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MODIFIED ELONGATED BETATRON ACCELERATOR

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With a manifestly covariant treatment, self-consistent equilibria are obtained for the relativistic Vlasov equation and the geometry of the MEBA. The equilibria are functions of the constant of the motion P_θ and the action integrals $\oint P_z\,dz$ and $\oint P_r\,dr$, which are abiabatic invariants. A radial space-charge limit is derived; comparisons are made with a longitudinal space charge limit and with recent experiments. With a linear perturbation analysis, a classification is made of the waves resonant with the beam accelerated in MEBA; the effects of particle motion along the symmetry axis are included. When the Doppler-shifted frequency $\Omega = \gamma(\omega \pm l\dot{\theta})$ seen by a particle is much less than the shift due to longitudinal motion $\omega_z = k_z v_z$, a component of the perturbed current that is nearly uniform along the symmetry axis is decoupled. This is the negative mass mode. With a spread of P_θ and I_z in the equilibrium distribution, the growth rate is reduced, but the system is unstable for a finite range of γ. Only a spread of I_r, the action variable of betatron oscillations, can produce stability for all values of γ. The price for stability is a reduction in the space-charge limit and an increase in emittance.

1. INTRODUCTION

The Modified Elongated Betatron Accelerator (MEBA)[1] attempts to increase the beam loading by storing more charge in more space. The elongated geometry greatly simplifies injection and trapping, because electrons leave the injector, move to the opposite mirror, are reflected and travel the length of the accelerator again before they can hit the injector. There is sufficient time to change the magnetic field to prevent this. The MEBA is illustrated in Fig. 1. The vacuum space that contains the electron beam has a hollow cylindrical shape of length L, inner radius r_i, and outer radius r_0.

The magnetic fields consist of a poloidal field B_z in the direction of the symmetry axis. It is produced by a long solenoid with additional turns at the ends to produce magnetic mirrors. There is an iron core or a separate flux coil to accelerate the electrons while maintaining the betatron condition $\langle B_z \rangle = 2B_{z0}$; B_{z0} is the poloidal field at the electron location and $\langle B_z \rangle$ is the mean field inside the electron orbit. There is also a toroidal magnetic field B_θ produced by conductors parallel to the symmetry axis. The magnetic fields increase together in time; the coils may be connected in series. The MEBA is a direct descendant of the

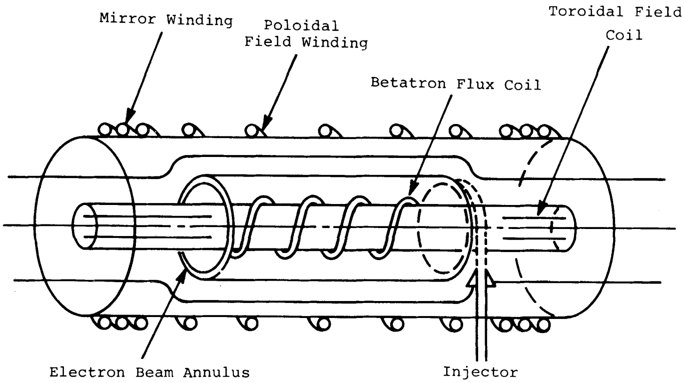

FIGURE 1 Schematic diagram of the MEBA.

Astron, which was developed at the Lawrence Livermore National Laboratory under the direction of N. Christofiles.[2]

The usual geometric parameters[1] are $L/r_0 \approx 20$ and $(r_i - r_0)/(r_i + r_0) \approx 1/5$. At injection the spread in P_z and P_θ are of order $r\Delta P_z/P_\theta \approx 0.1$, $\Delta P_\theta/P_\theta \approx 0.1$, where $P_\theta = (\gamma^2 - 1)^{1/2} r$, with $\gamma = 1.14$. The poloidal and toroidal magnetic fields are almost equal, i.e., $B_\theta/B_z = 0.9$–1.1.

