EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

INTENSITY ESTIMATION FOR CYCLOTRONS

Stig Holm,

CERN, Geneva, Switzerland.

ABSTRACT

A calculation of the minimum value of the focusing force near the centre in a cyclotron is made assuming magnetic as well as electric focusing. By equating the vertical space-charge force and the minimum focusing force, expressed as a function of the starting phase angle, the intensity limit is determined by an integration over the starting phase interval. The current is found to vary with the $^{3}/_{2}$-power of the dee voltage in CW cyclotrons, and proportionally to the dee voltage times the repetition rate in synchro-cyclotrons. In the weak focusing synchro-cyclotron, a high slope of the magnetic field in the centre is favoured. The optimum value of the initial equilibrium phase depends on the injected phase range as well as on the magnetic field law, and is found quite close to the phase of zero energy gain.

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1. INTRODUCTION

Mechanisms for current limitations in cyclotrons and synchro-cyclotrons have recently been discussed extensively[1-6]. Several reasons for the observed limitations, especially those in synchro-cyclotrons, have been put forward. The vertically directed fields from the accelerated charge can, if sufficiently strong, create beam losses against the electrodes near the centre[2]. Beam-loading effects might in some cases cause unstable accelerating conditions[5]. Azimuthal space-charge effects can modify the phase motion in synchro-cyclotrons and induce losses[6]. The above-mentioned limitation occurring at small radii seems in many cases to be the most fundamental one. In this paper it will therefore be taken as a basis for the estimation of obtainable currents.

Past calculations have been done without taking into account the electrostatic focusing. Depending on the onset of the magnetic force, one arrives at current laws which go from $I \sim V^3$ for an abrupt onset of the magnetic force at a certain radius, to $I \sim V^{5/3}$ for a parabolic shape of the magnetic field. In the following calculations, first-order electrostatic focusing will be included. It is assumed that the narrow-gap theory is applicable. This excludes the conventional open-source, wide-gap synchro-cyclotron, where the particles make hundreds of revolutions within the dee gap.

2. THE SPACE-CHARGE FORCE

For the range of starting phases, which are electrically focused, the total vertical focusing force decreases with radius to a minimum value. This occurs at a radius where the rate of decrease of the electric force equals the rate of increase of the magnetic one. The calculations given below are based on the assumption that the space-charge limitation takes place here. Particles in the main part of the useful starting-phase range have to travel several turns in order to reach this radius. The separation between the individual turns can then be neglected[7] when calculating the space-charge force, and the charge can be considered to be smeared-out radially. In the median plane near the centre, the accelerated ions occupy a sector-shaped area. With a beam height much smaller than the azimuthal dimension of the sector, the vertical space-charge force can be approximated by the force from an infinite sheet of charge[1,2]. In MKSA units this force is

$$F_{sp} = \frac{m\omega I}{2\epsilon_0 V_0} , \qquad (1)$$

where I is the instantaneous current, V_0 is the voltage gain per turn, m is the particle mass, and ω is the orbital angular frequency. In general, particles starting off from the centre with a given phase angle φ_0 may reach the critical point at a moment when the phase angle φ is different from φ_0. However, considering the motion to be unrelativistic, we have area conservation in a $\Delta(r^2)$, $\Delta\varphi$-plane. This means that a small surface element r dr dφ is constant during acceleration. As the space-charge force (1) is proportional to the density projected on the median plane one has in Eq. (1) $V_0 = V \cos \varphi_0$, where V is the maximum voltage gain per turn.

Under the more realistic assumption that the above-mentioned sector of charge has a vertical height comparable to the azimuthal width, the situation is more complicated since the azimuthal and vertical fields are of the same order of magnitude. The vertical field components at the edges of the sector will be considerably lower than in the infinite sheet approximation. The effect is most important at the upper end of the phase range where the focusing is strongest. Lacking a better representation of the space-charge force, we will use Eq. (1) with the modification $V_0 = V \cos \Phi$, with Φ equal to the phase angle in the middle of the starting phase range, which is assumed to extend from $\varphi_0 = 0$ to at most $\varphi_0 = 75°$.

3. THE $Q^2_{z\,min}$-POINT

The focusing force acting on a particle at a distance z above the median plane is

$$F = -m\omega^2 Q_z^2 \, z , \qquad (2)$$

where m is the particle mass, and Q_z^2 is the vertical betatron frequency in units of oscillations per turn. For the following calculation we write

$$Q_z^2 = \frac{eVc^2}{2\pi E_0 \omega^2} \cdot \frac{\sin\varphi}{r^2} + (K-1) \cdot \frac{\omega^2}{c^2} \cdot r^2 + \alpha r^2 \, , \qquad (3)$$

where E_0 is the rest energy of the particle, r is the radius, c is the velocity of light, and φ is the phase angle of the dee voltage when the particle passes the middle of the dee gap ($\varphi = 0$ when the gap traversal occurs at maximum accelerating voltage). The first term is due to first-order electric focusing, and the second term is due to the weak focusing of the magnetic field. Instead of the field index n, we have here used $K = 1 + nc^2/\omega^2 r^2$ [8]. We consider K to be constant at small radii, which is true for a parabolic shape of the mean magnetic field. The third term gives for $\alpha \neq 0$ the contribution from an azimuthally varying magnetic field. For the simplicity of the following calculations, we have chosen it to have the same radial dependence as the second term.

