on the volt-

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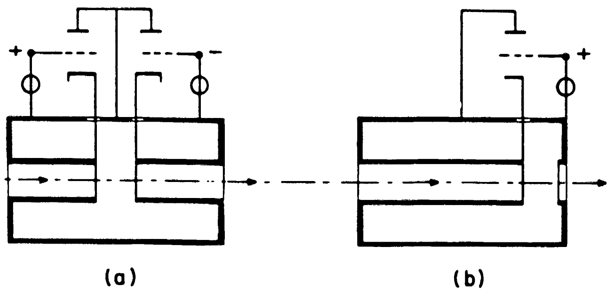

FIGURE 1 Schematic diagram of the cavity resonators for acceleration of particle beams. Two amplifier systems: (a) push-pull common anode, (b) single ended c-a.

age drop induced by the beam is fed into the amplifier input. Since the feedback is strong, so must be the signal.

In a single-ended amplifier chain, similar to the one studied by W. Hardek and W. E. Chyna,[2] this signal has the correct polarity to increase the current in the final tube and drive it eventually to heavy grid current conditions. In a push-pull amplifier, while the same happens in one of the final tubes, the other can be brought to cutoff, giving an overly large distortion of the accelerating voltage waveform.

This behavior can be understood if we consider that an increasing load requires an increasing current in both tubes. While the current change is positive in one tube, it is necessarily negative in the other, with the result that a tube must move towards cutoff.

It is this reality, more than anything else, that has suggested our choice of a single-ended final amplifier.

II. DC ANALYSIS

The detailed study of a power amplifier, either push-pull or single-ended, always implies the solution of a very complicated network. Moreover, since the current due to the particle beam has a shape quite different from a sinusoid,* the circuit analysis must be performed in transient conditions.

If we also want to take into account the nonlinear behavior of the electronic tubes, it is clear that the analysis becomes rather difficult and its complexity does not help us in understanding fully the mechanism described in the last section. Rather, it obscures the physical picture and will be done only where numerical results are required. (This is the case in the next section.) Instead, a dc analysis, with its inherent simplicity, may show clearly the intrinsic difference in operation of a push-pull amplifier vs a single-ended amplifier.

The ground configuration to be taken as a starting point for the analysis of both systems can be chosen among many possible ones. Generally, the "common-anode" configuration is considered, because it appears to give the lowest constant output impedance over a frequency range that other negative-feedback circuits are barely able to give.* Accordingly, we will start with this kind of configuration that has also the advantage of being easy to analyze by direct inspection.

We will assume in the following that the same resistive load, in parallel to a current generator (the beam), is being fed by a common-anode amplifier, consisting either of two tubes in push-pull or of a single tube (single-ended amplifier).

The two arrangements are shown in Figs. 2 and 3 respectively. Linear behaviour is assumed. Let us denote by V_{ru} and V_{rs} the voltages developed across the Z_p resistance in the cases, respectively, of an unsymmetric and symmetric configuration. It follows that

$$V_{ru} = F_u\left(V - \frac{\rho}{\mu} I_b\right)$$
$$V_{rs} = F_s\left(V - \frac{2\rho}{\mu} I_b\right), \qquad (1)$$

where μ/ρ is the tube transconductance and F_u and F_s are two form factors (see Appendix) that differ by a small amount from unity. I_b is the beam current.

It appears that the voltage drop due to a current generator equivalent to the beam is twice as large for a push-pull amplifier. This is an important point when dealing with heavy beam loads, but it is not critical, as can easily be shown. Much more important is the behavior of the steering voltages.

In an electron tube, the plate current is controlled both by the grid-to-cathode and plate-to-

* It is assumed, e.g., that the beam current pulses have a biased-cosine shape with a width approximately equal to $\frac{1}{3}$ to $\frac{1}{4}$ of the accelerating voltage period.

