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Supersymmetric Jarlskog Invariants: the Neutrino Sector

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We generalize the notion of the Jarlskog invariant to supersymmetric models with right–handed neutrinos. This allows us to formulate basis–independent necessary and sufficient conditions for CP conservation in such models.

I. INTRODUCTION

CP violation in the quark sector of the Standard Model (SM) is controlled by the Jarlskog invariant [1],

$$\mathrm{Im}\Big(\mathrm{Det}[Y^uY^{u\dagger}, Y^dY^{d\dagger}]\Big) \,, \tag{1}$$

which can also be written in the form [2],[3]

$$\mathrm{Im}\Big(\mathrm{Tr}[Y^uY^{u\dagger}, Y^dY^{d\dagger}]^3\Big) \,, \tag{2}$$

where $Y^{u,d}$ are the quark Yukawa matrices. This is a CP–odd quantity, invariant under quark basis transformations. CP violation is possible if and only if the Jarlskog invariant is non–zero (assuming $\bar{\theta}_{\rm QCD} = 0$). This is a simple and powerful result.

In the lepton sector, the situation is more complicated. Assuming that the smallness of the neutrino masses is explained by the seesaw mechanism [4]-[7], the effective neutrino mass matrix is of the Majorana type. It has different basis transformation properties compared to the Dirac case. This results in three independent CP phases and more complicated CP–odd invariants [8]. A recent discussion of this subject is given in [9]. Applications of the invariant technique to physics beyond the SM can be found in [10]-[13].

A generalization of the Jarlskog invariant to supersymmetric models was constructed in [14]. It was found that CP violation is controlled in this case by a different type of invariants containing an antisymmetric product of three flavour matrices. Applications of this approach were studied in [15]. In this work, we extend these results to SUSY models with right–handed neutrinos. As seen in the SM case, this brings in flavour objects with "unusual" transformation properties and leads to distinct physics.

In what follows, we first study CP–phases and invariants in the SM with three right–handed neutrinos. We differ from previous work in implementing the concise techniques of [14]. Within this formalism, we then construct the SUSY generalization, the Minimal Supersymmetric Standard Model (MSSM) with three right–chiral neutrino superfields, and give an example of possible applications.

II. SM WITH THREE RIGHT-HANDED NEUTRINOS

Consider an extension of the SM with three right-handed neutrinos. The relevant terms in the leptonic Lagrangian density are

$$\Delta\mathscr{L} \;=\; Y^e_{ij}\bar{l}_i e_j \mathcal{H} + Y^\nu_{ij}\bar{l}_i \nu_j \widetilde{\mathcal{H}} + \frac{1}{2}M_{ij}\bar{\nu}^c_i \nu_j + \mathrm{H.c.},$$

where l, e, ν and $\mathcal{H}$ denote the left-handed charged lepton doublet, the right-handed charged lepton singlet, the right-handed neutrino singlet and the Higgs doublet, respectively. $\widetilde{\mathcal{H}}$ is given by $i\tau_2\mathcal{H}^*$, where τ_2 is the second Pauli matrix. Y^e_{ij} is the charged lepton Yukawa matrix, Y^ν_{ij} is the Yukawa matrix for the neutrinos, and M_{ij} is the complex symmetric Majorana mass matrix for the right-handed neutrinos. i, j are

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the generation indices and the superscript c denotes charge conjugation.

The kinetic terms are invariant under unitary basis transformations

$$U(3)_l \times U(3)_e \times U(3)_\nu \; , \tag{3}$$

namely

$$\begin{aligned} l &\rightarrow U_l^\dagger \; l, & (4)\\ e &\rightarrow U_e^\dagger \; e, & (5)\\ \nu &\rightarrow U_\nu^\dagger \; \nu. & (6)\end{aligned}$$

This means that a theory with the flavour matrices transformed according to

$$\begin{aligned} Y^e &\rightarrow U_l^\dagger \; Y^e \; U_e, & (7)\\ Y^\nu &\rightarrow U_l^\dagger \; Y^\nu \; U_\nu, & (8)\\ M &\rightarrow U_\nu^T \; M \; U_\nu & (9)\end{aligned}$$

represents the same physical situation and is equivalent to the original one. With an appropriate choice of the phase convention, the CP operation amounts to complex conjugation of these matrices (see e.g.[16]),

$$\mathcal{M} \rightarrow \mathcal{M}^* \; , \tag{10}$$

where $\mathcal{M} = \{Y^e, Y^\nu, M\}$. If this operation can be "undone" by a symmetry transformation, no CP violation is possible.

Physical CP violation is controlled by CP–violating basis independent invariants *à la* Jarlskog. This allows one to formulate necessary and sufficient conditions for CP conservation in a basis independent way. On the other hand, it is also instructive to study CP violating phases in a specific basis, taking advantage of symmetries of the system. In what follows, we will pursue both of these approaches.

In seesaw models, the scale of the Majorana mass matrix is taken to be very large, around the GUT scale. In this case, the low energy theory is obtained by integrating out the right–handed neutrinos. This produces a dimension-5 operator involving the left–handed leptons and an effective coupling constant

$$m_{\rm eff} = Y^\nu M^{-1} {Y^\nu}^T \; , \tag{11}$$

which results in neutrino masses upon electroweak symmetry breaking. The apparent flavour symmetry of this low energy theory is

$$U(3)_l \times U(3)_e \tag{12}$$

with the transformation law

$$\begin{aligned} Y^e &\rightarrow U_l^\dagger Y^e U_e \; ,\\ m_{\rm eff} &\rightarrow U_l^\dagger m_{\rm eff} U_l^* \; . & (13)\end{aligned}$$

The number of independent CP phases can be obtained by a straightforward parameter counting. In the high energy theory, Y^e, Y^ν and M contain 9+9+6 = 24 phases. A unitary 3×3 matrix representing basis transformations has 6 phases, which means that 18 phases can be removed.[1] Thus we end up with six physical phases at high energies. In the low energy theory, Y^e and $m_{\rm eff}$ contain $9+6 = 15$ phases. 12 of them can be removed by unitary transformations, while three are physical. Clearly, the other three physical phases of the high energy theory are associated with the heavy neutrinos and cannot be observed at low energies. However, these can be relevant to CP violation at high energies, *e.g.* leptogenesis [17].

