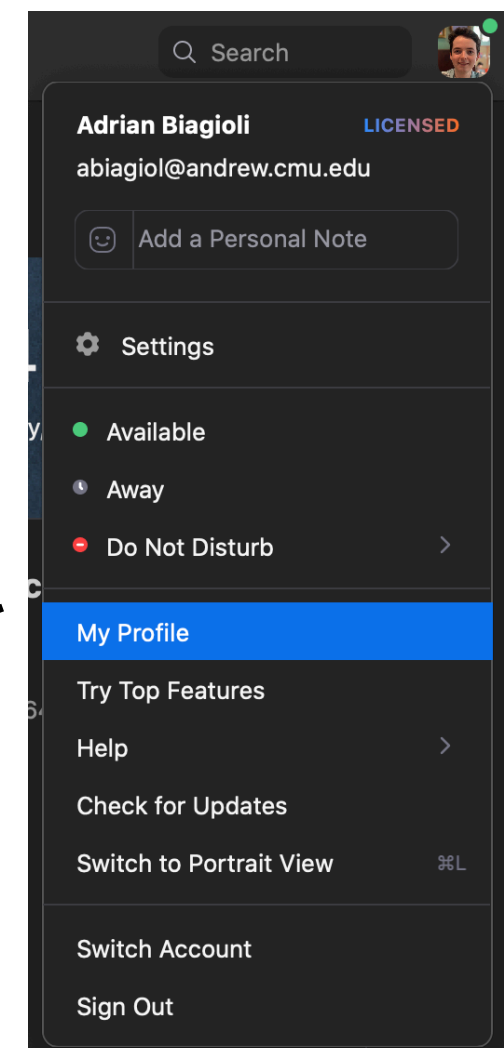
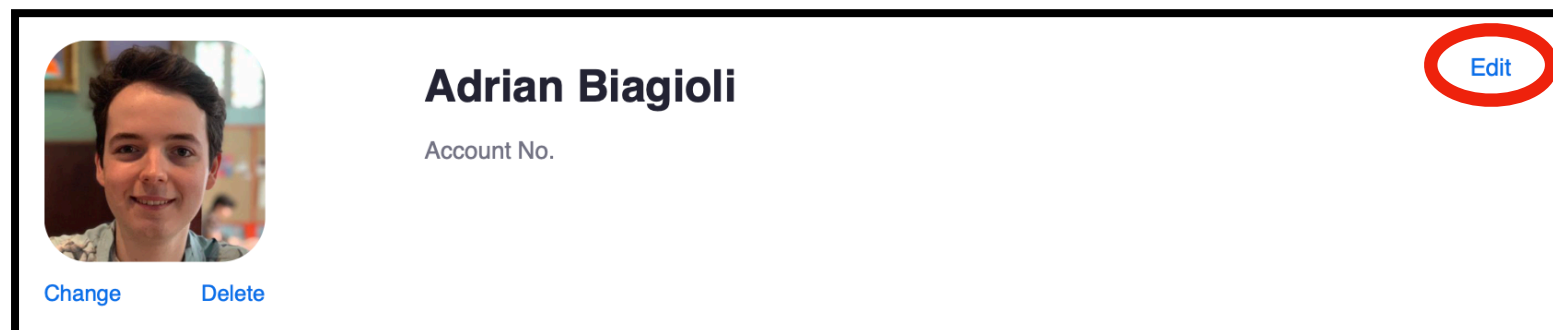


# Recitation will begin Shortly

- ▶ Make sure your microphone is on mute
- ▶ Get used to the raise hand feature on zoom, I will be paying attention to this so you can ask questions
  - Please make sure your name is set on zoom so I can call on you
  - You can also ask questions in the text chat



