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PRACTICAL HIGH-β INSERTIONS WITH APPLICATION TO A 1000-GeV SUPERCONDUCTING EXTENSION OF THE EUROPEAN ACCELERATOR

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In very high energy synchrotrons the final magnet apertures may be determined by the requirements of a high efficiency resonant extraction system. These apertures may be reduced by having an enhanced value of the β-function localized to the first extraction septum position. The effects of enhanced values of β at this position are determined for a 1000-GeV superconducting extension to the European Accelerator. Various methods of obtaining the required β-function are compared and a practical method based on a π-insertion is presented, which could have general application to very high energy accelerators.

1. INTRODUCTION

In the design study for a 1000-GeV superconducting extension to the European 300-GeV Accelerator,[1] the maximum aperture and hence the stored energy in the machine is determined by the aperture requirements[2] for a high efficiency (> 98 per cent), resonant extraction system. If an insertion could be found which had a high value of the β-function, localized to the region for the first septum position, then the extraction aperture requirements would be demagnified[3,4] in the normal cells of the machine by the factor:

$$\left[\frac{\beta_{\max}\text{ in a normal cell}}{\beta\text{ at the input to the first septum}}\right]^{1/2}.$$

For the insertion to be most effective, it is essential that the perturbation which produces the large value of β at the first septum position should not increase β in any dipole beyond $\beta_{\max}$ in a normal cell. For the 1000-GeV design study there is the added constraint that the insertion must be compatible with the conventional magnet lattice which has been designed for extraction at 300 GeV.

The effects of enhanced values of the β-function at the first septum position are first determined. After comparing various possible methods of obtaining the required β-function, a practical method based on a π-insertion is presented, which satisfies the above constraints and could have general application to very high energy accelerators.

2. CHOICE OF β VALUE

A schematic normalized phase space representation of third integral resonant extraction, at the input to the first septum S1, is illustrated in Fig. 1.

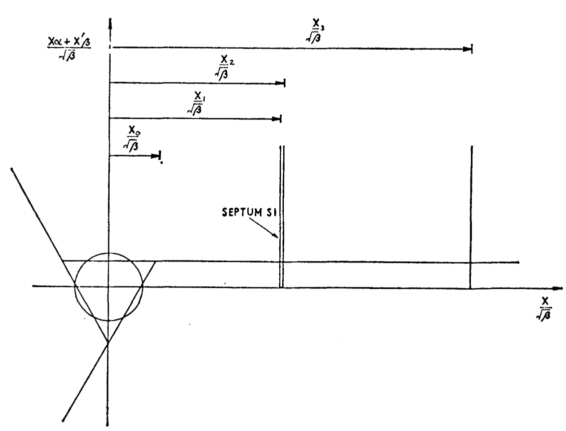

FIG. 1. Normalized phase space representation of third-integral extraction at the first extraction septum azimuth.

With reference to Fig. 1, Symon[5] gives the equation of particle motion along the separatrix as

$$\mathrm{d}x/\mathrm{d}\theta = k(x^2 - x_0{}^2),$$

where k is a constant related to the resonance driving term and x_0 is the displacement of the fixed point. The particle jump per three turns Δx, is given approximately by

$$\Delta x = 6\pi k(x^2 - x_0^2),$$
$$\Delta x \ll x.$$

Thus the particle density along the separatrix may be approximated to[2]

$$\rho = n/(x^2 - x_0^2)$$

where n = constant.

Hence the extraction efficiency is given by[2]

$$\eta = \frac{\log[(x_0+x_2)(x_0-x_3)/(x_0-x_2)(x_0+x_3)]}{\log[(x_0+x_1)(x_0-x_3)/(x_0-x_1)(x_0+x_3)]}. \quad (1)$$

From Eq. (1), it is possible to calculate for different β values at S1, the maximum aperture x_3 for given values of $(x_3 - x_1)$ and extraction efficiency. The aperture x_3 may then be transformed to β_{max} in a normal cell by multiplying by the factor $\{\beta_{max}/\beta_{S1}\}^{1/2}$.