In what follows, we study in more detail these CP phases and the corresponding invariants.

A. High–Energy Theory

1. CP phases

Let us first identify the physical CP phases in a specific basis assuming a general form of Y^e, Y^ν and M. The unitary transformations (7-9) allow us to bring the flavour matrices into the form

$$\begin{aligned} Y^\nu &= \text{real diagonal} \; ,\\ Y^e &= \text{Hermitian} \; , & (14)\\ M &= \text{symmetric} \; ,\end{aligned}$$

where the last equation is satisfied in any basis. This basis is defined only up to a diagonal phase transformation

$$\widetilde{U}_l = \widetilde{U}_e = \widetilde{U}_\nu = \text{diag}(\exp[i\alpha_1], \exp[i\alpha_2], \exp[i\alpha_3]) \; . \tag{15}$$

Under this residual symmetry, Y^e and M transform as

$$\begin{aligned} Y^e_{ij} &\rightarrow Y^e_{ij} \; \exp[\mathrm{i}(\alpha_j - \alpha_i)], & (16)\\ M_{ij} &\rightarrow M_{ij} \; \exp[\mathrm{i}(\alpha_i + \alpha_j)]. & (17)\end{aligned}$$

The physical CP phases must be invariant under these transformations. Since Y^e and M have 9 phases, only 6 of them are are physical.

The simplest invariant CP phase is a CKM–type phase which is the only one surviving the limit $M \rightarrow 0$. It is given by

$$\phi_0 = \arg[Y^e_{12} Y^e_{23} Y^{e*}_{13}] \; . \tag{18}$$

[1] If the Majorana mass matrix were absent, only 17 phases could be removed since a phase transformation proportional to the unit matrix leaves Y^e and Y^ν intact, which corresponds to a conserved lepton number.

The other five phases involve M. Three of them can be built entirely out of M,

$$\phi_1 = \arg[M_{11}M_{22}M_{12}^{*2}] \ , \quad (19)$$
$$\phi_2 = \arg[M_{22}M_{33}M_{23}^{*2}] \ , \quad (20)$$
$$\phi_3 = \arg[M_{11}M_{33}M_{13}^{*2}] \ , \quad (21)$$

while the remaining two involve Y^e as well,

$$\phi_4 = \arg[Y^e_{13}M_{13}M_{33}^*], \quad (22)$$
$$\phi_5 = \arg[Y^e_{23}M_{22}M_{23}^*]. \quad (23)$$

It should be clear by considering the independent matrix entries, that these phases are independent.

The necessary and sufficient conditions for CP conservation are given by

$$\phi_i = 0 \quad (24)$$

for $i = 0,..,5$, and the phases are understood mod π. If these conditions are satisfied, the flavour objects in Eq. (14) can be made real by choosing appropriate α_i. Then no CP violation is possible. Conversely, CP conservation implies that the flavour matrices are real in some basis. Then, the CP conserving Y^e, Y^ν and M are generated by the phase redefinitions in (15), leaving $\phi_i = 0$ intact.

2. CP Violating Invariants

Conditions for CP conservation can also be formulated in a basis independent way. To do that, one first forms matrices which are manifestly invariant under two of the unitary symmetries, then builds CP–odd traces out of them.

Consider the following Hermitian matrices

$$A \equiv Y^{\nu\dagger}Y^\nu, \quad (25)$$
$$B \equiv Y^{\nu\dagger}Y^eY^{e\dagger}Y^\nu, \quad (26)$$
$$C \equiv M^*M, \quad (27)$$
$$D \equiv M^*(Y^{\nu\dagger}Y^\nu)^*M. \quad (28)$$

In general, they are not diagonalizable simultaneously and transform as

$$\mathcal{M}_i \ \rightarrow \ U_\nu^\dagger \ \mathcal{M}_i \ U_\nu \ , \quad (29)$$

where $\mathcal{M}_i = \{A, B, C, D\}$. The simplest CP-odd invariants that can be formed out of this set are

$$\mathrm{Tr}[\mathcal{M}_i^p, \mathcal{M}_j^q]^n \ ,$$
$$\mathrm{Tr}[\mathcal{M}_i^p, \mathcal{M}_j^q, \mathcal{M}_k^r]^m \ , \quad (30)$$

where p, q, r are integer and n, m are odd; [...] denotes complete antisymmetrization of the matrix product. The first class ("J–type") of invariants is the familiar Jarlskog type, while the second class ("K–type") appears, for example, in supersymmetric models [14], see also Eqs. (84) below. These objects are CP-odd since the CP operation on the fields is equivalent to complex conjugation of the matrices, which is in turn equivalent to a transposition for Hermitian matrices. In a specific basis [for instance, (14)], these objects are functions of the six physical CP phases. In the non–degenerate case which we are considering, the vanishing of six independent invariants implies the vanishing of the physical CP phases. This means in turn that all possible CP violating invariants are zero and CP is conserved.

An admissible choice of independent invariants is[2]

$$\mathrm{Tr}[A, B]^3, \quad (31)$$
$$\mathrm{Tr}[A, C]^3, \quad (32)$$
$$\mathrm{Tr}[A, D]^3, \quad (33)$$
$$\mathrm{Tr}([A, C]B), \quad (34)$$
$$\mathrm{Tr}([A, D]B), \quad (35)$$
$$\mathrm{Tr}([A, D]C), \quad (36)$$

where we have used $\mathrm{Tr}[a, b, c] \propto \mathrm{Tr}[a, b]c$. The first invariant is proportional to the sine of the CKM–type phase ϕ_0, while the others depend in a complicated way on all of the phases (18)-(23). It is a non–trivial task to determine whether given invariants are mutually independent. To do that, we calculate the Jacobian

$$\mathrm{Det}\Big(\frac{\partial \mathscr{J}_i}{\partial \phi_j}\Big) \ , \quad (37)$$

where $\mathscr{J}_i$ are the invariants above. A non–zero Jacobian indicates that the objects are independent. We confirm that this is indeed the case.