# Assignment 3

## Overview

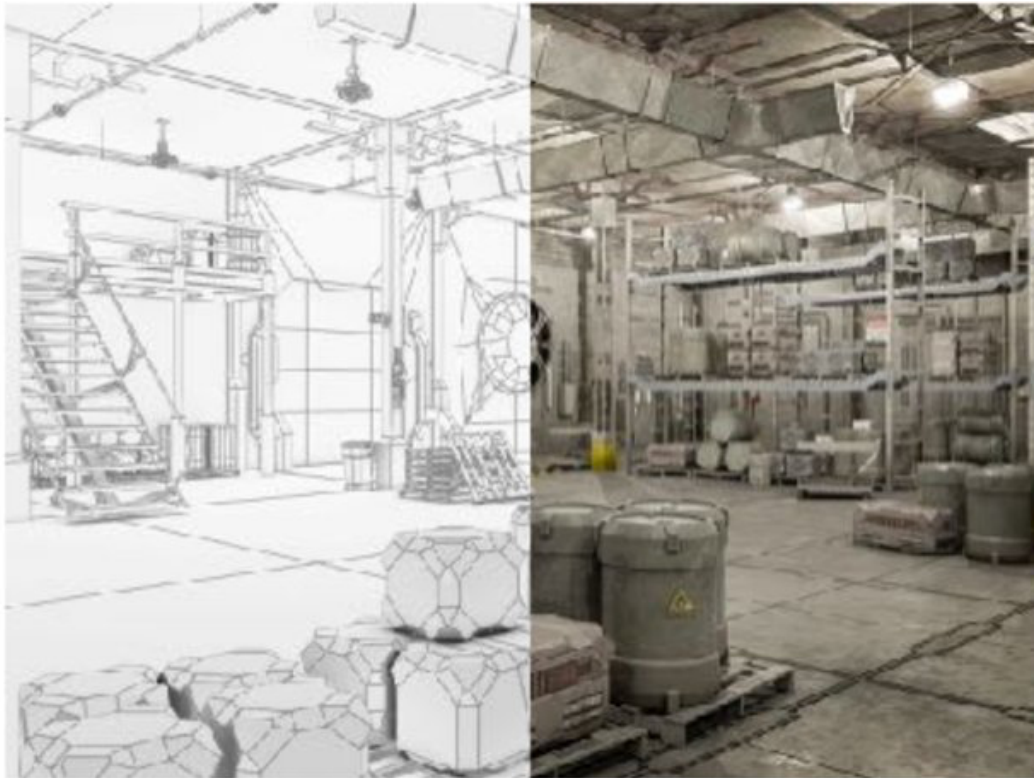
---

***Computer Graphics***  
***CMU 15-462/662***

# Assignment 3: Pathtracer

- ▶ Extension of the work you did in MeshEdit
  - Now that we can create meshes with Scotty3D, it's time to build a renderer that computes a realistic rendering of the scene
- ▶ Warning: Pathtracer will be difficult for different reasons than MeshEdit was difficult!
  - In MeshEdit we aimed to maintain the invariants of a complex data structure
    - ◎ Errors are more “obvious” and result in crashes/hangs
  - In PathTracer, we aim to maintain physical accuracy, but we aren't changing the scene at all
    - ◎ Errors are related to math or theory and the symptoms are usually visual and may not be obvious

# Rasterization vs Pathtracing



## Rasterization

Transform scene geometry via matrix operations to screen space, then use triangle fill algorithm.

*Optimized for performance*

DrawSVG (A1)