Using Eqs. (1), (2), and (3) one can find a "balance line" $z = \bar{z}(r)$ for which there is no vertical acceleration of the particle. Although this line is not a solution of the differential equation for the vertical motion, the solutions will exhibit oscillations around it[9]. Disregarding these oscillations, we seek the maximum point of the balance line, thus the minimum of Q_z^2. From Eqs. (1) and (2) we then get, with z = h, the half aperture,

$$I = 2\epsilon_0 V_0 \omega Q_{z\,min}^2 \, h \, . \qquad (4)$$

As the electrostatic focusing is phase-dependent, one has to take into account the phase motion during the first part of the acceleration, when calculating $Q_{z\,min}^2$. We do so using the phase-slip equation:

$$\frac{d\varphi}{2\pi} = \frac{\omega_s - \omega}{\omega} \cdot \frac{dE}{eV\cos\varphi} \, , \qquad (5)$$

where

ω_s = synchronous revolution frequency

ω = ion revolution frequency

E = energy.

When the particle starts at zero radius the phase is φ_0 and the synchronous revolution frequency (i.e. the frequency of the accelerating voltage) is ω_{so}. At the time t we have

$$\omega_s = \omega_{so} + \dot{\omega}_s t . \tag{6}$$

The connection between frequency-time derivative $\dot{\omega}_s$, and the equilibrium phase φ_s, is given by[10]

$$\dot{\omega}_s = -\frac{eV\omega^2 K}{2\pi E} \cos \varphi_s . \tag{7}$$

Here E is the total energy of the particle, for our purpose equal to the rest energy E_0. We write the revolution frequency ω as

$$\omega = \omega_0 + \Delta\omega , \tag{8}$$

where ω_0 is the frequency at r = 0, and $\Delta\omega$ is the change in frequency when the particle is at the radius r. Expressing this in radius we have first from the definition of K:

$$\frac{\Delta\omega}{\omega} = -K \frac{\Delta E}{E} .$$

For small $\Delta E/E$

$$\Delta E = \frac{E_o}{2c^2} \omega^2 r^2 ,$$

which gives

$$\Delta\omega = -\frac{K\omega^3}{2c^2} r^2 . \tag{9}$$

The connection between time t and radius r is given by

$$dt = \frac{2\pi E_o}{eV\ c^2} \frac{\omega r}{\cos \varphi} dr . \tag{10}$$

For the purpose of our calculation we may neglect the variation of $\cos \varphi$ with radius in Eq. (10). Thus

$$t = \frac{\pi E_0 \omega}{eV\ c^2\ \cos\varphi_0}\, r^2 \, . \tag{11}$$

The time derivative of the phase is equal to the difference between electric and ion revolution frequency. Thus at zero radius we have

$$\dot{\varphi}_0 = \omega_{s_0} - \omega_0 \, . \tag{12}$$

Using the expressions (6) to (12) and

$$dE = E_0 \frac{\omega^2}{c^2}\, r\, dr$$

we can integrate Eq. (5) and find

$$\sin\varphi = A + Br^2 + Cr^4 \tag{13}$$

where

$$A = \sin\varphi_0$$

$$B = \frac{\pi E_0 \omega\, \dot{\varphi}_0}{eV\ c^2}$$

$$C = \frac{\pi E_0 \omega^4 K}{4eV\ c^4}\left(1 - \frac{\cos\varphi_s}{\cos\varphi_0}\right) \, .$$

Introducing this result into Eq. (3) we get

$$Q_z^2 = \frac{A_0 A}{r^2} + A_0 B + (A_0 C + B_0)\cdot r^2 \, , \tag{14}$$

where

$$A_0 = \frac{eV\ c^2}{2\pi E_0 \omega^2}$$

$$B_0 = (K - 1)\cdot\frac{\omega^2}{c^2} + \alpha \, .$$

For $\sin\varphi_0 > 0$ we can now find a minimum of Eq. (14) by derivation with respect to r and multiplying with dr/dt. Now dr/dt = 0 when the phase reaches $\pi/2$. Assuming that this occurs after $dQ_z^2/dr = 0$ we have a minimum point at $dQ_z^2/dr = 0$. The radius at which this minimum occurs is $r_{min} = [A_0A/(A_0C + B_0)]^{1/4}$. With the constants inserted:

$$r_{min} = \sqrt{2}\frac{c}{\omega}\left(\frac{eV \sin\varphi_0}{\pi E_0 K}\right)^{1/4}\left[1 - \frac{\cos\varphi_s}{\cos\varphi_0} + 8\left(\frac{K-1}{K} + \frac{\alpha}{K}\frac{c^2}{\omega^2}\right)\right]^{-1/4} . \quad (15)$$

As a numerical example: $V = 60$ kV, $K = 2$, $\varphi_0 = 45°$, $\cos\varphi_s = 0.3$, and a magnetic field of 1.9 Wb/m² gives for protons $r_{min} = 8$ cm. The number of turns required to reach this radius is about 26.