* It has to be kept in mind that we are dealing here with 50 to 100 kW power amplifiers, and that therefore sophisticated feedback networks cannot be used.

cathode voltages. Let us denote by V_{gk} the time-dependent component of the voltage developed between grid and cathode of the single-ended amplifier, and with V_{gk_1} and V_{gk_2} the corresponding voltages in the push-pull amplifier. After some trivial algebra we obtain

$$V_{gk} = (1 - F_u)V_D + \frac{\rho}{\mu} F_u I_b$$

$$V_{gk_1} = \frac{1}{2}\left[(1 - F_s)V_D + \frac{2\rho}{\mu} F_s I_b\right] \qquad (2)$$

$$V_{gk_2} = -\frac{1}{2}\left[(1 - F_s)V_D + \frac{2\rho}{\mu} F_s I_b\right],$$

where V_D is the driving voltage applied to the tube in the unsymmetric arrangement, and $V_D/2$ the driving voltage applied, with opposite phases, to the two tubes in the symmetric case.

It appears clearly from Eqs. (2) that, once given the function V_D, the effect of the beam current is such that, while in the single-ended amplifier the grid-to-cathode voltage may increase (or decrease), in the push-pull it must be

$$V_{gk_1} = -V_{gk_2}. \qquad (3)$$

Equation (3) shows that, whatever the beam-current shape might be, the grid-to-cathode voltage in one tube is always opposite to the grid-to-cathode voltage in the other. In the real case, the nonlinearity of the tubes may invalidate the exact equality above, but the physics remains unchanged.

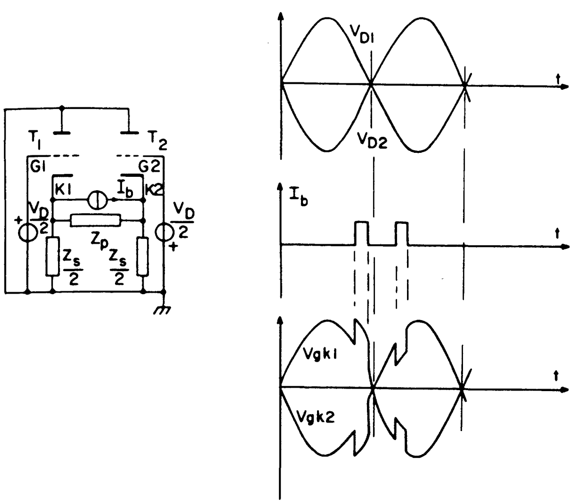

FIGURE 2 Push-push amplifier diagram for dc analysis. Curves V_D, I_b, V_{gk1}, V_{gk2}, as a function of time. Upon injection of a positive beam current pulse, conduction in tube No. 1 increases, and decreases in tube No. 2. Tube No. 2 may go to cutoff.

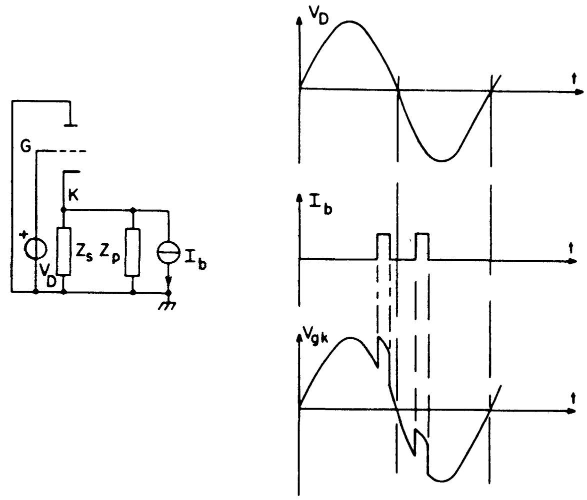

FIGURE 3 Single-ended amplifier diagram for dc analysis. Curves V_D, I_b, V_{gk} as a function of time. V_{gk} tends to increase and the conduction of the tube increases also.

It follows that an operating condition that would load a single-ended amplifier heavily would bring one of the two tubes that operate in the corresponding push-pull to cutoff.

In the case of a particle accelerator, the resonators are excited by sinusoidal voltages and the beam pulses cross the accelerating gaps with such phases as to fulfill the stability conditions and to gain on the average a positive energy from the resonator at each passage.