It is instructive to consider the above invariants in a specific basis, for example, where matrix A is diagonal,

$$A = \mathrm{diagonal} \ . \quad (38)$$

This basis is defined up to a rephasing

$$\tilde{U}_\nu = \mathrm{diag}(\exp[i\alpha_1], \exp[i\alpha_2], \exp[i\alpha_3]) \ . \quad (39)$$

The physical CP phases must be invariant under this residual symmetry and are of the form

$$\arg[B_{12}B_{23}B_{13}^*] \ , \ \arg[C_{12}C_{23}C_{13}^*] \ , ... \quad (40)$$
$$\arg[B_{12}C_{12}^*] \ , \ \arg[B_{23}C_{23}^*] \ , ... \quad (41)$$

For N independent Hermitian objects one can form $3N - 5$ independent invariant phases and all of the

[2] We drop the Im(...) for each invariant in the following.

invariants depend on these $3N-5$ variables. This can be understood by parameter counting: N Hermitian matrices contain $3N$ phases and unitary basis transformations U_ν absorb $6-1=5$ of them since the overall phase transformation leaves all the matrices intact. The explicit dependence of the invariants on these phases has been studied in [14].

In our case, there appear to be seven phases according to this argument. However, not all of our Hermitian matrices are completely independent as they are built out of three flavour matrices. One of the phases is a function of the others and we have six truly independent CP phases as explained in the previous subsection. These are rather complicated functions of the expressions (40) and (41), except

$$\phi_0 \propto \arg[B_{12}B_{23}B_{13}^*] \; . \tag{42}$$

Note that if we chose only three Hermitian matrices A, B, C to work with, we could only extract four CP phases regardless of how many invariants we would write. So, some information is lost when constructing Hermitian objects. It is thus necessary to include a further matrix D, which brings in additional input. To show that this is sufficient, one must calculate the Jacobian (37).

The necessary and sufficient conditions for CP conservation in the non–degenerate case are

$$\mathscr{J}_i = 0 \; , \tag{43}$$

where $\mathscr{J}_i$ are the invariants (31)-(36). This is equivalent to Eq.(24).

B. Low–Energy Theory

1. CP Phases

At low energies, we have two flavour matrices Y_e and m_{eff}. Using the unitary freedom (13), we bring them into the form

$$\begin{aligned} m_{\text{eff}} &= \text{real, positive and diagonal,} \\ Y^e &= \text{Hermitian} \; . \end{aligned} \tag{44}$$

In the non–degenerate case, there is no residual freedom in this basis due to the Majorana character of m_{eff}. The three physical phases are therefore

$$\phi_1^{\text{eff}} = \arg[Y_{12}^e] \; , \tag{45}$$
$$\phi_2^{\text{eff}} = \arg[Y_{23}^e] \; , \tag{46}$$
$$\phi_3^{\text{eff}} = \arg[Y_{13}^e] \; . \tag{47}$$

Alternatively, one can choose a basis in which Y^e is diagonal,

$$\begin{aligned} Y^e &= \text{real, positive and diagonal,} \\ m_{\text{eff}} &= \text{symmetric} \; , \end{aligned} \tag{48}$$

where the second equation is satisfied in any basis. The residual freedom is

$$\widetilde{U}_l = \widetilde{U}_e = \text{diag}(\exp[i\alpha_1], \exp[i\alpha_2], \exp[i\alpha_3]), \tag{49}$$

such that the three physical phases are of the form

$$\arg[(m_{\text{eff}})_{ii}(m_{\text{eff}})_{jj}(m_{\text{eff}})_{ij}^{*2}] \tag{50}$$

for $i \neq j$.

It is conventional to separate these phases into so-called Majorana and Dirac ones. This can be done by expressing m_{eff} as

$$m_{\text{eff}} = U \; (\text{real diagonal}) \; U^T \; , \tag{51}$$

where U is unitary. Five of its phases can be factored out [18]

$$\begin{aligned} U &= \text{diag}(\exp[i\alpha_1], \exp[i\alpha_2], \exp[i\alpha_3]) \\ &\times \; U' \; \text{diag}(1, \exp[i\Phi_1], \exp[i\Phi_2]) \; , \end{aligned} \tag{52}$$

with U' containing a single phase which cannot be factored out in this form. The phases α_{1-3} are unphysical and can be removed by the residual symmetry transformations $m_{\text{eff}} \to \tilde{U}_l^\dagger m_{\text{eff}} \tilde{U}_l^*$. The "Majorana" phases $\Phi_{1,2}$ as well as the "Dirac" phase δ in U' are unaffected by this phase redefinition and are physical. They enter the PMNS matrix and thus contribute to the W-boson–lepton–lepton vertex [19]-[21].

The necessary and sufficient conditions for CP conservation in the non–degenerate case are given by

$$\phi_i^{\text{eff}} = 0 \tag{53}$$

for $i = 1, 2, 3$ which is equivalent to $\Phi_1 = \Phi_2 = \delta = 0$ (the phases are understood mod π).

2. CP Violating Invariants

As in the previous subsection, we first construct Hermitian matrices transforming under one of the unitary symmetries only. At low energies, U_l is the relevant symmetry and we choose

$$\begin{aligned} &\mathcal{A} = Y^e Y^{e\dagger} \; , \\ &\mathcal{B} = m_{\text{eff}} m_{\text{eff}}^* \; , \\ &\mathcal{C} = m_{\text{eff}} (Y^e Y^{e\dagger})^* m_{\text{eff}}^* \; . \end{aligned} \tag{54}$$

They all transform as

$$\mathcal{M}_i \to U_l^\dagger \; \mathcal{M}_i \; U_l \; , \tag{55}$$

where $\mathcal{M}_i = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$. We first note that generally $\mathcal{A}, \mathcal{B}, \mathcal{C}$ are not diagonalizable in the same basis. Second, they contain $3 \times 3 - 5 = 4$ invariant phases, three of which are independent and related to ϕ_i^{eff}. Again, using two Hermitian matrices, *e.g.* $\mathcal{A}$ and $\mathcal{B}$, would only allow us to extract information about a single phase, so it is necessary to consider $\mathcal{C}$ as well.