Introducing Eq. (15) into Eq. (14) we find

$$Q^2_{z\,min} = \frac{1}{2}\left(\frac{eV\,K}{\pi E_0}\right)^{1/2}\left[1 - \frac{\cos\varphi_s}{\cos\varphi_0} + 8\left(\frac{K-1}{K} + \frac{\alpha}{K}\cdot\frac{c^2}{\omega^2}\right)\right]^{1/2} \cdot \sin\varphi_0^{1/2} + \frac{\dot\varphi_0}{2\omega} . \quad (16)$$

The terms $1 - (\cos\varphi_s/\cos\varphi_0)$ and $\dot\varphi_0/2\omega$ are due to the slip in phase which occurs between $r = 0$ and $r = r_{min}$. The interesting range of $\dot\varphi_0$ lies inside the phase stability region ($K \neq 0$), that is

$$|\dot\varphi_0| \leq \omega\left(\frac{eV\,K}{\pi E_0}\right)^{1/2} \cdot F^{1/2}(\varphi_0,\varphi_s)$$

where

$$F(\varphi_0,\varphi_s) = \sin\varphi_0 + \sin\varphi_s - (\varphi_0 + \varphi_s)\cos\varphi_s . \quad (17)$$

In this region the accuracy of Eq. (16) is generally in the order of a few per cent for φ_0 ranging from 0-75° except for high φ_0 when $\dot\varphi_0 > 0$. A particle with these starting conditions may reach $\varphi = \pi/2$ and begin to be decelerated before the radius as given by Eq. (15) is reached. $Q^2_{z\,min}$ occurs then at $\varphi = \pi/2$ and is larger than the value given by Eq. (16). We neglect this effect here as the mentioned starting conditions are present only in synchro-cyclotrons, and then for a small fraction of the captured particles.

4. CW CYCLOTRON

The total current is found from Eqs. (16) and (4) by averaging over one period. Assuming $K \neq 0$ we have for the interval (φ_1, φ_2)

$$\bar{I} = \frac{\varepsilon_0 \omega \cos \Phi h}{2\pi} \left(\frac{eK}{\pi E_0} \right)^{1/2} \cdot V^{3/2} \int_{\varphi_1}^{\varphi_2} \left[G^{1/2}\left(\varphi_0, \frac{\pi}{2}\right) + X \right] d\varphi_0 \qquad (18)$$

where we have used

$$G(\varphi_0, \varphi_s) = \left[1 - \frac{\cos \varphi_s}{\cos \varphi_0} + 8\left(\frac{K-1}{K} + \frac{\alpha}{K} \frac{c^2}{\omega^2} \right) \right] \sin \varphi_0 \qquad (19)$$

$$\dot{\varphi}_0 = \omega \left(\frac{eV K}{\pi E_0} \right)^{1/2} \cdot X \ . \qquad (20)$$

For negative $\dot{\varphi}_0$, φ_1 is determined from

$$G^{1/2}(\varphi_1, \varphi_s) = - X \ ,$$

as the integrand always must be positive. With other parameters being constant, we have in Eq. (18) a $V^{3/2}$ dependence. (However, with a given arrangement of the extracting electrodes in the centre, φ_2 is likely to increase with dee voltage. Assuming a conical aperture in the centre, the same is true for h, which is the aperture at the $Q^2_{z\,min}$-point. With $h \sim r$, one would have $h \sim V^{1/4}$ from Eq. (15), thus at least $\bar{I} \sim V^{7/4}$.)

In the isochronous case (K = 0) one arrives at

$$\bar{I} = \frac{\varepsilon_0 \omega \cos \Phi h V}{\pi} \int_{\varphi_1}^{\varphi_2} \left\{ \left[\frac{2eV}{\pi E_0} \left(\alpha \cdot \frac{c^2}{\omega^2} - 1 \right) \sin \varphi_0 \right]^{1/2} + \frac{\dot{\varphi}_0}{2\omega} \right\} d\varphi_0 \qquad (21)$$

5. SYNCHRO-CYCLOTRON

The time during which a particle of a given starting phase φ_0 can be captured into phase-stable orbits is[10]

$$\Delta t(\varphi_0) = \frac{4}{\omega}\left(\frac{\pi E_0}{eV\,K}\right)^{1/2} \cdot \frac{L(\varphi_0,\varphi_s)}{\cos\varphi_s} \quad . \tag{22}$$

During this time, particles in the interval $(\varphi_0,\ \varphi_0 + d\varphi_0)$ are captured within a fraction $d\varphi_0/2\pi$ of each RF cycle. We can therefore write the captured current as

$$di = I \cdot \Delta t\ fm \cdot \frac{d\varphi_0}{2\pi} \tag{23}$$

where fm is the repetition rate of the acceleration cycle and I is the current supplied during each $d\varphi_0$ interval. Assuming that I is limited by space charge, we have from Eqs. (4), (16), (19), (20), and (22)

$$di = \frac{2}{\pi}\, fm \cdot \epsilon_0\ V \cos\Phi\ h\left(G^{1/2}(\varphi_0,\varphi_s) + X\right) \cdot \frac{L(\varphi_0,\varphi_s)}{\cos\varphi_s}\, d\varphi_0 \quad . \tag{24}$$