In the equivalent scheme, that situation arises when the center of mass of the beam pulse crosses the gap when the voltage time derivative is negative, and when the injected current has the proper direction to counteract, in the load, the current injected by the amplifier.

Figure 2 shows the grid-to-cathode voltage waveforms, modified by the beam current to be accelerated. A pulse of rectangular shape is assumed. The sinusoidal driving voltage is also shown for comparison.

Figure 3 shows analogous curves for the single-ended amplifier. It is clear from the figures that if the single-ended amplifier is led to high grid current, then in the push-pull one tube behaves similarly while the other is driven to cutoff.

Class-2 operation (grid current) is not troublesome, especially with modern high-power transmitting tubes. Cutoff means instead the temporary absence of one tube from the circuit. The entire amplifier network becomes unsymmetric and the resonator waveform is not only severely reduced in amplitude, but is also highly distorted.

The above argument applies exactly to tetrodes and pentodes, because the grid-to-cathode drive is the only determinant of the plate current. As for triodes, where the plate-to-cathode voltage also controls the anode current, the analysis is more complicated, but leads to the same conclusions.

III. NUMERICAL EXAMPLES

As we said above, the solution of a nonlinear network is always rather complicated and the numerical results can scarcely deepen our knowledge of the physics involved. Nevertheless, some simplifying assumptions that do not change the physics may render the analysis simpler and very meaningful.

Accordingly, the following assumptions were made:

(a) The amplifiers are excited with ideal voltage generators.
(b) Stray elements are not considered.
(c) The tubes operate as ideal triodes until the cutoff condition is reached. Then both plate conductance and transconductance of the tube concerned are made vanishingly small.

Some comments are appropriate:

The first two assumptions do not interfere very much with the behavior of the circuits being considered and represent only a simplification. (Of course, in a technical project all the passive elements are to be considered and the resulting system must be solved with computer aid.) The third assumption does not fit with a real situation, but only represents the "worst case"

Figure 4(a) and (b) shows two circuits to be analyzed under the previous assumptions with

TABLE I

Parameters Used in the Numerical Calculations for a Push-Pull vs Single-Ended Amplifier[a]

Resonator parameters (only the fundamental mode is considered)	$C = 2.10^{-8}\ F$ $L = 2.298\ 10^{-5}\ H$ $R = 6{,}000\ \Omega$
Beam pulse (biased cosine) duration	$\Delta t = \frac{T}{3} = 1.42\ 10^{-6}$ sec
Tube parameters (ML-7560)	$GM = 0.083$ $\rho = 720$ ohm $E_p = 16{,}000\ V$ $E_b = -120\ V$
Drive voltage	$V_{Do} = \pm 6{,}000\ V\ [P - P]$ $= 12{,}000\ V\ [S - E]$

[a] In order to stress the differences in behaviour of a push-pull and a single-ended amplifier, we have employed the same polarization voltages in the former as in the latter. In reality, a push-pull would have used half voltages.

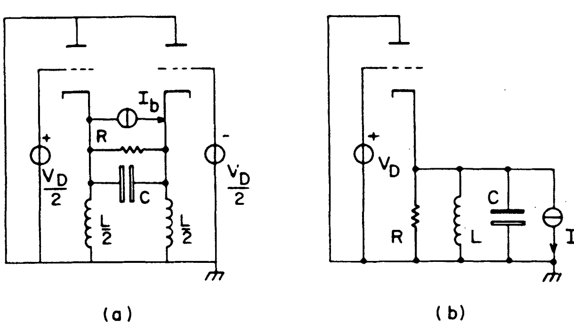

FIGURE 4 Simplified circuit of two amplifiers for numerical analysis: (a) push-pull, (b) single-ended.