A small-scale experiment has been carried out that demonstrated injection, trapping, confinement, and acceleration. An experiment to demonstrate extraction has been proposed.[3] A theory to predict scaling to a large accelerator is described in this paper. It has not previously been developed because of the unusual geometry that complicates the analysis.

2. ANALYSIS

The procedure is to solve self-consistently the relativistic Liouville equation,

$$\frac{\partial f}{\partial X^\mu}\frac{\partial h}{\partial P_\mu} - \frac{\partial h}{\partial X^\mu}\frac{\partial f}{\partial P_\mu} = [f, h] = 0, \tag{1}$$

and Maxwell's equations,

$$\Box A^\mu = \frac{4\pi e^2 J^\mu}{mc^2}, \tag{2}$$

where (X^μ, P_μ) are the position and momentum four-vectors; $f(X^\mu, P_\mu)$ is the distribution function; h is the Hamiltonian, given by[4]

$$h = -\tfrac{1}{2} g^{\mu\nu}(P_\mu + A_\mu)(P_\nu + A_\nu) = -\tfrac{1}{2}, \tag{3}$$

$g^{\mu\nu}$ is the metric tensor, and A_μ is the vector potential.

The only constant of the motion is P_θ. For the radial and axial coordinates r

and z, we assume that all quantities are slowly varying, so that

$$I_z = \oint P_z dz, \qquad I_r = \oint P_r dr \tag{4}$$

are adiabatic invariants. Any function of I_r, I_z, and P_θ is a solution of Eq. (1). We select a suitable function and calculate

$$J^\mu = \int d^4P(P^\mu + A^\mu)f. \tag{5}$$

Some examples of the distribution of I_r are illustrated in Fig. 2, along with the resultant current distribution, which is proportional to $rg(X)$, where $X = r - r_A$ and r_A is the orbit center. The result of the calculation is the average radial

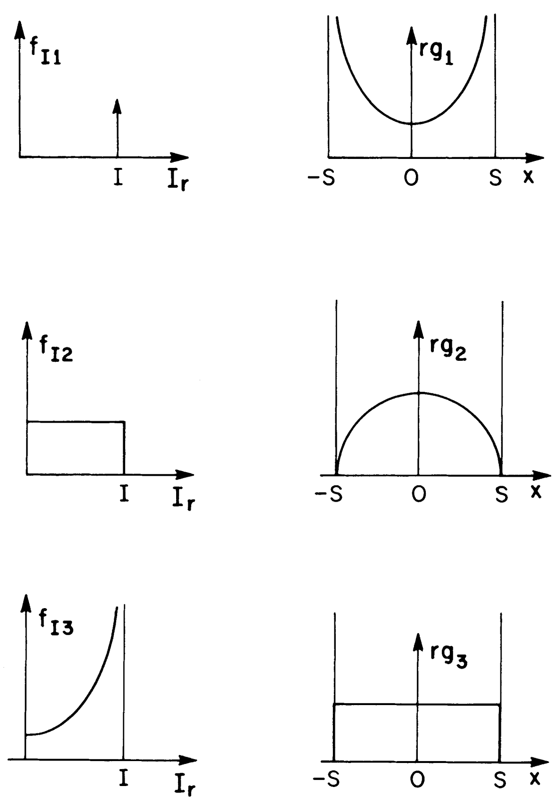

FIGURE 2 The distribution function f_I and the corresponding current-density distribution $rg(\omega, x)$.

oscillation frequency

$$\omega_{rA}^2 = (\partial^2 h/\partial r^2)_{r=r_A}$$
$$= \left\{ -\left(\frac{\partial A_4}{\partial r}\right)^2 + \left(\frac{\partial A_z}{\partial r}\right)^2 + \frac{(P_\theta + A_\theta)^2}{r^4} + \left[\frac{P_\theta}{r^2} - \frac{\partial}{\partial r}\left(\frac{A_\theta}{r}\right)\right]^2 \right\}_A$$
$$- \frac{\nu}{\gamma_A r_A}\left(\frac{\omega_{rA}}{I}\right)^{1/2}, \tag{6}$$

where $\nu = Ne^2/mc^2$ is the Budker parameter;

$$N = \int_{r_A - s}^{r_A + s} n(r)\, 2\pi r\, dr \tag{7}$$

is the line density; $n(r)$ is the electron density; r_A is the beam average radius; and $2s$ is the beam thickness. For an equilibrium to exist, ω_{rA}^2 must be real, which sets a limit on ν or on the circulating current. This is the space-charge limit.