The maximum value of β in a normal cell of the 300-GeV lattice is $\beta_{max} \simeq 100$ m. Therefore in Fig. 2, the normal cell maximum semi-aperture requirements for extraction from the 300-GeV-type lattice are plotted against β_{S1}, for various values of $(x_3 - x_1)$ and an extraction efficiency of 98 per cent. It is assumed that the beam emittance E is 1.0 μmrad at 200 GeV and 0.5 μmrad at 1000 GeV, and that the septum thickness is 0.15 mm. The displacement of the fixed point is given by Symon[5] as

$$x_0 = (\pi E\beta_{S1}/\sqrt{3})^{1/2}.$$

In general there are constraints on the solutions as presented in Fig. 2. The septum must be positioned outside the injected beam profile. Also, sufficient distance should be allowed between the fixed point x_0 and the septum x_1, to obtain a reasonable jump growth rate. Therefore, the values of $(x_1 - x_0)$, corresponding to the parameters for Fig. 2, are plotted in Fig. 3.

From Fig. 2, it can be seen that for each value of β_{S1}, there is a broad minimum in the range of values of $(x_3 - x_1)$, from 12.5 mm to 15.0 mm,

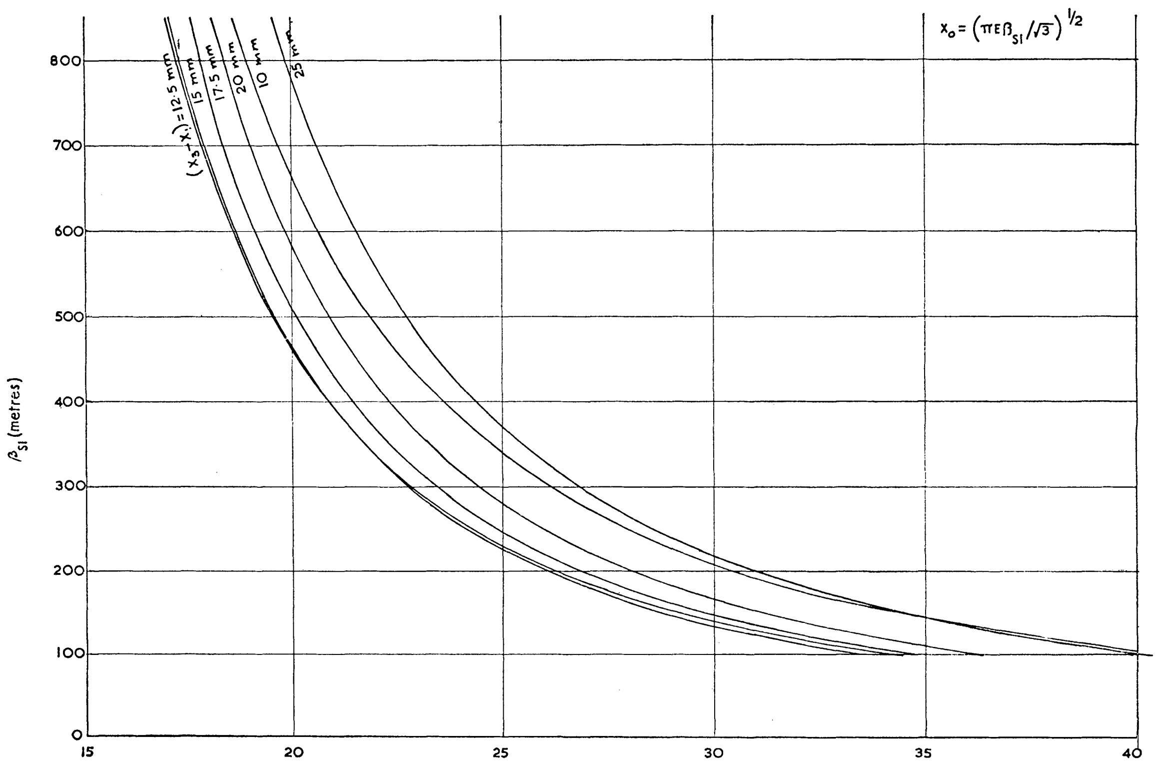

FIG. 2a. Maximum semi-aperture in a normal cell required at extraction, versus β_{S1}, for different septum apertures $(x_3 - x_1)$. The calculations are for an extraction efficiency η, of 98 per cent.