The CP–odd invariants can be chosen as

$$\mathrm{Tr}[\mathcal{A},\mathcal{B}]^3 \ , \quad (56)$$

$$\mathrm{Tr}[\mathcal{A},\mathcal{C}]^3 \ , \quad (57)$$

$$\mathrm{Tr}([\mathcal{A},\mathcal{B}]\mathcal{C}) \ . \quad (58)$$

In the non–degenerate case, they are all independent and can be used to extract ϕ_i^{eff}. This is established by calculating the Jacobian: $\mathrm{Det}\left(\frac{\partial \mathcal{J}_i}{\partial \phi_j^{\mathrm{eff}}}\right)$. We thus have three necessary and sufficient conditions for CP conservation or violation.

As expected, the Jarlskog–type invariant (56) is independent of the Majorana phases and is proportional to the Dirac phase,

$$\mathrm{Tr}[\mathcal{A},\mathcal{B}]^3 \propto \sin\delta \ . \quad (59)$$

It vanishes in the limit of degenerate eigenvalues or vanishing mixing angles. The other invariants are complicated functions of the Dirac and Majorana phases.

The necessary and sufficient conditions for CP conservation in the non–degenerate case are

$$\mathcal{J}_i = 0 \ , \quad (60)$$

where $\mathcal{J}_i$ $(i=1,2,3)$ denote the invariants (56)-(58).

C. Degenerate Case

So far we have assumed that there are no degenerate eigenvalues in any of the matrices and that the mixing angles are non–zero. It is however instructive to consider the special case, where all the low-energy neutrino mass eigenvalues are equal, *i.e.* there exists a basis such that

$$m_{\mathrm{eff}} = m \times \mathbf{1} \ , \quad (61)$$

where $\mathbf{1}$ is a 3×3 unit matrix and m is real. In that case, the special basis (44) is defined up to a real orthogonal transformation

$$\tilde{U}_l = \tilde{U}_e = O \ , \ OO^T = \mathbf{1} \ , \quad (62)$$

which retains the Hermiticity of Y^e. Due to this residual symmetry, the ϕ_i^{eff} are not all independent and can be parametrized by a single phase [22].

This becomes more transparent in the other special basis (48), where Y_e is real and diagonal. This basis must be unitarily related to the basis (61) and thus m_{eff} is given by

$$m_{\mathrm{eff}} = m\ U_l^\dagger U_l^* = \text{symmetric unitary} \ . \quad (63)$$

A symmetric unitary matrix can be parametrized by four phases (and two angles) [23]. Indeed, three of them can be factored out as [18]

$$\begin{aligned}&\mathrm{diag}(\exp[i\alpha_1],\exp[i\alpha_2],\exp[i\alpha_3])\ \ U' \ \times \\ &\mathrm{diag}(\exp[i\alpha_1],\exp[i\alpha_2],\exp[i\alpha_3]) \ ,\end{aligned} \quad (64)$$

while the symmetric unitary matrix U' contains a single phase. The explicit form of U' can be found in [22]. The phases α_{1-3} are removed by the residual phase symmetry (49) in this basis, leaving a single physical phase.

Thus, in this degenerate case there is one physical Majorana phase. This phase has to be Majorana since the Jarlskog invariant $\mathrm{Tr}[\mathcal{A},\mathcal{B}]^3$ vanishes. [Both $\mathcal{A}$ and $\mathcal{B}$ are diagonal in the basis (63).] We observe that the only non–vanishing invariant is (57). In the basis where m_{eff} is diagonal, it is given by (up to a factor) [22]

$$\mathrm{Tr}[Y^eY^{e\dagger},(Y^eY^{e\dagger})^*]^3 \quad (65)$$

and is invariant under the residual orthogonal symmetry (62). It is non–zero in general since $\mathcal{A}$ and $\mathcal{A}^*$ are not diagonal in the same basis.

This analysis can be carried over to the "high energy theory" case in a straightforward albeit tedious way.

III. MSSM WITH THREE RIGHT–HANDED NEUTRINOS

The leptonic part of the most general proton-hexality [24] (or R-parity) conserving renormalizable superpotential is given by

$$\begin{aligned}\mathcal{W}_{\mathrm{leptonic}} &= -\hat{\mathcal{H}}_2 Y^\nu_{ij}\hat{L}_i\hat{N}_j + \hat{\mathcal{H}}_1 Y^e_{ij}\hat{L}_i\hat{E}_j \\ &\quad +\frac{1}{2}M_{ij}\hat{N}_i\hat{N}_j \ .\end{aligned} \quad (66)$$

Here $\hat{L}$, $\hat{E}$ and $\hat{N}$ are the left-chiral superfields describing the lepton doublet, a charge conjugate of the right–handed electron and a charge conjugate of the right–handed neutrino, respectively. $\hat{\mathcal{H}}_1$ and $\hat{\mathcal{H}}_2$ are the Higgs doublet superfields. The relevant soft SUSY breaking terms are

$$\begin{aligned}\Delta V_{\mathrm{soft}} &= (-\mathcal{H}_2 A^\nu_{ij}\tilde{l}_i\tilde{n}^*_j + \mathcal{H}_1 A^e_{ij}\tilde{l}_i\tilde{e}^*_j \\ &\quad +\frac{1}{2}B_{ij}\tilde{n}_i\tilde{n}_j + \mathrm{H.c.}) \\ &\quad + M^{l\,2}_{ij}\ \tilde{l}_i\tilde{l}^*_j + M^{\nu\,2}_{ij}\ \tilde{n}_i\tilde{n}^*_j + M^{e\,2}_{ij}\ \tilde{e}_i\tilde{e}^*_j \ ,\end{aligned} \quad (67)$$

where $\tilde{l}$, $\tilde{e}^*$ and $\tilde{n}^*$ are the scalar components of $\hat{L}$, $\hat{E}$ and $\hat{N}$, respectively. $\mathcal{H}_1$ and $\mathcal{H}_2$ denote the Higgs doublets.