From Eqs. (12) and (20) one has $X \sim \omega_{so} - \omega_0$. As the assumption $d\omega_{so}/dt$ = const is valid during the capture time, we can average X in Eq. (24) from its value at the beginning and at the end of the capture period. The region within which X may vary can be seen in Fig. 1, which shows the initial phase space with focusing and capture areas in a typical case. Only the overlapping part of the areas is useful. Particles start to be captured at $X = 2L - F^{1/2}$. For $\varphi_{11} \leq \varphi_0 \leq \varphi_{12}$, where φ_{11} and φ_{12} are given by $2L(\varphi_{11},\varphi_s) - F^{1/2}(\varphi_{11},\varphi_s) = -G^{1/2}(\varphi_{11},\varphi_s)$ and $G^{1/2}(\varphi_{12},\varphi_s) = F^{1/2}(\varphi_{12},\varphi_s)$, respectively, focusing is lacking when $X \leq -G^{1/2}$. Here the effective capture time is thus proportional to $2L - F^{1/2} + G^{1/2}$ instead of 2L [as in Eq. (22)] and the average value of X is $L - (F^{1/2} + G^{1/2})/2$. The angle φ_{12} is generally $<< 1$ so we have from Eqs. (17) and (19) approximately

$$\varphi_{12} = \frac{K}{8} \cdot \frac{\sin\varphi_s - \varphi_s\cos\varphi_s}{K - 1 + \alpha \cdot \frac{c^2}{\omega^2}} \quad . \tag{25}$$

For $\varphi_0 \geq \varphi_{12}$, $Q^2_{z\,min}$ is positive during the whole capture time, Eq. (22), which ends at $X = -F^{1/2}$, so X averages to $L - F^{1/2}$. Introducing this result into Eq. (24) with the proper modifications of the factor 2L, we find the current by integrating over the starting-phase interval (φ_1, φ_2)

$$i = \frac{2}{\pi}\,\epsilon_0\, fm \cos\Phi\, h \cdot V \left[\int_{\varphi_{11}}^{\varphi_{12}} \left(\frac{G^{1/2}(\varphi_0,\varphi_s) - F^{1/2}(\varphi_0,\varphi_s)}{2} + L(\varphi_0,\varphi_s) \right)^2 \cdot \frac{d\varphi_0}{\cos\varphi_s} \right.$$

$$\left. + \int_{\varphi_{12}}^{\varphi_2} \left(G^{1/2}(\varphi_0,\varphi_s) + L(\varphi_0,\varphi_s) - F^{1/2}(\varphi_0,\varphi_s) \right) \cdot \frac{L(\varphi_0,\varphi_s)}{\cos\varphi_s}\, d\varphi_0 \right] . \qquad (26)$$

We have assumed that $\varphi_1 < \varphi_{11}$ and $\varphi_2 > \varphi_{12}$. Other parameters being constant, the current is thus proportional to the dee voltage. However, for a given final energy, the repetition rate can be increased proportionally to the dee voltage, which would give the current a V^2-dependence.

The two integrands (denoted Int_1 and Int_2, respectively) are shown in Fig. 2 for $\cos\varphi_s = 0.3$. $K < 1$ corresponds to an average field increasing and $K > 1$ to a field decreasing with radius. It should be remarked that φ_s as well as K are initial values, valid in the region of the first phase oscillation. The figure shows the importance of the higher starting phases which are subject to both better focusing and capture. Note also that for small α $(\alpha < \omega^2/c^2)$ the value of the integrand increases with K, making a high slope of the magnetic field at small radii desirable in a conventional weak-focusing synchro-cyclotron.

In Figs. 3 to 6 the integration has been performed and the current, normalized to $i_N = (2/\pi)\,\epsilon_0\, fm \cos\Phi\, h \cdot V$, is shown as a function of $\cos\varphi_s$ for three different starting phase ranges. As a numerical example, we may use data from the Orsay synchro-cyclotron: $V = 44$ kV, $K = 2$, $fm = 455$ p/sec, $h = 2.5 \times 10^{-2}$ m, and starting-phase range 0-45°. Choosing the maximum point in the curve Fig. 3 we find $i = 5.2\ \mu A$ [actually measured value $4.6\ \mu A$[11]].

In order to find the condition which optimizes the current i, the variation of repetition rate with the initial K and $\cos\varphi_s$ must be known.

However, this will depend on the magnetic field law and on the final energy. Assuming frequency-time curves optimized by the adiabatic theorem[10], the repetition rate can, in general, be higher with a higher K in the centre. The decrease of fm with cos φ_s in the region of the optima in the figures will also be less important when K is high, so the real current optimum in the case without flutter will occur at smaller cos φ_s than in the low K flutter situations. This makes the high K case without flutter (Fig. 3) more attractive than it appears to be, compared to the low K flutter cases (Figs. 4 to 6).