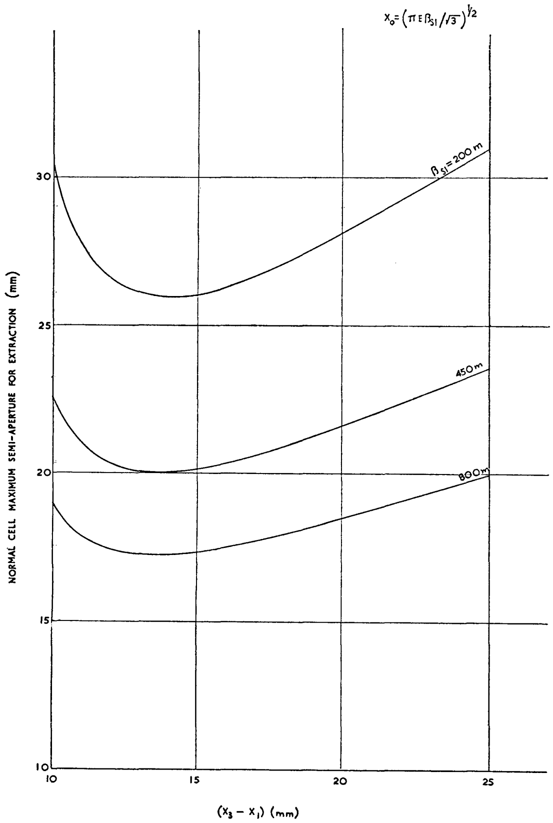

FIG. 2b. Maximum semi-aperture in a normal cell required at extraction versus extraction septum aperture (x_3-x_1), for different values of β_{S1}. (η = 98 per cent.)

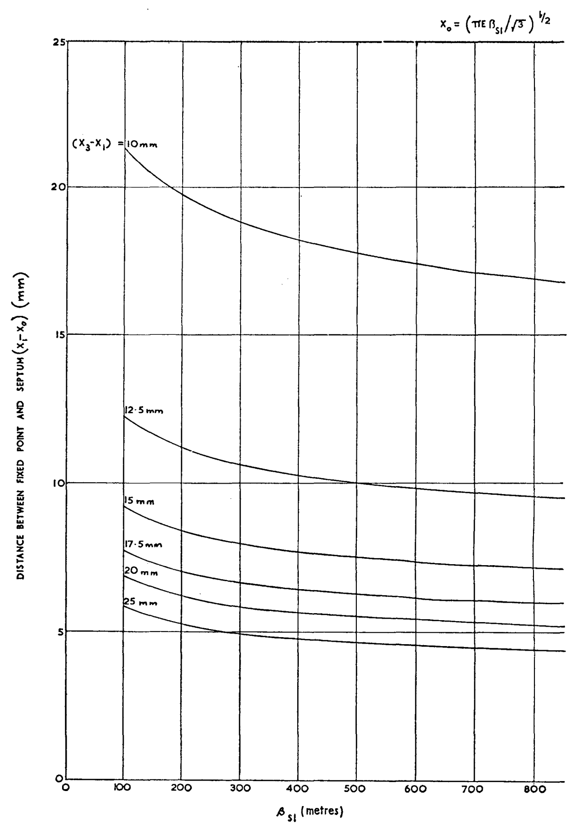

FIG. 3. Distance available for particle jump growth at the extraction septum (x_1-x_0), versus β_{S1}, for different values of the septum apertures (x_3-x_1). (η = 98 per cent.)

which minimizes the aperture in the normal cell. Also it can be seen that the aperture in the normal cell decreases as β_{S1} increases, but the rate of decrease falls quite rapidly as β_{S1} increases.