As in the SM, the flavour symmetry is

$$U(3)_l \times U(3)_e \times U(3)_\nu \ , \quad (68)$$

which now applies to superfields.[3] The transformation law of the flavour structures is

$$
\begin{aligned}
Y^\nu &\to U_l^\dagger\ Y^\nu\ U_\nu\ , && (69)\\
Y^e &\to U_l^\dagger\ Y^e\ U_e\ , && (70)\\
A^\nu &\to U_l^\dagger\ A^\nu\ U_\nu\ , && (71)\\
A^e &\to U_l^\dagger\ A^e\ U_e\ , && (72)\\
M^{l\,2} &\to U_l^\dagger\ M^{l\,2}\ U_l\ , && (73)\\
M^{\nu\,2} &\to U_\nu^\dagger\ M^{\nu\,2}\ U_\nu\ , && (74)\\
M^{e\,2} &\to U_e^\dagger\ M^{e\,2}\ U_e\ , && (75)\\
M &\to U_\nu^T\ M\ U_\nu\ , && (76)\\
B &\to U_\nu^T\ B\ U_\nu\ . && (77)
\end{aligned}
$$

These objects altogether contain $4\times 9+3\times 3+2\times 6 = 57$ complex phases. The symmetry transformations eliminate 3×6 of them such that we end up with 39 physical CP phases.[4]

In what follows, we classify the corresponding CP phases and CP–odd invariants.

A. SUSY CP Phases and CP–odd Invariants

In the supersymmetric basis corresponding to (14) where Y^ν is real and diagonal, and Y^e is Hermitian, the additional invariant CP phases due to the SUSY flavour structures are given by

$$
\begin{aligned}
&\arg\Big(Y^e_{ij}\ A^{\{e,\nu\}*}_{ij}\Big) &&\to\ 18\ ,\\
&\arg\Big(Y^e_{ij}\ M^{\{e,\nu,l\}\ 2\ *}_{ij}\Big) &&\to\ 9\ ,\\
&\arg\Big(M_{ij}\ B^*_{ij}\Big) &&\to\ 6\ .
\end{aligned} \qquad (78)
$$

These are invariant under the transformations (15).

In the Standard Model, as a next step, we constructed simple Hermitian objects which *all* transformed under only one of the symmetries (3). In the MSSM, this approach leads to very cumbersome expressions. We thus construct three separate groups of Hermitian objects, which each transform under only one unitary symmetry, respectively. These are presented in Table I. We find that this set is sufficient to determine all physical phases of the system in the non–degenerate case. Before we write down the CP–odd invariants, let us study what CP phases these Hermitian matrices are sensitive to.

$U(3)_l$	$U(3)_e$	$U(3)_\nu$
$Y^eY^{e\dagger}$	$Y^{e\dagger}Y^e$	$Y^{\nu\dagger}Y^\nu$
$Y^\nu Y^{\nu\dagger}$	$A^{e\dagger}A^e$	$A^{\nu\dagger}A^\nu$
$A^eA^{e\dagger}$	$Y^{e\dagger}A^e+\text{H.c.}$	$A^{\nu\dagger}Y^\nu+\text{H.c.}$
$A^\nu A^{\nu\dagger}$	$M^{e\,2}$	$M^{\nu\,2}$
$Y^eA^{e\dagger}+\text{H.c.}$		M^*M
$A^\nu Y^{\nu\dagger}+\text{H.c.}$		$M^*(Y^{\nu\dagger}Y^\nu)^*M$
$M^{l\,2}$		$B^*(Y^{\nu\dagger}Y^\nu)^*B$
		$B^*M+\text{H.c.}$

TABLE I: The minimal set of Hermitian flavour objects.

Consider for example Column 3. In the basis where $Y^{\nu\dagger}Y^\nu$ is diagonal, the CP phases invariant under the residual symmetry (15) are of the type

$$
\begin{aligned}
&\arg((\mathcal{M}_i)_{12}(\mathcal{M}_i)_{23}(\mathcal{M}_i)^*_{13})\ , && (79)\\
&\arg((\mathcal{M}_i)_{12}(\mathcal{M}_{i+1})^*_{12})\ ,.., && (80)
\end{aligned}
$$

where $\mathcal{M}_i$ are the Hermitian matrices of the third Column of Table I. Given $N>1$ independent Hermitian matrices, one can construct $3N-5$ independent invariant phases. These can be chosen as one CKM–type phase (79) and the rest of the form (80). In this fashion, we obtain 19 invariant phases from Column 3. However, as we have seen in the SM case, one has to be cautious in determining the correct number of *independent* phases, and not too many, since there are certain relations among these matrices.

In order to make the choice of Hermitian objects in Table I plausible and to better understand the counting of independent phases, consider first the hypothetical special case, when the only non–zero quantities are Y^e, Y^ν and $M^{\nu\,2}$. In the basis (14) with $M=0$, using the above counting arguments, we then obtain only four physical independent phases. These *can not* be recovered from the Hermitian quantities in the three columns of Table I. It is only possible to get one phase of the form (79) in Column 1, and another phase of the same type from Column 3. In order to construct the four phases, it is thus necessary to include a more complicated Hermitian object, $Y^{\nu\dagger}Y^eY^{e\dagger}Y^\nu$, in Column 3, as we did in Sect. I. This brings in three extra phases, two of which are independent. This shows that, in the special case, extra Hermitian objects may have to be included.