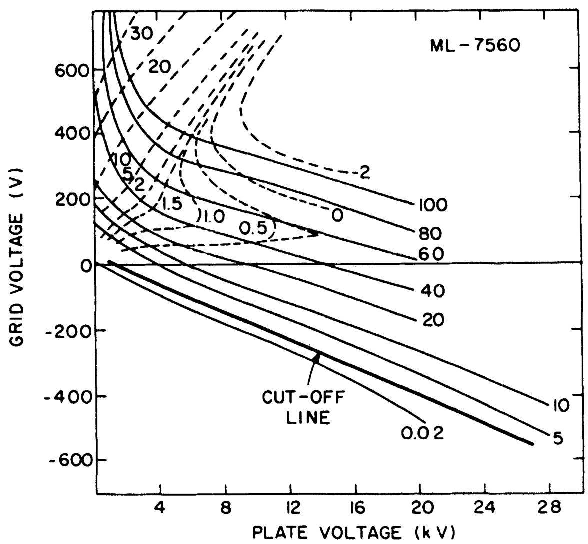

FIGURE 5 Constant-current characteristics for the tube ML-7560 used as an example in the numerical calculations. Solid lines: plate-current lines. Dashed lines: grid current. A possible cutoff straight line is also shown.

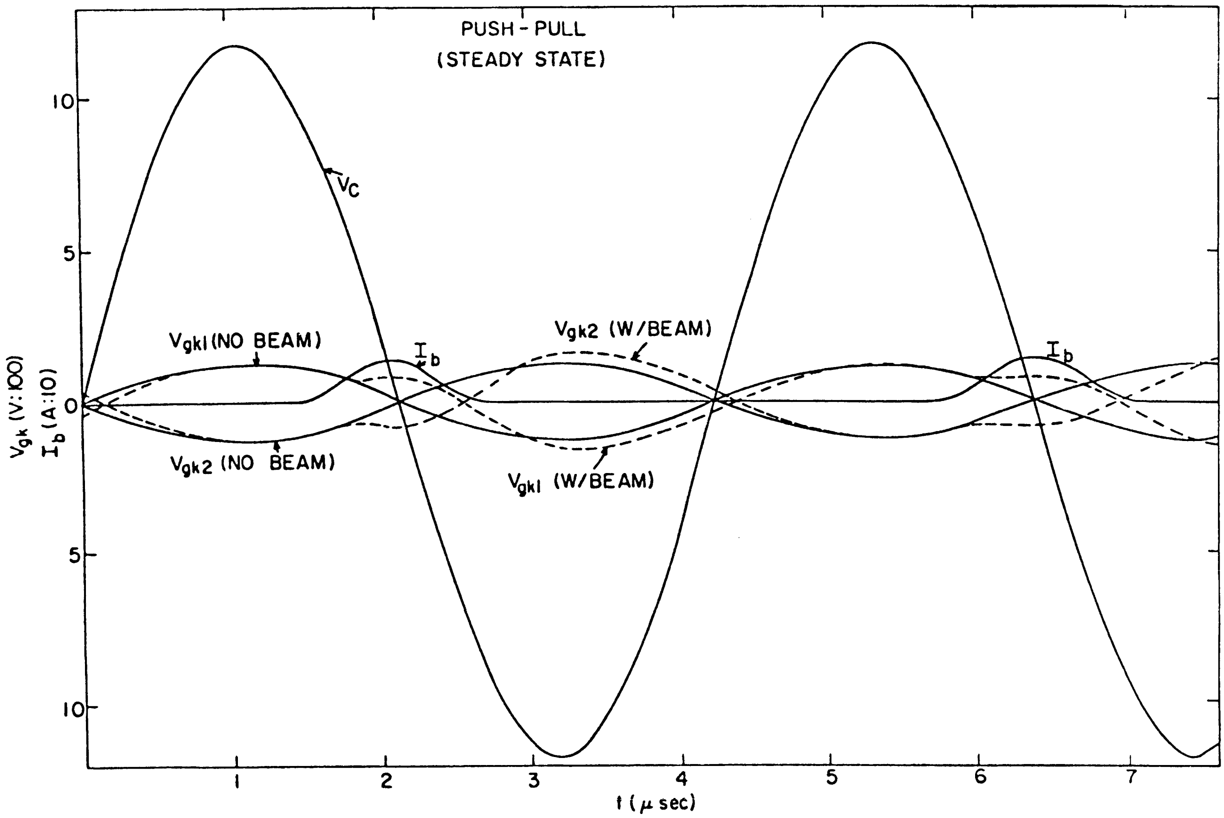

FIGURE 6 Results of a numerical analysis of a push-pull amplifier. Beam with peak current of 14 A at $\phi = 0°$. No cutoff occurs. The curves V_c, V_{gk1}, V_{gk2}, shown as a function of time refer to steady-state conditions.