To illustrate this we might use some data from Ref. 10, where the acceleration time to 600 MeV has been calculated for a magnetic field which approximates the field of the CERN synchro-cyclotron ($K \approx 2$ in the centre) and for some semi-isochronous cases ($K < 1$). For $\alpha = 0$, $K = 5$ and starting-phase range 0-75° (Fig. 3) we find, using the repetition rate as calculated for the approximative CERN SC-field and adding 10% due to the higher K, that the optimum current is found at the initial cos φ_s to be slightly less than 0.05. Using cos $\varphi_s = 0.05$ and $eV = 60 \times 10^3$ we get fm = 495 p/sec (25% flyback time). With h = 3 cm we obtain from Fig. 3 the current i = 50 μA.

With $\alpha = 2m^{-2}$, $K = 0.5$ (Fig. 6), and a repetition rate calculated for a semi-isochronous field ($K = 0.8$ at full energy) we get a flat optimum at cos $\varphi_s = 0.1$, for which fm = 270 p/sec. Using h = 2 cm (aperture somewhat reduced in order to obtain $\alpha = 2m^{-2}$) the current (from Fig. 6) becomes i = 58 μA, which is not much more than the previous figure. It must be remembered, though, that in the semi-isochronous case the required frequency variation for the dee voltage is considerably smaller. This would in practice make it possible to raise the current limit by increasing the dee voltage.

6. LIMITATIONS OCCURRING AFTER THE BUCKET IS FORMED

During the first inward phase oscillation the particles may again come to the region where the electric focusing is effective. The minimum value of Q_z^2 experienced must, however, always be greater than that given by Eq. (16). The space-charge force during this motion is determined from the captured charge and the bucket area[6]. Using here also the infinite sheet approximation and the charge as calculated above, it is found that with the largest starting-phase range (0-75°) a further intensity limitation might occur for these particles swinging back closest to the centre. The effect has not been fully evaluated, but it is felt that this limitation is of minor importance as only a small fraction of the particles come back close enough to the centre.

7. CONCLUSION

It has been shown that the minimum value of the square of the vertical betatron frequency Q_z occurring near the centre of a cyclotron, can be expressed as a function of the starting-phase angle. This permits a calculation of the space-charge limit at small radii in cyclotrons by an integration over the starting-phase interval. Combined with the theory for capture, the maximum charge which can be accelerated in a synchro-cyclotron, is found. The given curves will permit a quantitative estimate of the space-charge limited current in synchro-cyclotrons, and a comparison of the merits of some different schemes.

The calculations of the minimum values of Q_z^2 are approximative in the sense that only first-order electrostatic focusing is considered. Including higher-order terms, on the other hand, would only extend the usable phase range a few degrees near zero phase, which would be unimportant, particularly for the synchro-cyclotron case, where phases around zero are generally badly captured. A more precise calculation of the space charge forces remains to be done.

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Figure captions

Fig. 1 : Initial phase space with focusing and capture areas. Angle φ_{11} is defined at the crossing of the lines $2L - F^{1/2}$ and $-G^{1/2}$, and φ_{12} at the crossing of $-G^{1/2}$ and $-F^{1/2}$.

Fig. 2 : Current integrands versus starting phase for $\cos \varphi_s = 0.3$. Curves with $\alpha = 2m^{-2}$ are plotted for $B = 1.9$ Wb/m². For other field values, they are valid for $\alpha = 5.44 \cdot \omega^2/c^2$. K specifies the shape of the average magnetic field in the region of the first phase oscillation. Due to the electric focusing, only positive phases give a contribution to the current.

Fig. 3 : Current as a function of $\cos \varphi_s$ for the weak focusing case $(\alpha = 0)$.

Fig. 4 : Current as a function of $\cos \varphi_s$.

Fig. 5 : Current as a function of $\cos \varphi_s$.

Fig. 6 : Current as a function of $\cos \varphi_s$.