3. STORED ENERGY

The main reason for obtaining a reduction in the apertures in the normal cells of the machine is to minimize the stored energy: the stored energy in a superconducting machine is approximately proportional to the square of the total aperture. Thus before translating the apertures of Fig. 2 into stored energy, extra aperture must be added for closed orbit distortion, sagitta, synchrotron oscillations and bad field allowance. For the 1000-GeV extension this extra aperture amounts to 18.6 mm, comprising 5 mm C.O. distortion, 3.2 mm sagitta, 0.4 mm synchrotron oscillation (assuming $\Delta p/p \simeq 0.1\times 10^{-3}$ at extraction) and 10 mm bad field allowance.

Figure 4 shows the variation of stored energy with β_{S1} for (x_3-x_1) in the range 12.5 mm to 15 mm and for two designs of machine. The first design assumes all the dipoles have identical circular apertures defined by $\beta=100$ m. The second design assumes that half the dipoles are as above and half have apertures defined by β = 50 m. The stored energies are normalized to the design 1 machine with $\beta_{S1}=100$ m. It can be seen from Fig. 4 that significant savings in stored energy may be obtained for values of β_{S1} up to approximately 450 m. Above this value of β_{S1} only a very small further saving may be obtained. With reference to Fig. 3, it can be seen that for this value of β_{S1} and (x_3-x_1) in the range 12.5 mm to 15 mm, the

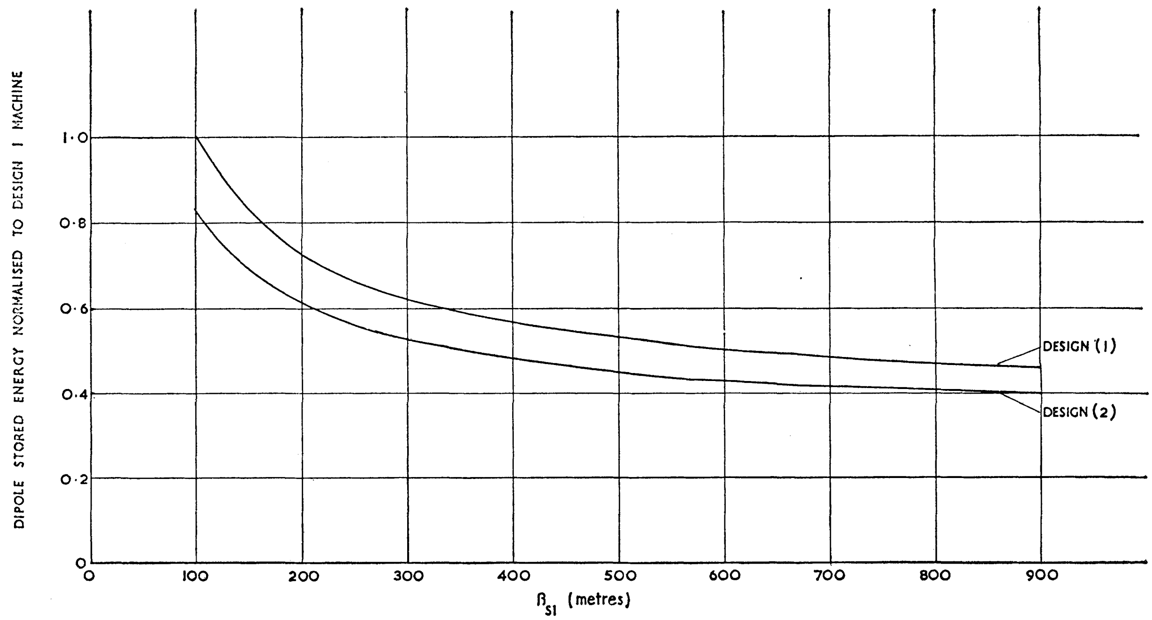

FIG. 4. Variation of stored energy with β_{S1} for the two 1000-GeV machine designs described in Sec. 3.

distance available for jump growth ranges from 10 mm to 7.5 mm.