parameters fit to the ISABELLE acceleration system. Figure 4(a) shows the push-pull geometry in a simplified diagram. The network equations to be integrated are

$$\left(\frac{1}{\rho_1} + GM_1\right) L\frac{dI_1}{dt} + C\frac{dV_c}{dt} = GM_1 V_D - I_b - I_1 - \frac{V_c}{R}$$

$$\left(\frac{1}{\rho_2} + GM_2\right) L\frac{dI_2}{dt} - C\frac{dV_c}{dt} = -GM_2 V_D + I_b - I_2 + \frac{V_c}{R} \qquad (4)$$

$$L\frac{dI_1}{dt} - L\frac{dI_2}{dt} = V_c,$$

where the indices 1 and 2 refer to tubes 1 and 2, and $1/\rho$ is the plate conductance of either tube, GM is the transconductance of either tube, I_1, I_2 are the currents in either half of the accelerating cavity, L is the total inductance, C the capacity, R the shunt resistance, V_c is the accelerating voltage across the cavity, V_D is the (sinusoidal) tube drive voltage, and I_b is the (biased cosine) beam current pulse.

Equations (4) have been integrated numerically by means of a predictor-corrector integration routine, to find the solution at regime with no beam load, and with beam pulses periodically applied across the cavity.* At each integration

* It is known from the theory of the numerical solution of a system of differential equations that, when the system has a high degree of symmetry (as it is the present case for $\rho_1 = \rho_2$, $GM_1 = GM_2$. . .) it is not easy to obtain the convergence of the integration routine at each step. Here results good to a high approximation (e.g., 10^{-5} to the theoretical value, calculated for no beam load) were obtained by rewriting Eqs. (4) in a different, non-symmetrical form, by introducting new auxiliary variables.