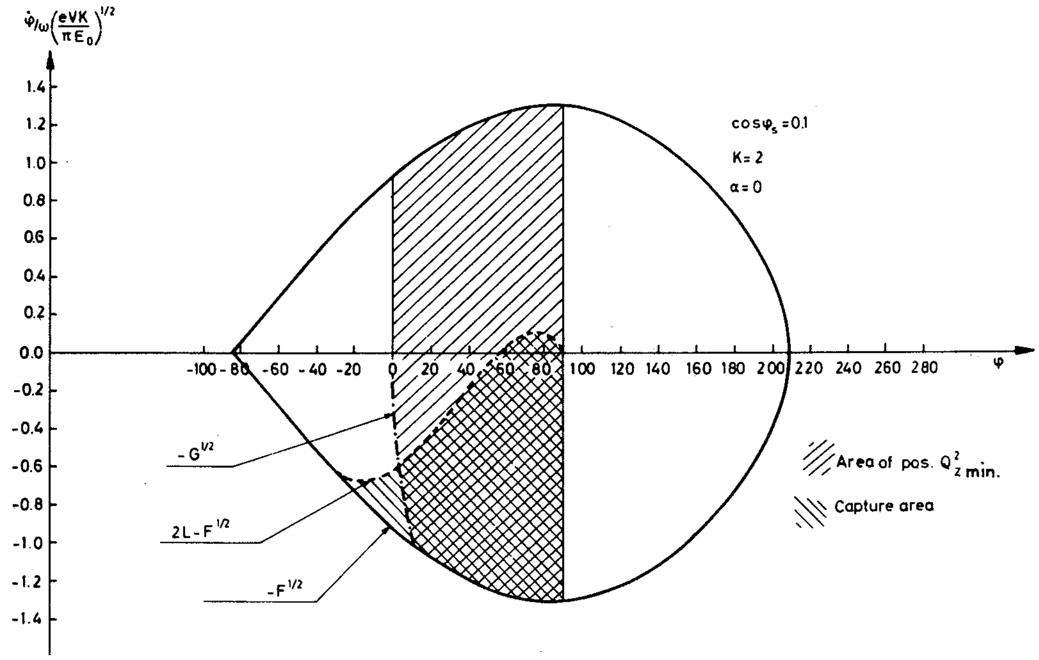

FIG.1 Initial phase space with focussing and capture areas

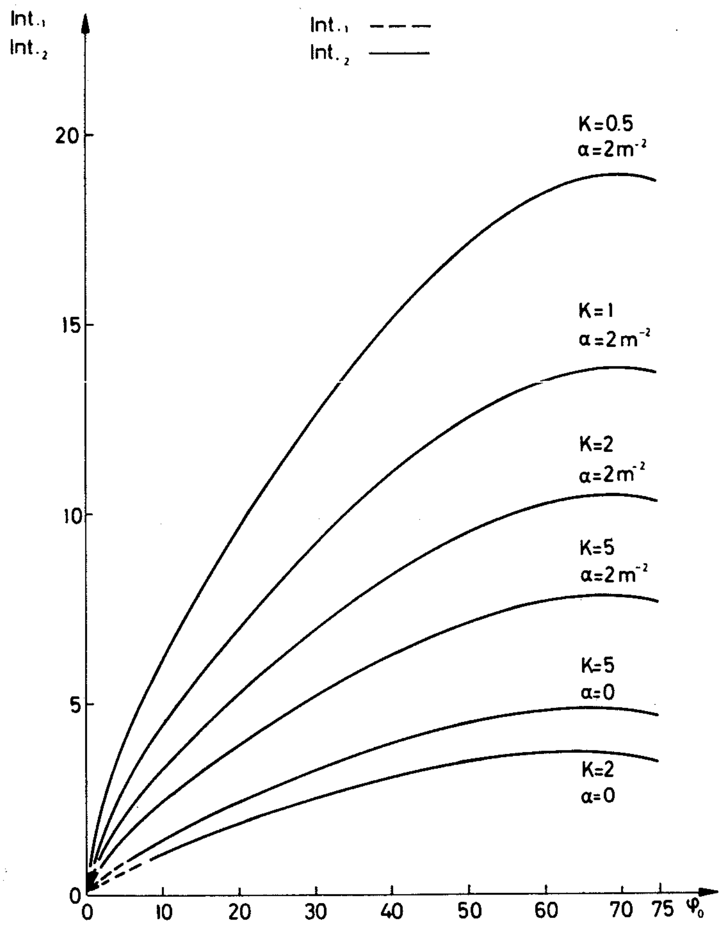

FIG.2 Current integrands versus starting