4. OBTAINING HIGH β-VALUES

In order to meet the constraint of compatibility with the 300-GeV conventional magnet lattice design, an 'overlay' type insertion was first studied. By 'overlay' is meant the inclusion of elements within the existing lattice which are excited at or near full energy to produce the desired β-value at S1. It was found that to obtain convergent solutions for this type of insertion β became unacceptably large in some of the normal cell dipoles. Thus although it may be possible to save stored energy this would be at the expense of extra types of dipole. A further complication with this type of insertion is the maintenance of beam stability during switch-on.

Therefore, to obtain the required β_{S1} value without perturbing the normal cells, the straight sections of the 300-GeV lattice design must be replaced by special matched insertions of the same length, i.e. $\simeq$ 128 m (two normal lattice periods). Also to leave maximum space for the extraction elements it is important that β should increase to the desired value in as short a distance as possible at the beginning of the insertion. Thus the most advantageous insertion point in the normal lattice is immediately before the quadrupole which is focusing in the plane of extraction. At this point β is near β_{max} and increasing and to ensure a rapid growth in β the first matching quadrupole in the insertion should be defocusing.

5. MATCHED INSERTIONS WITH HIGH β-VALUE

In theory a matched insertion may have any phase advance, but those with 2π or π have the distinct advantage that they are matched in transverse phase space for all input conditions.

5.1. 2π-*Insertions*

Besides matching the transverse phase space for all input conditions, the 2π-insertion also has the apparent advantage that the dispersion parameters α_p and α_p' are also matched. However, α_p may be considerably enhanced within the insertion particularly when high β-values are required.

An example of this type of insertion with a high β-value has been reported by Neyret and Parain.[6] Their insertion required 192 m of straight section