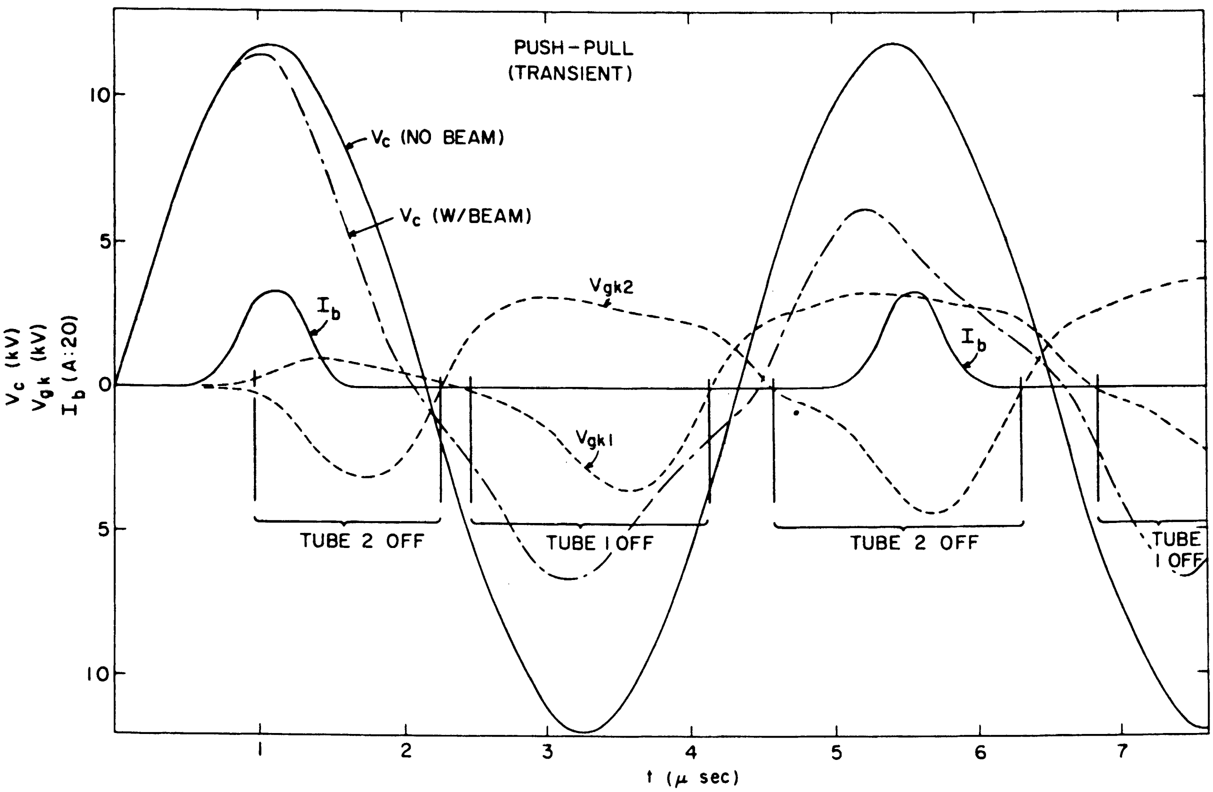

FIGURE 7 Results of a numerical analysis of a push-pull amplifier. Beam with peak current of 65 A at $\phi = -90°$. Tubes Nos. 2 and 1 cut-off alternately.
The curves V_c, V_{gk1}, V_{gk2}, are shown as a function of time in a transient regime soon after a beam has begun to be accelerated, to show clearly how the voltage period tends to decrease by a factor of $1/\sqrt{2}$.

step the conditions for cutoff of either tube were checked, by verifying whether the inequality

$$V_{pk} + \alpha \cdot V_{gk} + \beta \geq 0 \quad (5)$$

held. Here V_{pk} and V_{gk} are the plate-to-cathode and the grid-to-cathode voltages of a tube and α and β coefficients defining a straight line on the constant-current plate - grid characteristics chart of the tubes chosen (Fig. 5). The parameter set used in the calculation is given in Table I.

When condition (5) was not fulfilled in one of the two tubes in the push-pull arrangement, the tube was cut off and therefore excluded from the circuit, simply by assuming, in the calculation, that from that time on,

$$\frac{1}{\rho} \rightarrow 0, \quad GM \rightarrow 0 \quad (6)$$

with a time constant very short compared with a V_D period.

Figures 6 and 7 show some results of the calculations for the push-pull amplifier. Figure 6 gives the voltage V_c across the accelerating cavity as a function of time and the grid-to-cathode voltages in both tubes, without and with beam load. The beam-pulse shape is also shown for comparison.

For this figure, the beam shape chosen was a biased-cosine and the phase of the pulse with respect to the accelerating voltage was chosen at 0° (cavity voltage from accelerating to decelerating). Beam current was kept low enough (14 A peak) not to cause any cutoff. From the figure, drawn for steady state conditions, the deformation induced by the beam on the grid-to-cathode voltage of both tubes is clearly apparent.

The effect of a much higher current (like the one assumed to be ISABELLE's goal, (40 A peak)), is to drive tube No. 2 rapidly to cutoff, with tube No. 1 following to cutoff soon afterwards.