Next let us consider the more involved case, where apart from Y^e, Y^ν and $M^{\nu\,2}$, also $A^\nu\neq 0$. Again, by our counting argument, we then have 13 physical independent phases from the remaining Hermitian objects in Table I in the supersymmetric basis corresponding to (14). In order to construct the extra phases, we can now write down additional Hermitian matrices $A^\nu A^{\nu\dagger}$ and $A^\nu Y^{\nu\dagger}+\text{H.c.}$ in the first column, as well as $A^{\nu\dagger}A^\nu$ and $A^{\nu\dagger}Y^\nu+\text{H.c.}$ in the third column. These extra

[3] Fermions and sfermions are transformed in the same fashion in order to avoid flavour mixing at the super–gauge vertices.
[4] If the Majorana matrices were absent, we would get $45-17=28$ physical CP phases.

objects restore the deficit encountered above, *i.e.* we can now recover 13 physical phases from the Hermitian objects. The naïve counting gives seven phases for Column 1 and seven phases for Column 3, which is too many. However, of the matrices

$$A^\nu A^{\nu\dagger}\ ,\ A^\nu Y^{\nu\dagger}+\text{H.c.}\ ,\ A^{\nu\dagger}A^\nu\ ,\ A^{\nu\dagger}Y^\nu+\text{H.c.}$$

only three are independent. One of these matrices, say $A^{\nu\dagger}Y^\nu+\text{H.c.}$, can be reconstructed from the others [14]. In other words, the nine phases of A^ν can be derived from the nine phases of the three Hermitian matrices. This means that the CKM–type phase associated with $A^{\nu\dagger}Y^\nu+\text{H.c.}$, namely

$$\arg\Big((A^{\nu\dagger}Y^\nu+\text{H.c.})_{12}(A^{\nu\dagger}Y^\nu+\text{H.c.})_{23}(A^{\nu\dagger}Y^\nu+\text{H.c.})^*_{13}\Big) \tag{81}$$

is not an independent phase and should not be counted. Although it may seem that $A^{\nu\dagger}Y^\nu+\text{H.c.}$ should be excluded altogether, this is not correct since it allows us to restore the (otherwise missing) phases of $M^{\nu\,2}$ through the rephasing invariant combinations

$$\arg\ \Big((M^{\nu\,2})_{12}(A^{\nu\dagger}Y^\nu+\text{H.c.})^*_{12}\Big)\ ,\ \textit{etc.} \tag{82}$$

The other three phases can be chosen as

$$\arg\ \Big((A^{\nu\dagger}A^\nu)_{12}(A^{\nu\dagger}Y^\nu+\text{H.c.})^*_{12}\Big)\ ,\ \text{etc.} \tag{83}$$

We thus end up with six phases from the Hermitian matrices of Column 3 and seven phases from those of Column 1. Similar considerations apply when adding A^e to Column 2, where the CKM-type phase for $A^{e\dagger}Y^e+\text{H.c.}$ is not independent.

In the Dirac case, where only $M=B=0$ in (66), (67), *i.e.* also $M^l, M^\nu, M^e\neq 0$, these are the only complications and we get 28 phases from the Hermitian objects of Table I. Adding a non–trivial Majorana mass M results in five further physical phases. This is because, in the basis (14), M adds six phases while its overall phase can be eliminated by the residual symmetry transformation, which leaves Y^e and Y^ν invariant. To recover these five phases from the Hermitian objects, we must add two entries in Column 3, M^*M and $M^*(Y^{\nu\dagger}Y^\nu)^*M$. This adds six invariant phases of the type (80), five of which are independent. Finally, inclusion of B brings in six more physical phases of the type (80) in the basis (14), all of which are independent. Correspondingly, we add $B^*(Y^{\nu\dagger}Y^\nu)^*B$ and $B^*M+\text{H.c.}$ to Column 3, which are sensitive to these phases. Note that the object of the form $B^*M+\text{H.c.}$ is necessary as it depends on the *physical* relative phase between B and M. In the end, the first, second and third Column provide 16, 6 and 17 independent phases, respectively.

The above choice of the Hermitian objects is not unique and there are many other possibilities. In particular, one may replace $A^{\nu\dagger}A^\nu$ in the third Column with $Y^{\nu\dagger}Y^eY^{e\dagger}Y^\nu$. In that case, the limit "soft terms" $\to 0$ reproduces the SM Hermitian matrices of Eqs.(25-28). On the other hand, our choice is similar to the quark sector Hermitian objects of Ref.[14]. These choices are equivalent in the non–degenerate case.

The CP–odd invariants are constructed out of the Hermitian objects transforming under one of the unitary symmetries in Eq. (68), respectively. These can be chosen as one Jarlskog–type invariant and the rest K–invariants. The former is sensitive to the cyclic product of phases of a each matrix while the latter are sensitive to the relative phases between Hermitian matrices [14]. Thus we have 39 independent invariants in the non–degenerate case,

$$\begin{aligned}&J(H_1,H_2)\ ,\\ &K(H_i^p,H_j^q,H_k^r)\ ,\end{aligned} \tag{84}$$

where $J(A,B)\equiv\text{Tr}[A,B]^3$, $K(A,B,C)\equiv\text{Tr}[A,B,C]$ and p,q,r are integers. In each invariant, only matrices H_a belonging to the same column appear. In the Appendix, we give an explicit example of 39 independent invariants. To prove that they are independent functions of the 39 physical phases (78) and (18-23), we have calculated the Jacobian

$$\text{Det}\Big(\frac{\partial J_i}{\partial\phi_j}\Big)\ , \tag{85}$$

where J_i denotes collectively all the invariants (84) and ϕ_i are the physical phases. We find that the Jacobian is non–zero. Thus, all the physical phases can be determined from these invariants.

We note that the traditional Jarlskog invariants $\text{Tr}[H_i^p,H_j^q]^r$ *are not* sufficient to describe CP violation in supersymmetry. This is seen most easily in the case of three Hermitian matrices A,B,C (which can be, for example, $Y^eY^{e\dagger}$, $Y^\nu Y^{\nu\dagger}$ and $M^{l\,2}$). This system has four physical phases, however there are only three independent Jarlskog–type invariants Tr $[A,B]^3$, Tr $[B,C]^3$ and Tr $[C,A]^3$. All higher order Jarlskog–type invariants are proportional to these three. This means that one CP phase cannot be picked up by such invariants and even if all of them vanish, CP violation is possible. It is thus necessary to include the K–type invariants [14].