phase for cos $\varphi_s = 0.3$

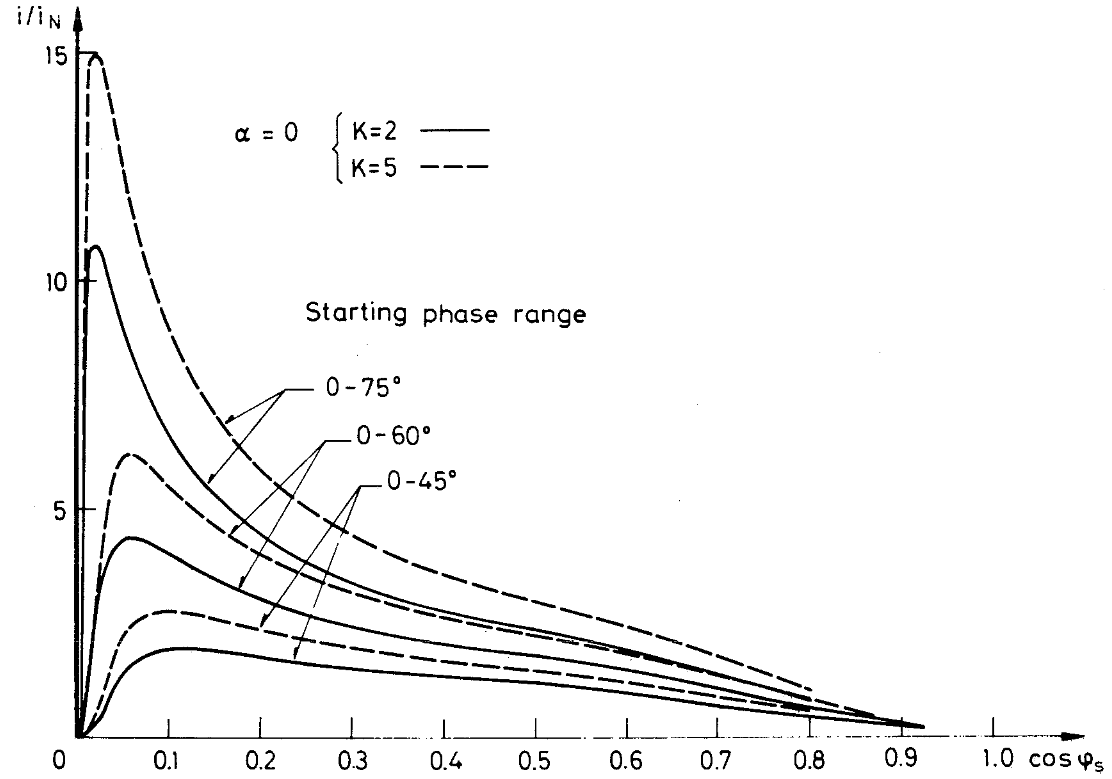

FIG. 3 Current as a function of cos φ_s

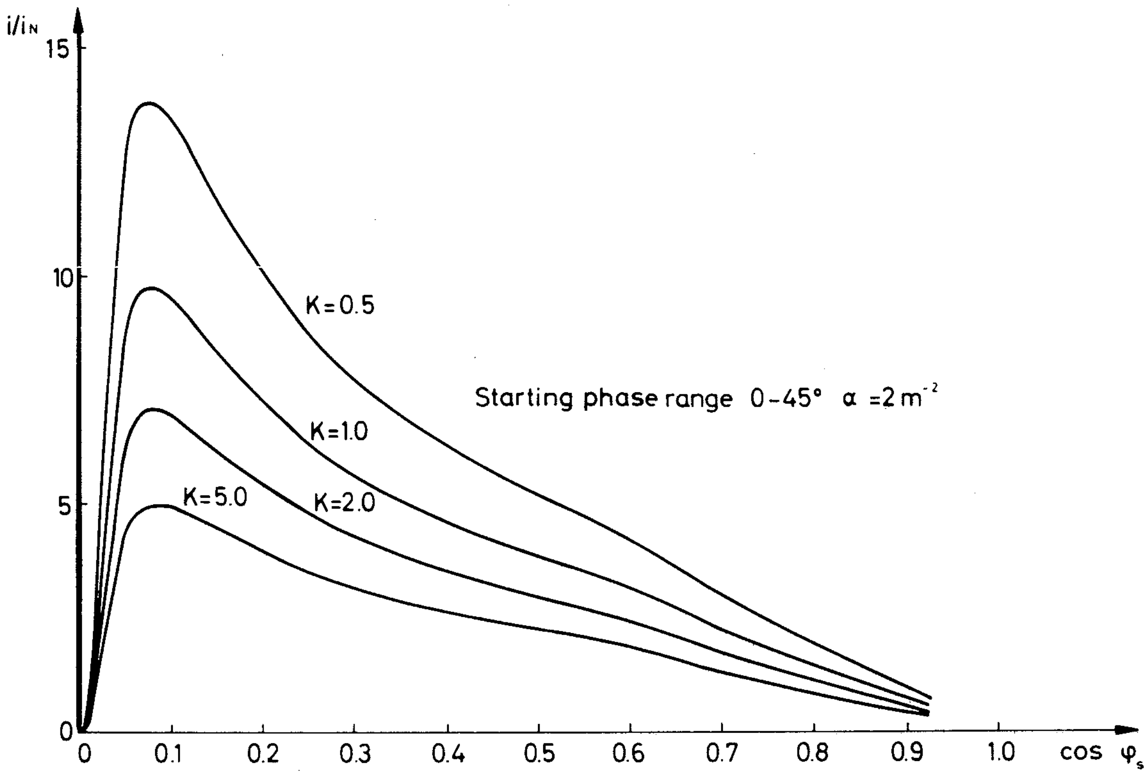

FIG.4 Current as a function of cos φ_s