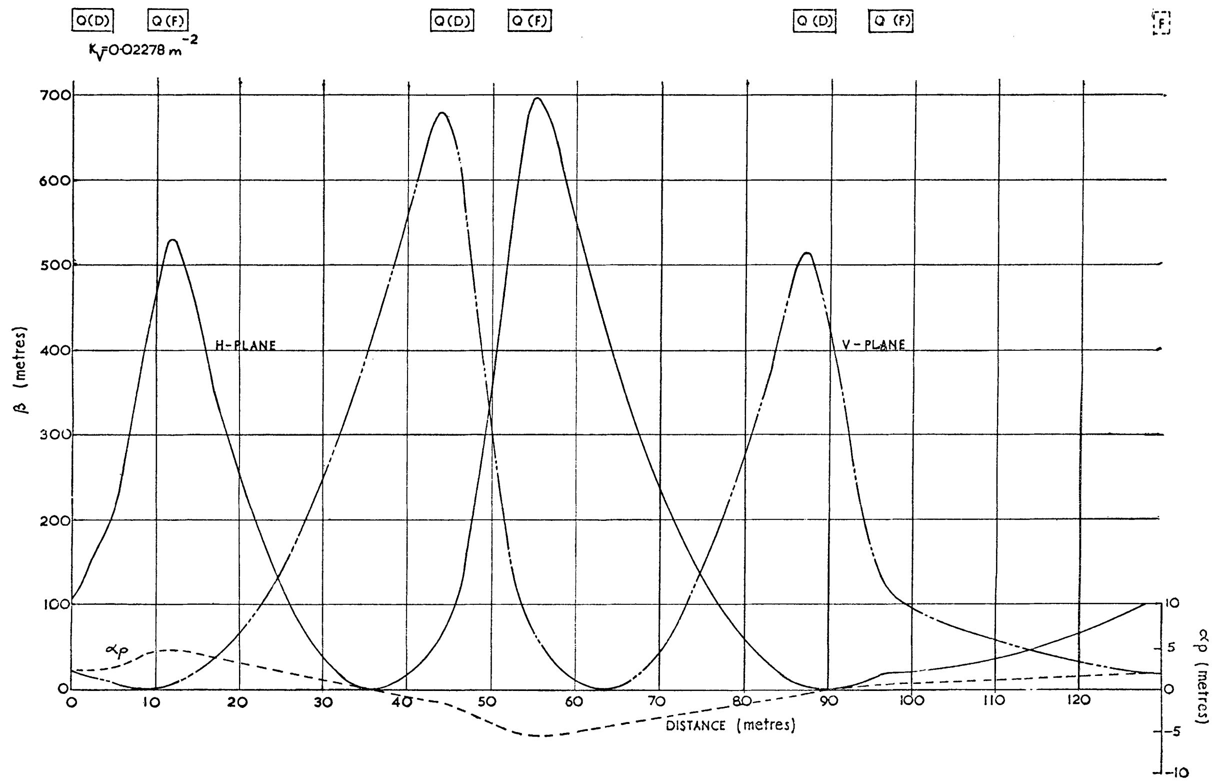

FIG. 5. Betatron and momentum functions within a three doublet, 2π high-β insertion.

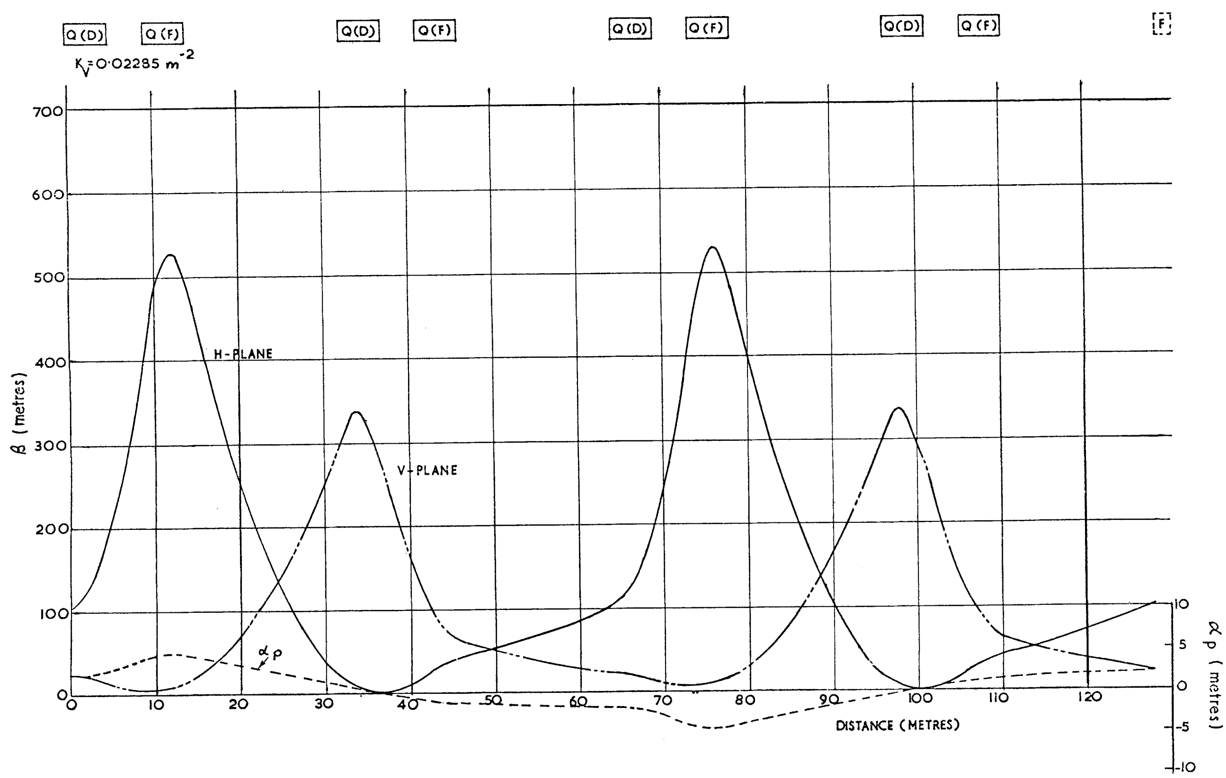

FIG. 6. Betatron and momentum functions within a four doublet, 2π high-β insertion.

and does not satisfy the criteria outlined in the previous section.