The necessary and sufficient conditions for CP–conservation in the non–degenerate case amount to vanishing of the invariants (84). In that case, the 39 physical phases vanish and in some basis all the flavour objects are real. Clearly, there can then be no CP violation and any higher order CP–odd invariant, e.g. Tr $[A,B,C,D,E,..]$, would vanish as well.

We will not discuss here the degenerate case in detail. Suffice it to say that additional conditions such

$U(3)_l$	$U(3)_e$
$Y^e Y^{e\dagger}$	$Y^{e\dagger} Y^e$
$A^e A^{e\dagger}$	$A^{e\dagger} A^e$
$Y^e A^{e\dagger} + \text{H.c.}$	$Y^{e\dagger} A^e + \text{H.c.}$
$M^{l\,2}$	$M^{e\,2}$
$m_{\text{eff}} m_{\text{eff}}^*$	
$m_{\text{eff}} (Y^e Y^{e\dagger})^* m_{\text{eff}}^*$	

TABLE II: The minimal set of Hermitian flavour objects in the low energy theory.

as Im(Tr $(A^e Y^{e\dagger})^n) = 0$, *etc.* arise [14].[5]

B. Low Energy Theory

Below the seesaw scale M, one can integrate out the right–handed neutrinos as superfields. The resulting theory is the MSSM supplemented with the dimension-5 operator $\hat{L}\hat{\mathcal{H}}_2\hat{L}\hat{\mathcal{H}}_2$ (which is proton hexality and R-parity invariant) generating the left–handed neutrino masses. The flavour objects in the low–energy theory are Y^e, m_{eff} and the soft terms A^e, $M^{l\,2}$, $M^{e\,2}$.

In the basis (44), there is no residual rephasing freedom and the extra SUSY CP phases are

$$\begin{aligned} &\arg(A^e_{ij}) \ \ \to 9 \ , \\ &\arg(M^{l\,2}_{ij}) \to 3 \ , \\ &\arg(M^{e\,2}_{ij}) \to 3 \ , \end{aligned} \tag{86}$$

such that altogether we have 18 physical phases. The corresponding basis invariants are built out of the Hermitian matrices of Table II. 18 independent invariants can be chosen to be of the form (84) with H_i being the matrices belonging to the same column of Table II, respectively. Their independence is established by calculating the Jacobian with respect to the physical CP phases. An example of such invariants is given in the Appendix. The necessary and sufficient conditions for CP–conservation in the non–degenerate case amount to the vanishing of 18 independent invariants.

1. Observables and CP–odd Invariants

Physical observables are (complicated) functions of the basis invariants. An example relevant to CP violation in neutrino oscillations can be found in [25]. Here, let us illustrate this connection with a simple example of the neutralino–induced electron EDM (see [26] for recent analyses). In generic SUSY models, it is often expressed in terms of the "mass insertion" $(\delta^e_{LR})_{11}$ [27],

$$\Delta d_e \propto \text{Im}(\delta^e_{LR})_{11} \ , \tag{87}$$

with

$$(\delta^e_{LR})_{11} \approx \frac{\langle \mathcal{H}_1 \rangle A^e_{11}}{\tilde{m}^2} \ , \tag{88}$$

where we have neglected the μ–term contribution. $\tilde{m}$ is the average slepton mass and the A–terms are calculated in the basis where the charged lepton masses are diagonal and real.

To understand the connection to CP–odd invariants, let us assume a simple form for the A–terms in this basis,

$$A^e = \begin{pmatrix} A^e_{11} & A^e_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{89}$$

Calculating the K–invariants with Hermitian matrices of Table II, Column 2, we find

$$\begin{aligned} \text{Tr}\ \left([Y^{e\dagger} Y^e, (Y^{e\dagger} A^e + \text{H.c.})] A^{e\dagger} A^e \right) \\ \propto \sin(\arg\ (A^e_{11} Y^{e*}_{11})) \ . \end{aligned} \tag{90}$$

We thus conclude that it is this invariant that controls the electron EDM.

A few comments are in order. First, note the appearance of the reparametrization invariant phase arg $(A^e_{11} Y^{e*}_{11})$. Second, this phase cannot be "picked up" by any Jarlskog–type invariant. This is because the A–matrix is effectively 2×2 and the CKM–type phases vanish. Finally, if $A^e_{12} = 0$, A^e and Y^e are diagonal simultaneously. In this (special) case, the K–invariants vanish and CP violation comes from CP–odd invariants based on anti–Hermitian objects like Tr $[(A^e Y^{e\dagger})^n - h.c.]$.

In general, even if all of the soft terms are real in some basis, that does not guarantee absence of dangerous SUSY contributions to EDMs. The SM flavour structures Y^e and m_{eff} may contain complex phases such that the reparametrization invariant phases are non–zero. In other words, K–invariants can be non–zero even if the soft terms are real. This is similar to the quark sector where the CKM phase can result in large EDMs in the presence of real soft terms [28].

[5] We are working under the assumption that different matrices are not diagonal in the same basis. In the degenerate case, this is not true and all J– and K–invariants can vanish even though there is physical CP violation. CP–odd invariants sensitive to the corresponding CP phases are, for example, Tr $[(A^e Y^{e\dagger})^n - \text{h.c.}]$.

IV. CONCLUSION

We have constructed a generalization of the Jarlskog invariant to supersymmetric models with right–handed neutrinos. We find that CP violation in supersymmetric models is controlled by CP–odd invariants of the conventional Jarlskog–type ("J–invariants") as well as those involving antisymmetric products of three Hermitian matrices ("K–invariants"), which cannot be expressed in terms of the former.

The presence of right–handed neutrinos brings in new features, in particular, Majorana–type CP phases in supersymmetric as well as soft terms. The corresponding CP–odd invariants are built out of Hermitian objects involving a product of two or four flavour matrices as opposed to 2 in the Dirac case. This complicates the analysis, on the one hand, but allows for interesting features, on the other hand. For example, CP violation is possible even if the neutrinos are all degenerate in mass.

We have identified 39 physical CP phases and corresponding CP–odd invariants which control CP violation in the lepton sector of the MSSM with right–handed neutrinos. Below the seesaw scale, the low energy theory is described by 18 CP phases which can again be linked to 18 independent CP invariants. This allows us to formulate basis–independent conditions for CP conservation in the non–degenerate case.