## Pathtracing

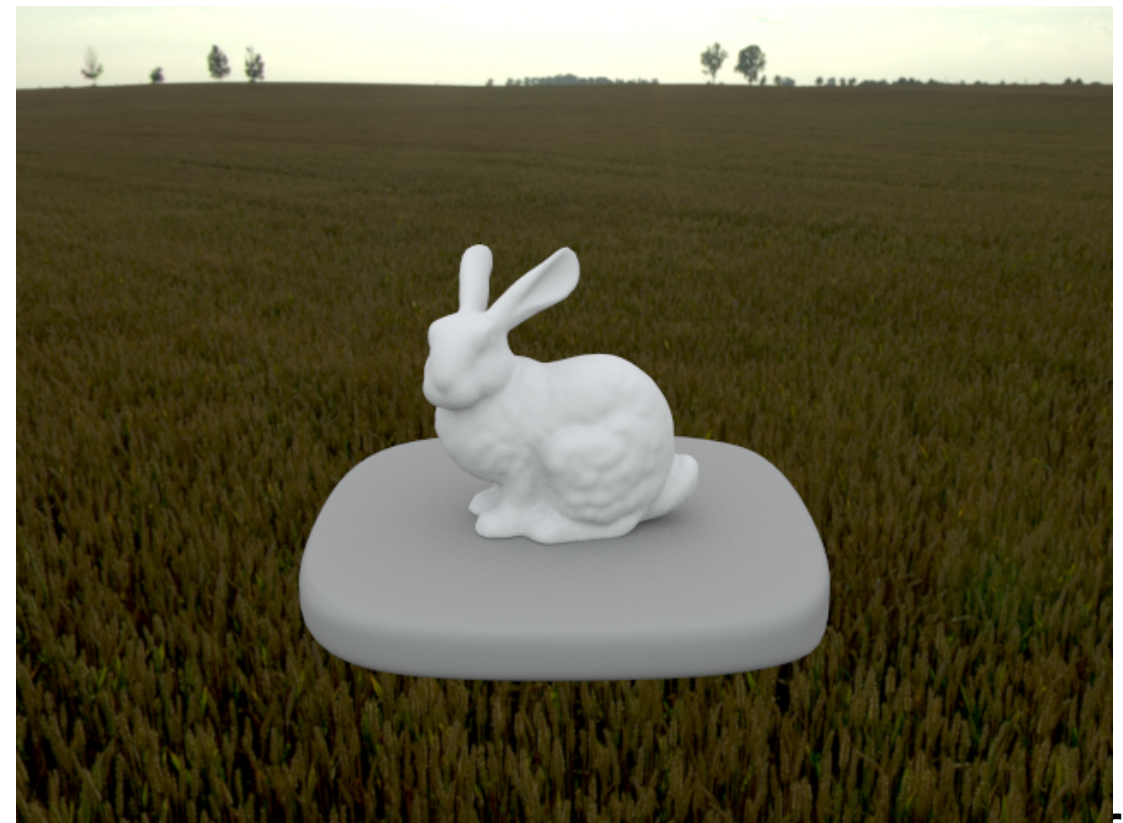
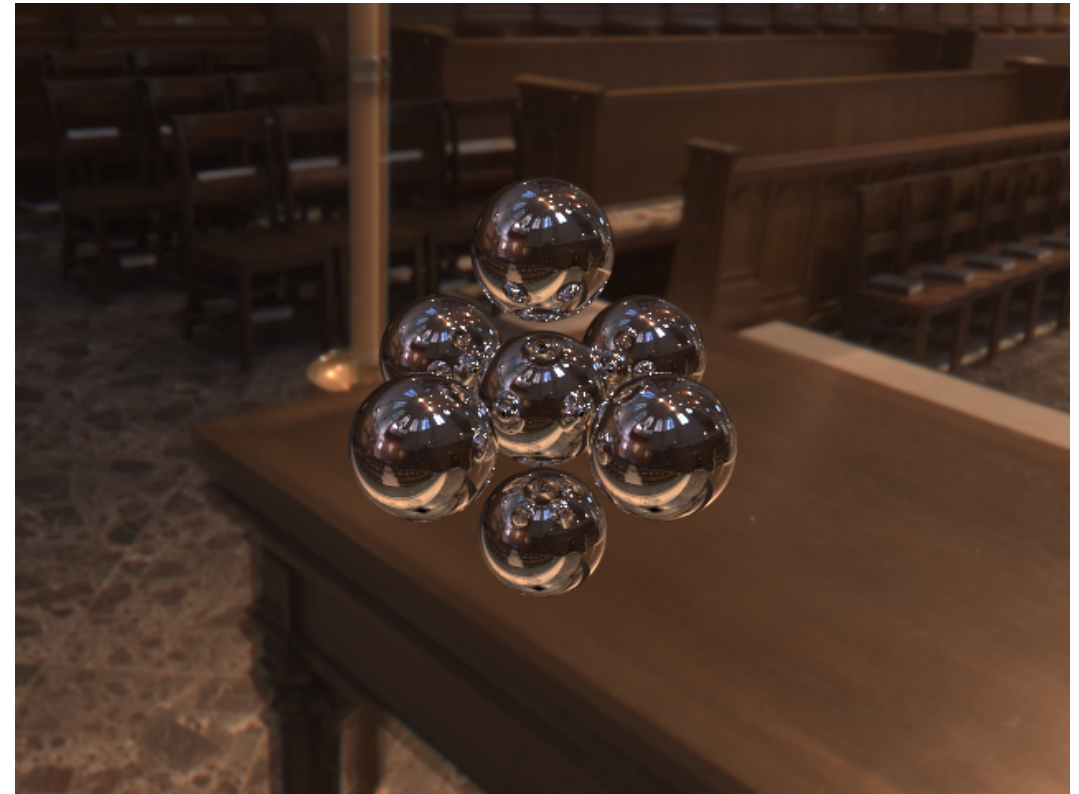
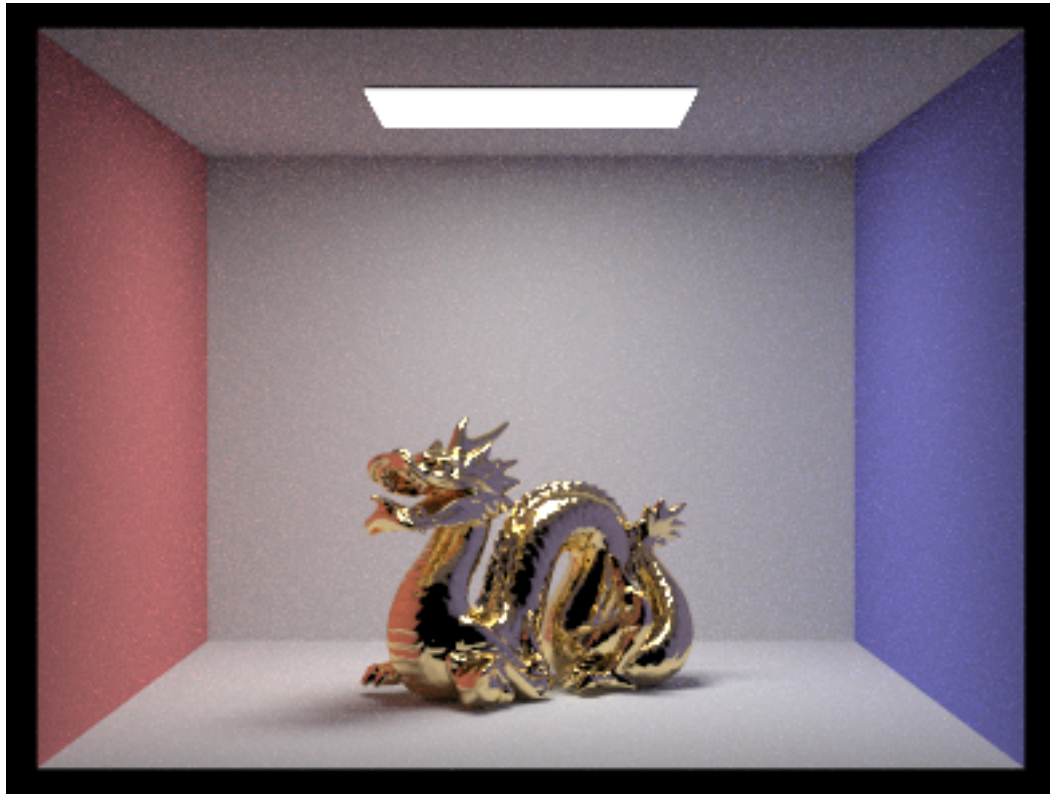
Bounce simulated rays of light throughout your scene randomly for each pixel, and illuminate if it eventually intersects a light.