High-β 2π-insertions were also studied for the combined function lattice of the Nina Booster.[3]

Figures 5 and 6 show two possible solutions for a high β-value 2π-insertion within 128 m using three doublets and four doublets respectively. The quadrupole parameter K_v, is defined by the relation

$$K_v = \text{quadrupole gradient } (g)/\text{beam rigidity } (B\rho)$$

The three-doublet insertion provides three useful straight sections, each of 28.7 m. From Fig. 5, it can be seen that the septum S1 may be placed immediately after the first doublet, as in this position β is approximately 500 m. However, there is a subsequent maximum in β at the second doublet and this occurs for all useful values of β_{S1}. It should be noted that by moving the doublets, β_{S1} may be varied at will: the 2π phase advance is always assured provided the periodicity is maintained within the insertion.

The aperture of the quadrupoles in the above insertion is determined by the maximum β subsequent to β_{S1}: this difficulty may be avoided by using four doublets positioned periodically for the insertion; see Fig. 6. However, the useful straight sections become 4×18 m.

A disturbing feature of the 2π insertions is the possible enhancement of space charge effects in the region of the extremely low β values ($\beta_{min} \simeq 1$ m). Also although α_p in the normal cells is unaffected $\{(\alpha_p)_{max} \simeq 3.8 \text{ m}\}$, it is enhanced within the insertion $\{(\alpha_p)_{S1} \simeq 4.3 \text{ m}\}$.

5.2. π-*Insertions*

A π-insertion using two doublets positioned periodically, is illustrated in Fig. 7. This provides two useful straight sections each of 52 m.

A high value of β_{S1} may be achieved without a subsequent maximum. Again β_{S1} may be varied at will, provided the doublet periodicity is maintained within the insertion. As shown in Fig. 7