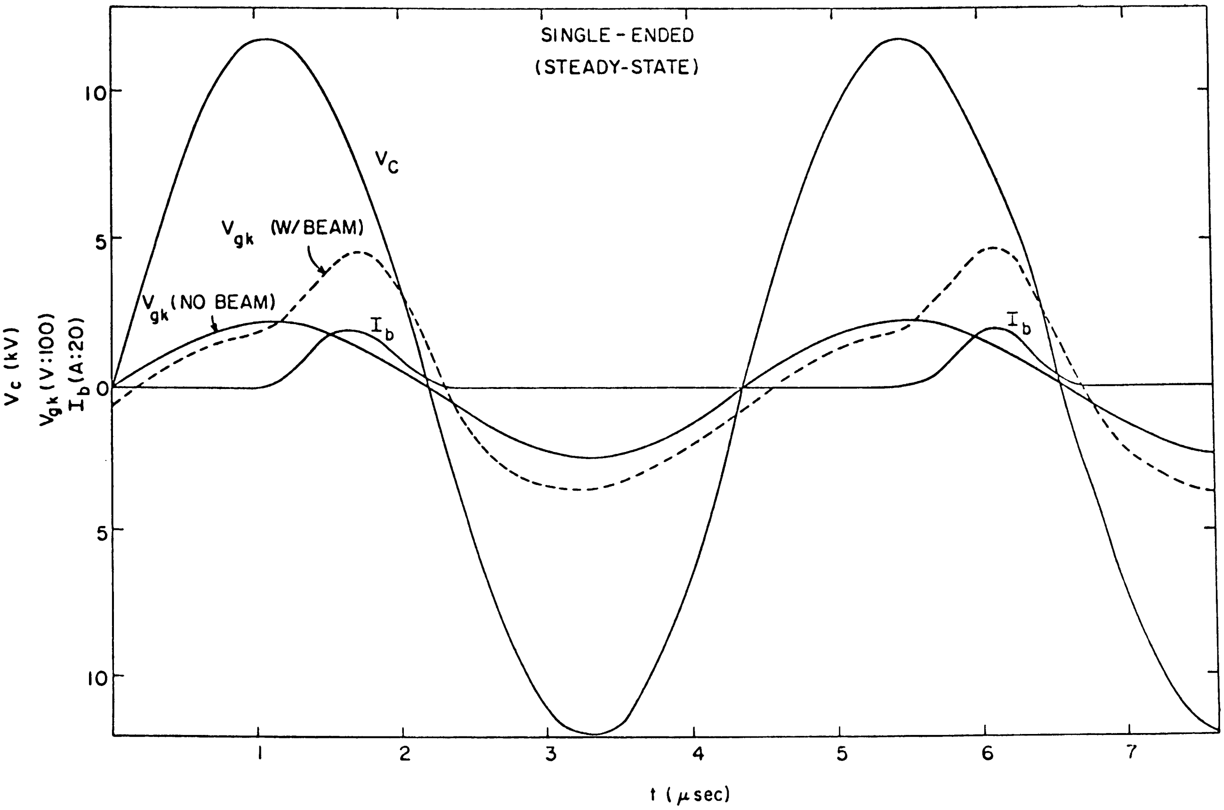

FIGURE 8 Results of a numerical analysis of a single-ended common-anode amplifier. Beam with peak current at 40 A at $\phi = -45°$. No cutoff occurs. In the same conditions, the push-pull amplifier is unable to produce acceleration of the beam.

An interesting effect arises for some values of the beam current and pulse phase. An example is shown in Fig. 7, for a phase of $-90°$: tube No. 2 goes rapidly to cutoff and comes back into operation again. Then tube No. 1 goes to cutoff and comes back, the whole process repeating itself at the next period. The voltage V_c across the cavity, is much reduced and the acceleration process is so upset as to become unpractical.

It is interesting to note that when tube No. 2 is cut off, the frequency of the voltage across the cavity tends to increase by a factor $\sqrt{2}$. This can be explained readily by inspection of the circuit of Fig. 4(a). With both tubes in operation, the two inductors are equally and strongly loaded by the tubes. When a tube is cut off, the inductor connected with that tube is no longer short-circuited by the tube itself, while the other is still practically short-circuited.

Figure 8 shows analogous curves for the case of a single-ended cathode-follower amplifier. The parameters used in this calculation are also given in Table I. The differential equations for this case are similar to Eqs. (4) and are therefore omitted.