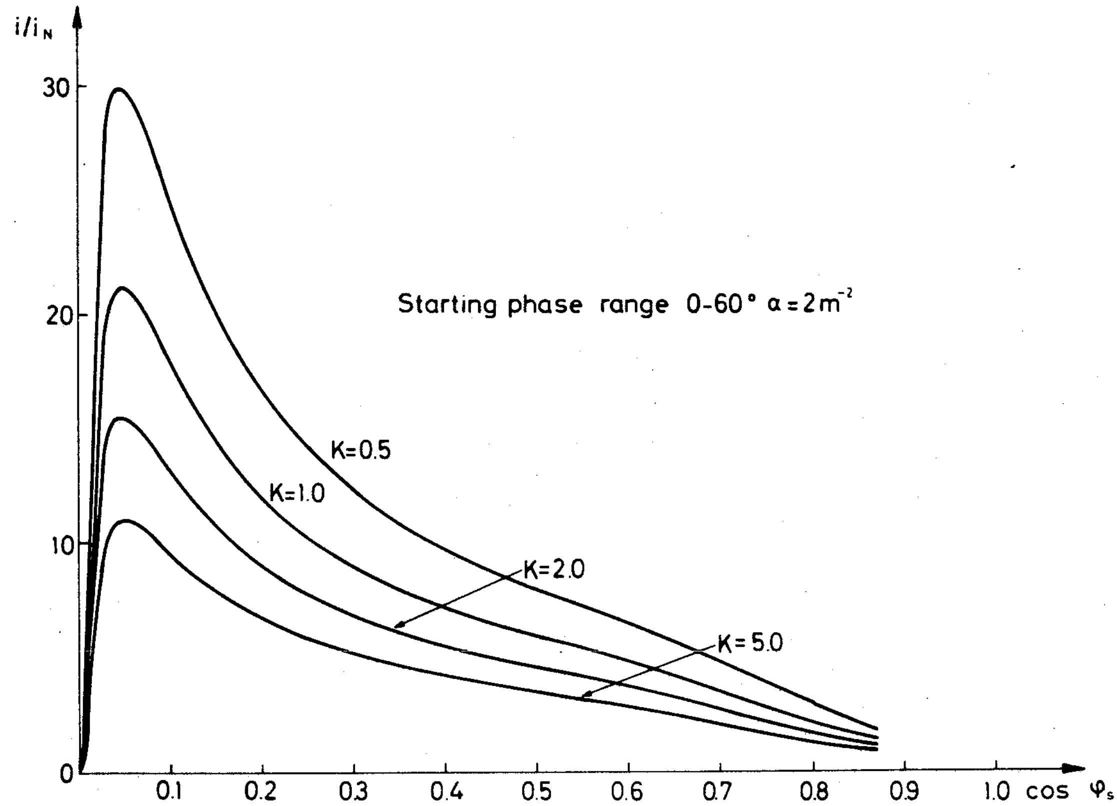

FIG. 5 Current as a function of cos φ_s

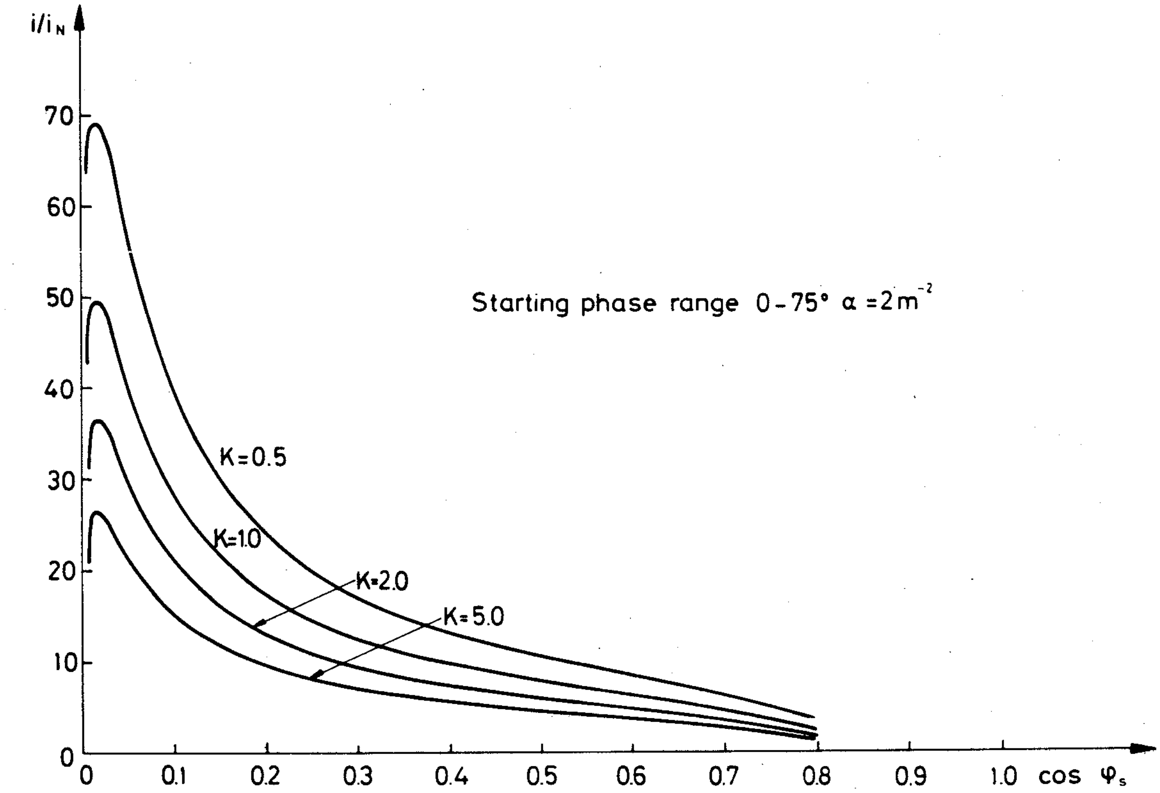

FIG.6 Current as a function of cos φ_s