*Optimized for realism*

Pathtracer (A3)



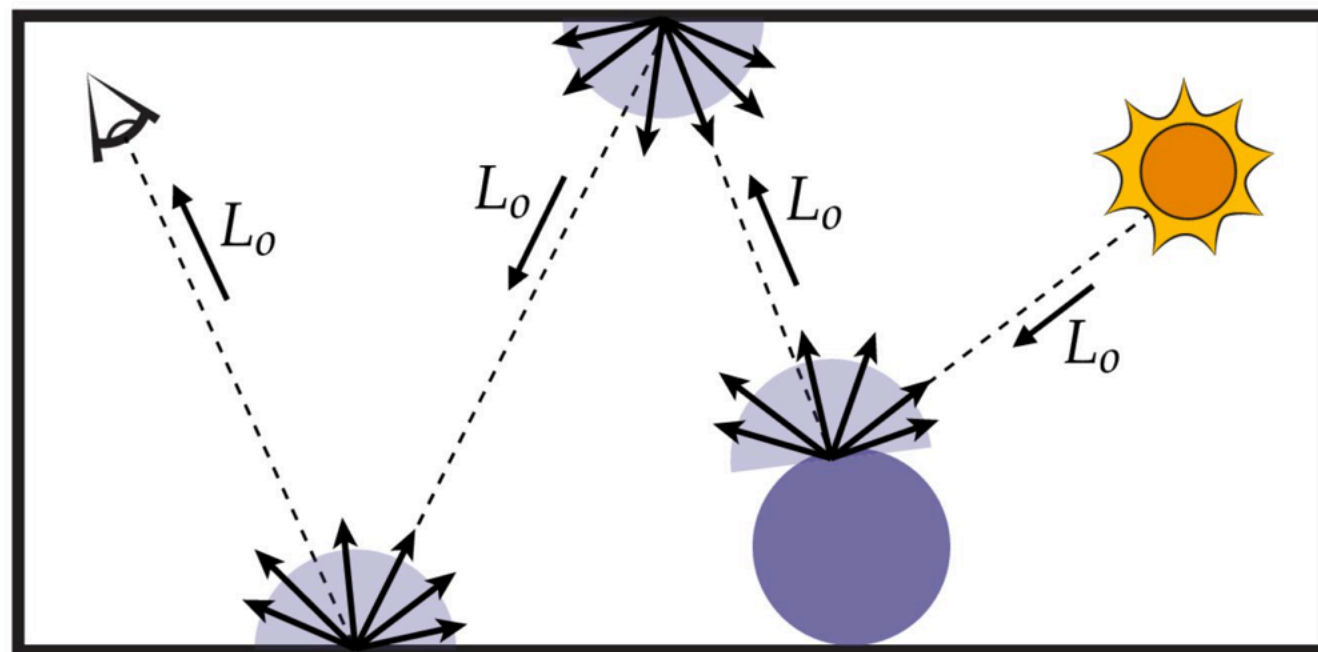
# Example Scotty3d Output



# The Big Picture

# End Goal

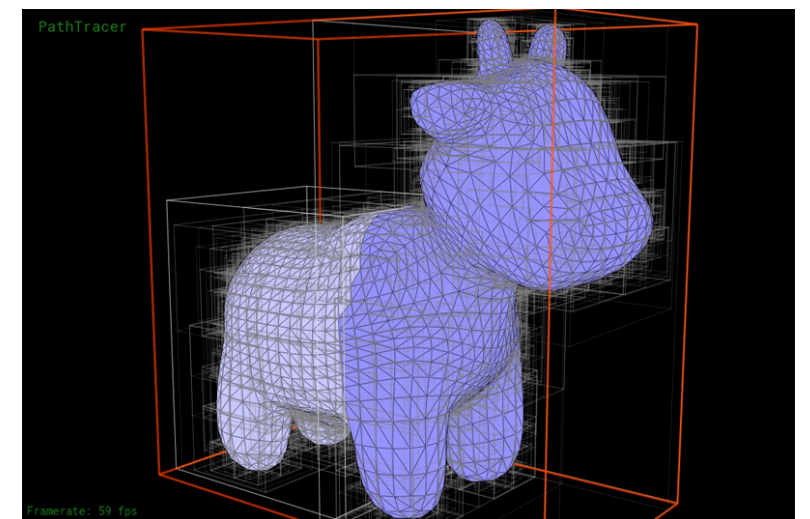
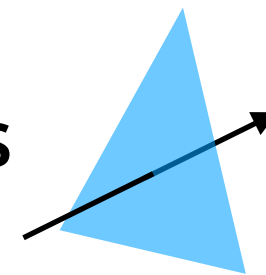
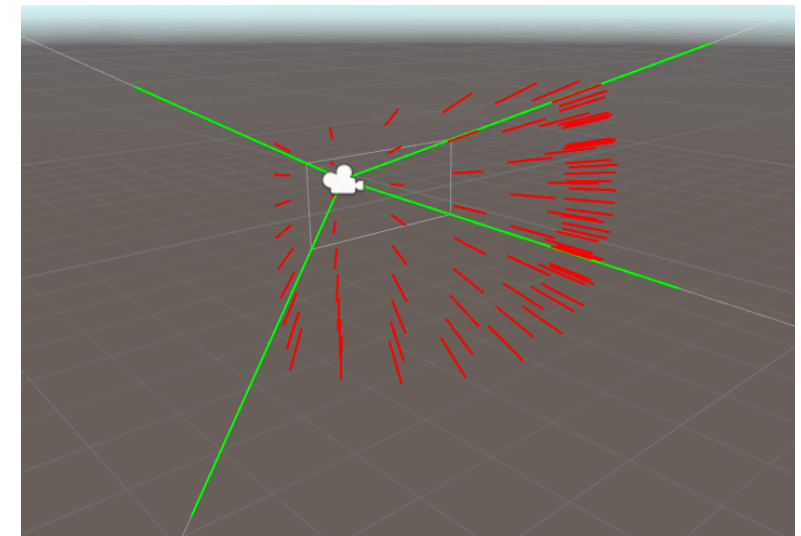
- ▶ You are tasked with building a *pathtracer*, which simulates rays of light bouncing around your scene and eventually “into the camera”
  - (Small detail: we will actually “start” our rays at the camera origin and bounce it around the scene until we hit a light)
  - Over the next few weeks we will dive into the physics of Color, Radiometry, the “Rendering Equation,” and more details that are important in designing a pathtracer
  - Up to this point you are ready to complete the assignment up to + including Task 4 (shadow rays).