Here the same high current that produces cutoff in the push-pull amplifier (and that therefore cannot be accelerated) does not do any harm to

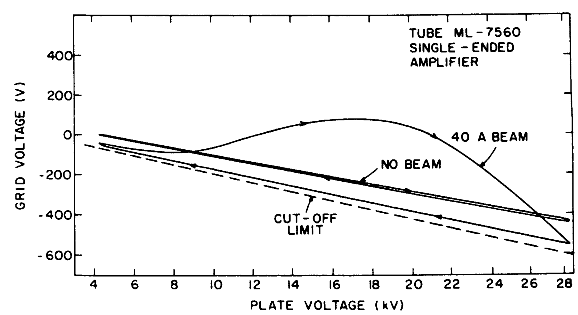

FIGURE 9 Results of a numerical analysis of a single-ended amplifier. Path of the working point of the tube in the constant-current grid voltage/plate voltage chart. With a 40 A peak beam, the closed curve grazes, but does not cross the cutoff limit straight line.

the single-ended amplifier. Only the grid-to-cathode voltage rises abnormally and the tube approaches saturation.

The behavior of the single-ended amplifier with and without beam load appears clearer from Fig. 9, where grid-to-cathode voltage vs plate-to-cathode voltage is shown. The cutoff straight-line limit appears also in the figure.

With no beam, the working point of the tube oscillates back and forth on a line almost parallel to the cutoff limit. When a beam is injected into the cavity for acceleration, the line becomes a closed loop that grazes cutoff, without trespassing it for the present choice of parameters.

APPENDIX

Since we are interested in the comparison between push-pull and single-ended amplifiers, we select a load that could simulate exactly the two conditions. With reference to Figs. 2 and 3, Z_s represents the cavity series impedance and Z_p the gap capacitance. For a dc analysis both are resistors. Z_s equals the resistive shunt impedance, while Z_p should increase indefinitely, so that it can be assumed to be very high.

The equation for the two cases are

(a) Push-pull:

$$
\begin{aligned}
V_{\text{gap}} &= V_c = V_{k_1} - V_{k_2} \\
V_c &= \left(V_D - \frac{2\rho}{\mu} I_b\right) F_s \\
V_{gk_1} &= \frac{V_D}{2}(1 - F_s) + \frac{\rho}{\mu} I_b F_s \\
V_{gk_2} &= -V_{gk_1} \\
F_s &= \frac{\mu}{\mu + 1} \, \frac{Z_p Z_s/(Z_p + Z_s)}{2\rho/(\mu + 1) + Z_p Z_s/(Z_p + Z_s)} \\
R_{\text{eq}} &= \frac{2\rho}{\mu + 1}
\end{aligned}
\tag{7}
$$

(b) Single-ended

$$
\begin{aligned}
V_{\text{gap}} &= V_c = V_k \\
V_{\text{gap}} &= \left(V_D - \frac{\rho}{\mu} I_b\right) F_u \\
V_{gk} &= V_D(1 - F_u) + \frac{\rho}{\mu} I_b F_u \\
F_u &= \frac{\mu}{\mu + 1} \cdot \frac{Z_p Z_s/(Z_p + Z_s)}{\rho/(\mu + 1) + Z_p Z_s/(Z_p + Z_s)} \\
R_{\text{eq}} &= \frac{\rho}{\mu + 1}
\end{aligned}
\tag{8}
$$

(c) Cathode follower derived from the push-pull with tube No. 2 cut-off

$$
\begin{aligned}
V_{\text{gap}} &= (V_D - Z_s I_b \beta) F_c \\
V_{gk} &= \frac{V_D}{2}\left[1 - \left(\frac{Z_s}{Z_p} + 2\right) F_c\right] + 2 I_b \frac{\rho}{\mu} F_c \\
F_c &= \frac{\mu}{\mu + 1} \cdot \frac{\frac{1}{4} Z_p Z_s/(Z_p + Z_s)}{\rho/(\mu + 1) + \frac{1}{4} Z_s (2Z_p + Z_s)/(Z_p + Z_s)} \\
\beta &= 1 + \frac{1}{\mu} + \frac{4\rho}{\mu Z_s}
\end{aligned}
\tag{9}
$$

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1. ISABELLE—A proton-proton colliding facility, Report BNL-50718, January 1978.
2. T. W. Hardek and W. E. Chyna, IEEE Trans. Nucl. Sci. **NS-26,** 3959 (1979).