Physical observables are in general complicated functions of CP–odd invariants, which we illustrate with an example of the electron EDM. SUSY CP violation and, in particular, dangerous EDM contributions, are possible even if the soft supersymmetry breaking terms are real in some basis.

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APPENDIX A: INDEPENDENT CP–ODD INVARIANTS

Let us label matrices of the first column of Table I by X_i, second – Y_i, and third – Z_i, where i refers to the row number. Then the 39 independent invariants can be chosen as

$$\mathrm{Tr}[X_1, X_2]^3, \tag{A1}$$
$$\mathrm{Tr}[X_1, X_2]X_3, \tag{A2}$$
$$\mathrm{Tr}[X_1^2, X_2]X_3, \tag{A3}$$
$$\mathrm{Tr}[X_1, X_2^2]X_3, \tag{A4}$$
$$\mathrm{Tr}[X_1, X_2]X_4, \tag{A5}$$
$$\mathrm{Tr}[X_1^2, X_2]X_4, \tag{A6}$$
$$\mathrm{Tr}[X_1, X_2^2]X_4, \tag{A7}$$
$$\mathrm{Tr}[X_1, X_2]X_5, \tag{A8}$$
$$\mathrm{Tr}[X_1^2, X_2]X_5, \tag{A9}$$
$$\mathrm{Tr}[X_1, X_2^2]X_5, \tag{A10}$$
$$\mathrm{Tr}[X_1, X_2]X_6, \tag{A11}$$
$$\mathrm{Tr}[X_1^2, X_2]X_6, \tag{A12}$$
$$\mathrm{Tr}[X_1, X_2^2]X_6, \tag{A13}$$
$$\mathrm{Tr}[X_1, X_2]X_7, \tag{A14}$$
$$\mathrm{Tr}[X_1^2, X_2]X_7, \tag{A15}$$
$$\mathrm{Tr}[X_1, X_2^2]X_7. \tag{A16}$$

$$\mathrm{Tr}[Y_1, Y_3]Y_2, \tag{A17}$$
$$\mathrm{Tr}[Y_1^2, Y_3]Y_2, \tag{A18}$$
$$\mathrm{Tr}[Y_1, Y_3^2]Y_2, \tag{A19}$$
$$\mathrm{Tr}[Y_1, Y_3]Y_4, \tag{A20}$$
$$\mathrm{Tr}[Y_1^2, Y_3]Y_4, \tag{A21}$$
$$\mathrm{Tr}[Y_1, Y_3^2]Y_4. \tag{A22}$$

$$\mathrm{Tr}[Z_1, Z_3]Z_2, \tag{A23}$$
$$\mathrm{Tr}[Z_1^2, Z_3]Z_2, \tag{A24}$$
$$\mathrm{Tr}[Z_1, Z_3^2]Z_2, \tag{A25}$$
$$\mathrm{Tr}[Z_1, Z_3]Z_4, \tag{A26}$$
$$\mathrm{Tr}[Z_1^2, Z_3]Z_4, \tag{A27}$$
$$\mathrm{Tr}[Z_1, Z_3^2]Z_4, \tag{A28}$$
$$\mathrm{Tr}[Z_1, Z_3]Z_5, \tag{A29}$$
$$\mathrm{Tr}[Z_1^2, Z_3]Z_5, \tag{A30}$$
$$\mathrm{Tr}[Z_1, Z_3^2]Z_5, \tag{A31}$$
$$\mathrm{Tr}[Z_1, Z_3]Z_6, \tag{A32}$$
$$\mathrm{Tr}[Z_1^2, Z_3]Z_6, \tag{A33}$$
$$\mathrm{Tr}[Z_1, Z_3]Z_7, \tag{A34}$$
$$\mathrm{Tr}[Z_1^2, Z_3]Z_7, \tag{A35}$$
$$\mathrm{Tr}[Z_1, Z_3^2]Z_7, \tag{A36}$$
$$\mathrm{Tr}[Z_1, Z_3]Z_8, \tag{A37}$$
$$\mathrm{Tr}[Z_1^2, Z_3]Z_8, \tag{A38}$$
$$\mathrm{Tr}[Z_1, Z_3^2]Z_8. \tag{A39}$$

Similarly, labelling entries of the first column of Table II by A_i and those of the second column by B_i, we have the following 18 independent invariants:

$$\mathrm{Tr}[A_1, A_6]^3, \tag{A40}$$
$$\mathrm{Tr}[A_5, A_1]A_6, \tag{A41}$$
$$\mathrm{Tr}[A_5^2, A_1]A_6, \tag{A42}$$
$$\mathrm{Tr}[A_5, A_1]A_2, \tag{A43}$$
$$\mathrm{Tr}[A_5^2, A_1]A_2, \tag{A44}$$
$$\mathrm{Tr}[A_5, A_1^2]A_2, \tag{A45}$$
$$\mathrm{Tr}[A_5, A_1]A_3, \tag{A46}$$
$$\mathrm{Tr}[A_5^2, A_1]A_3, \tag{A47}$$
$$\mathrm{Tr}[A_5, A_1^2]A_3, \tag{A48}$$
$$\mathrm{Tr}[A_5, A_1]A_4, \tag{A49}$$
$$\mathrm{Tr}[A_5^2, A_1]A_4, \tag{A50}$$
$$\mathrm{Tr}[A_5, A_1^2]A_4, \tag{A51}$$
$$\mathrm{Tr}[B_1, B_3]B_2, \tag{A52}$$
$$\mathrm{Tr}[B_1^2, B_3]B_2, \tag{A53}$$
$$\mathrm{Tr}[B_1, B_3^2]B_2, \tag{A54}$$
$$\mathrm{Tr}[B_1, B_3]B_4, \tag{A55}$$
$$\mathrm{Tr}[B_1^2, B_3]B_4, \tag{A56}$$
$$\mathrm{Tr}[B_1, B_3^2]B_4. \tag{A57}$$

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