Can there be an elegant spin-orbital decomposition of the nucleon magnetic moment?

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Recently, Mekhfi [Phys. Rev. D **72**, 114014 (2005)] remarked that when studying spin-orbital separation of the nucleon magnetic moment with the Gordon decomposition, one should keep a time-derivative term because the quark fields depend on time. We clarify that this term vanishes identically in a rigorous formulation of the nucleon magnetic moment, which then can be elegantly separated into a spin part related to quark tensor charge, and an orbital part related to quark convection angular momentum. In a quark model description of the nucleon, however, such a time-derivative term might contribute because it is hard to construct a true Hamiltonian eigenstate of relativistic interacting quarks.

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In Ref [1], we used the Gordon decomposition to derive an elegant relation between magnetic moment and angular momentum for a relativistic system. This relation unambiguously separates the nucleon intrinsic magnetic moment into a spin part related to the quark tensor charge, and an orbital part related to the quark "convection" angular momentum. Recently, Mekfhi [2] remarked that our derivation erroneously omitted a time-derivative term. We supplement here why this term vanishes identically when one rigorously studies the intrinsic magnetic moment of a particle, either fundamental or composite. We also call attention that in a phenomenological model description of a composite particle this term might nevertheless be nonzero, because the model wave function may not be a true Hamiltonian eigenstate. In such a case one must be cautious at which magnetic moment formula to use.

Gordon decomposition separates the Dirac vector current $j^\mu = \bar\psi\gamma^\mu\psi$ into a convection part and a spin part:

$$\bar\psi\gamma^\mu\psi = \frac{i}{2m}\bar\psi\overleftrightarrow{D}^\mu\psi + \frac{1}{2m}\partial_\nu(\bar\psi\sigma^{\mu\nu}\psi) \equiv j_C^\mu + j_S^\mu, \quad (1)$$

where m is the mass of the Dirac field, $\overleftrightarrow{D}^\mu = D^\mu - \overleftarrow{D}^\mu$ is the covariant derivative. The Gordon decomposition follows directly from the equation of motion. In the case of free field, D^μ is replaced with ∂^μ.

In [1], we remarked that the time-derivative term in Eq. (1) does not contribute to the nucleon magnetic moment. However, Mekhfi argues in [2] that this time-derivative term cannot be thrown away because the quark fields depend on time in a nucleon. To clarify this issue, one must keep in mind that the *intrinsic* magnetic moment of a particle is defined in its Hamiltonian eigenstate with momentum close to zero [3]. In such states, a time-derivative term can be discarded identically: For any Heisenberg operator $\mathcal{O}$, we have the Heisenberg equation of motion $\partial_t\mathcal{O} = i[H, \mathcal{O}]$, where H is the *total* Hamiltonian of the system. When taking expectation value in an eigenstate of H, $\partial_t\mathcal{O}$ vanishes for any operator $\mathcal{O}$.

The above nonperturbative conclusion can be verified perturbatively if one knows how to construct a Hamiltonian eigenstate of the particle. For example, one can easily check by straightforward calculation that at 1-loop order the time-derivative term in Eq. (1) does not contribute to the anomalous magnetic moment of the electron (despite that the electron field is nontrivially time-dependent when interacting with the photon field).

After dropping the time-derivative term in Eq. (1), we can elegantly decompose the magnetic moment operator $\vec\mu = \frac{1}{2}\int d^3x\vec r \times \vec j$ into an orbital part and a spin part [1]:

$$\vec\mu = \frac{1}{2m}\int d^3x\vec r \times \bar\psi\frac{1}{2i}\overleftrightarrow{D}\psi + \frac{1}{2m}\int d^3x\bar\psi\vec\Sigma\psi \equiv \vec\mu_L + \vec\mu_S. \quad (2)$$

The advantage of this expression is that the spin part is related to the quark tensor charge, which can be accessed experimentally and calculated reliably with lattice QCD [1]. However, one should not promptly employ this expression in a phenomenological quark model calculation. The dilemma is that it is hard to construct a true eigenstate of the total Hamiltonian of a relativistic interacting system, hence a time-derivative term $\partial_t\mathcal{O} = i[H, \mathcal{O}]$ might be nonzero in this model and Eq. (2) might be invalid.

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