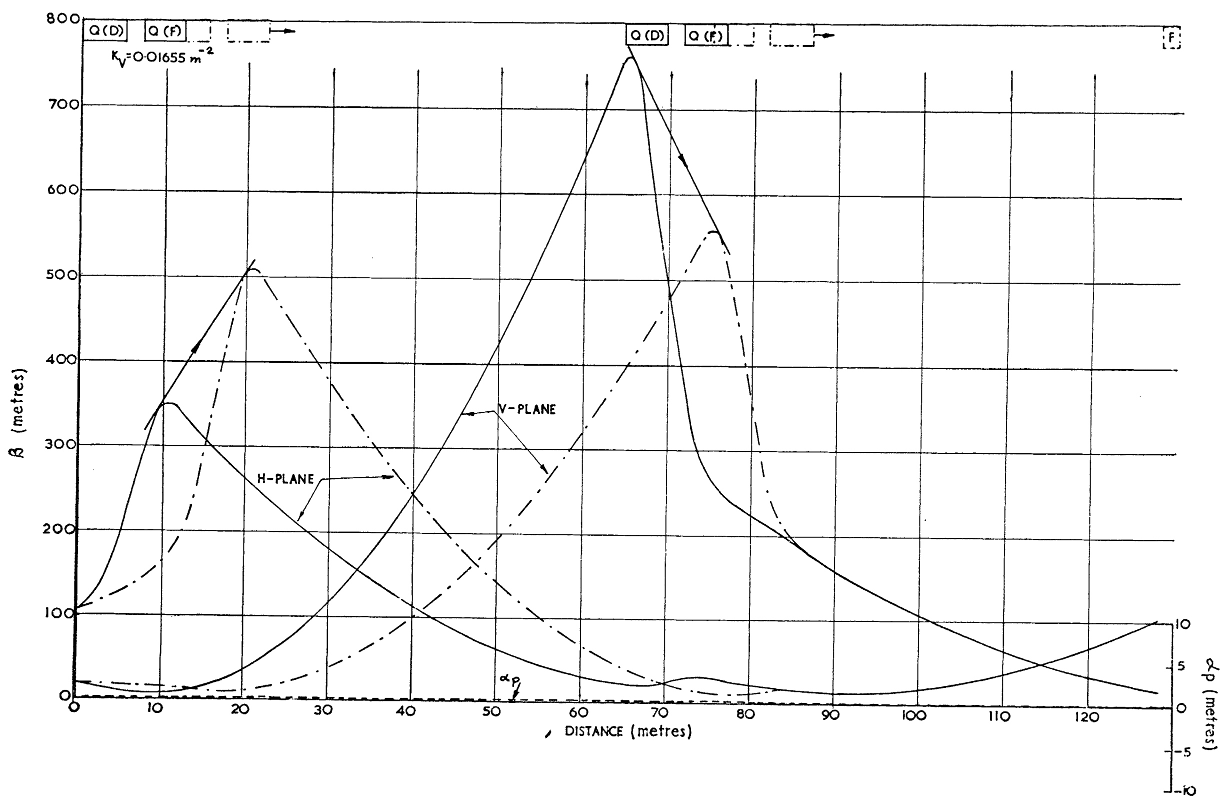

FIG. 7. Betatron and momentum functions within a two doublet, π high-β insertion. The chain-dot lines show the betatron functions with the doublets displaced 10 m down the straight section.

for example, β_{S1} may be varied from 350 m to 500 m by moving the doublets 10 m down the straight. The available straight is then finally divided into three sections of 10 m, 52 m and 42 m respectively. The minimum value of β is approximately 10 m.

Although this insertion does not match α_p and α_p', the mismatch in a normal cell is small and within the insertion $\alpha_p \simeq 0$ m. Without the provision of straight sections $(\alpha_p)_{max} \simeq 3.8$ m: with this insertion and a six-fold superperiodicity $(\alpha_p)_{max} \simeq 4.2$ m in a normal cell. A similar value of $(\alpha_p)_{max} \simeq 4.2$ m is obtained when the straights are provided by the omission of bending magnets from two normal cells (i.e. 'FODO insertion'). A further advantage with this insertion is that the matching quadrupoles require considerably lower gradients than those necessary for the 2π-type.

It can be seen therefore, that the π-insertion satisfies all the criteria for a practical high-β insertion and in addition provides two very long straight sections for extraction purposes. For the 1000-GeV extension, preliminary calculations show that savings in stored energy of up to 50 per cent may be made and the provision of the two very long straight sections is extremely favourable for extraction at 1000 GeV.

6. CONCLUSIONS

For a 1000-GeV superconducting extension to the European 300-GeV machine it is possible to save up to 50 per cent in machine stored energy by arranging to have β at the first extraction septum about 450 m while β_{max} in a normal cell is about 100 m. Above this value of β at the first septum position the savings in stored energy diminish rapidly and may be offset by practical difficulties in providing the necessary large aperture quadrupoles in the insertion.

The use of a matched insertion with a π-phase advance, suitably positioned in the normal lattice, is considered a practical method of providing the required high value of the β-function at the first septum position without perturbing the β-function in the normal lattice. The use of such an insertion in a possible 1000-GeV extension,[1] could result in a final machine stored energy of approximately 300 MJ. The insertion produces only a very small mismatch in α_p and in addition the value of α_p within the insertion is sensibly zero.

In this paper only radial extraction has been considered. However, by applying the same criteria, a similar insertion may be found for vertical extraction.

In addition to savings in stored energy, the arrangement of the insertion quadrupoles provides two very long straight sections which simplify extraction at 1000 GeV.

The use of a π-insertion to produce high β may allow significant aperture reductions in machines in which the final apertures are set by extraction requirements. However, by suitable choice of the insertion point, they may also simplify ejection from machines in which the apertures are determined by injection or transition requirements. This possibility is being studied for a conventional-magnet 300-GeV-type lattice.

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