Stability is investigated by means of a linear perturbation expansion about the above equilibria. The details of the calculations are available elsewhere.[5]

3. RESULTS

The space-charge limit is given by

$$\gamma = \frac{\nu\left[2 + \dfrac{r_A}{s(\gamma^2 - 1)}\right]}{1 + \left[\dfrac{B_\theta(r_A)}{B_z}\right]^2 - \dfrac{1}{\gamma^2 - 1}\left(\dfrac{I r_A}{s^2}\right)^2}, \tag{8}$$

where $B_\theta(r_A)$ is the toroidal magnetic field at the average radius, B_z is the poloidal magnetic field, and $\gamma = \gamma_A$ is the average relativistic energy factor required at injection to obtain the charge represented by ν. I is the maximum value of I_r. Initially, we neglect $I \approx 0$. For $\gamma \approx 1$, $[r_A/s(\gamma^2 - 1)] \gg 2$ and the formula reduces to the usual scaling for a modified betatron. For $\gamma \gg 1$, ν scales linearly with γ rather than with γ^3. The other new feature is that finite values of I_r reduce the space-charge limit.

The stability analysis shows that a cold beam would always have a range of γ for which the beam has a negative mass instability. A spread in P_θ and I_z in the equilibrium distribution reduces the growth rate, but the beam remains unstable for a finite range of γ. Complete stability can be obtained only with a finite value of the radial action I. This, in turn, will modify the space-charge limit of Eq. (8). With the present theory, it is possible to determine the injection γ and the quantity I corresponding to stable confinement of the charge ν. From I the emittance can be determined.

TABLE I

Comparison of experimental results[1] with predictions for a large MEBA

	Experiment	Large MEBA
Average radius of beam (r_A), cm	6	100
Length of beam (L), cm	80	600
Outside wall radius (r_0), cm	9.5	102.5
Inside wall radius (r_i), cm	4.5	97.5
Beam thickness ($2s$), cm	0.5	2
Magnetic field ratio $[B_\theta(r_A)/B_z]$	1	2
Mirror ratio (M)	0.05	0.15
Budker parameter (ν)	0.041	14
Total charge (Q)		
Radial limit nC (Q_r)	1850	5×10^6
Longitudinal nC (Q_l)	350	5×10^6
Observed nC	100	
Total circulating current (I)		
Observed, A	100	
Required, kA		240
Injection		
Radial (γ_r)		7.8
Longitudinal (γ_l)		5.4
Observed γ	1.1	

There is another space-charge limit due to action of the space-charge fields in the axial direction. Confinement requires that $e\Delta\Phi < \mu\Delta B_z$ where $\mu = P_\perp^2/2\gamma m B_z$ is the magnetic moment of the electron, ΔB_z is the change in magnetic field over half the length, $L/2$, of the beam. $\Delta\Phi$ is the corresponding change in electric potential Φ. According to this criterion, the minimum injection γ is

$$\gamma = \frac{4\nu}{M} \frac{\ln (r_0/r_A)}{[1 - (1/\gamma^2)]}, \tag{9}$$

where $M = 1 + \Delta B_z/B_z$ is the mirror rate.

In Table I, the predictions of Eqs. (8) and (9) are compared with experimental results.[1] Predictions of the γ required for injection are made for a large MEBA. Some revision of these calculations will be required because of the radial action I necessary for stability.

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