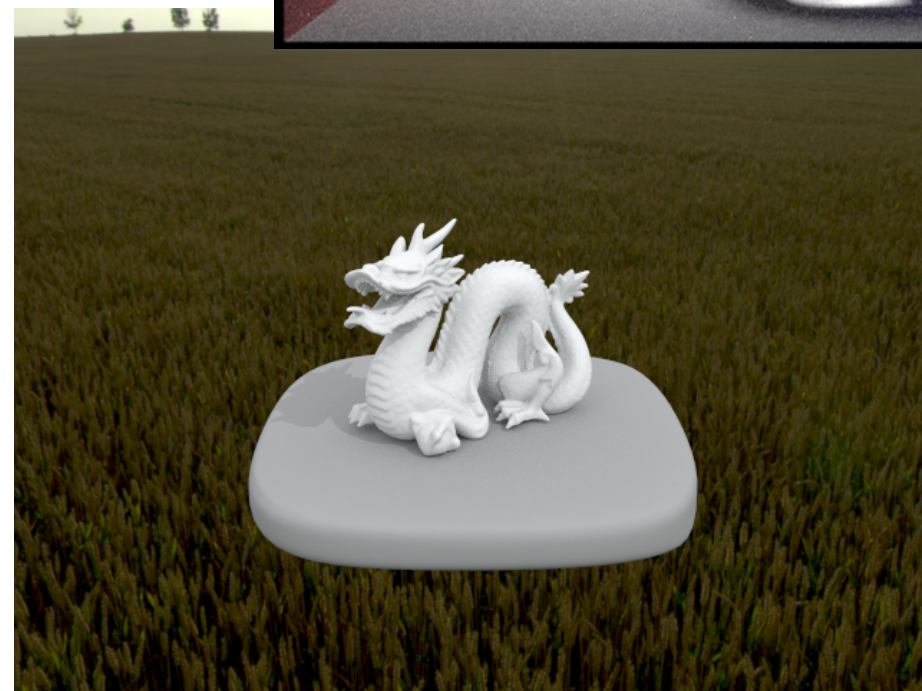
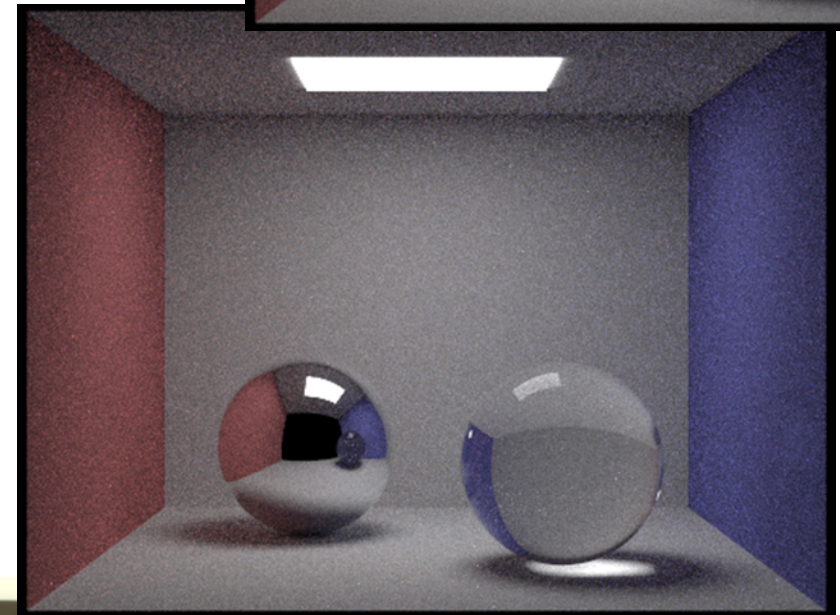
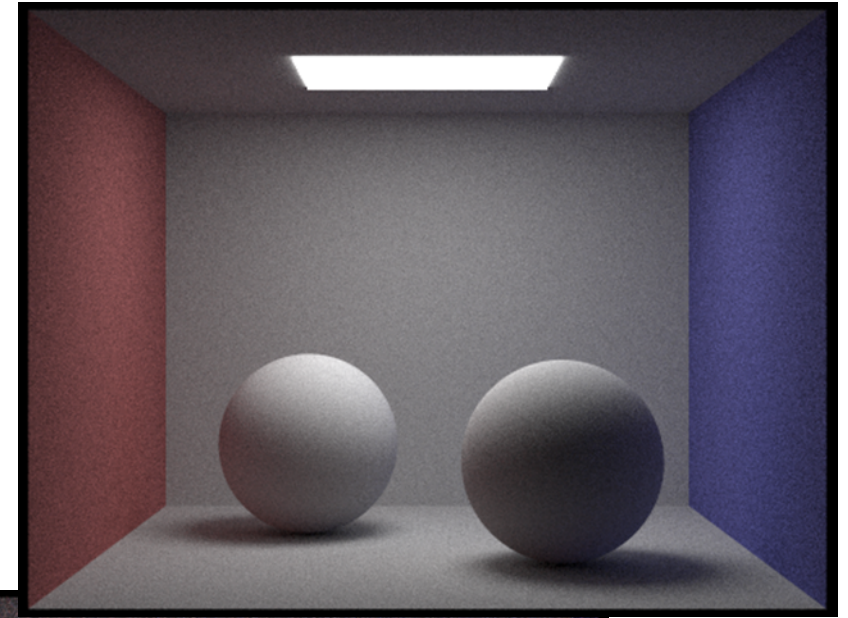
# Overview of Tasks

- ▶ Task 1: Generate the initial rays to send from the camera
- ▶ Task 2: Compute ray-primitive intersection
  - You need to support triangles and spheres
- ▶ Task 3: Accelerate ray-scene intersection queries using a Bounding Volume Hierarchy (BVH)
- ▶ Task 4: Implement direct lighting with shadows



# Overview of Tasks

- ▶ Task 5: Support indirect illumination via path tracing
- ▶ Task 6: Support non-diffuse materials (mirror, glass)
- ▶ Task 7: Support environment lighting via a texture

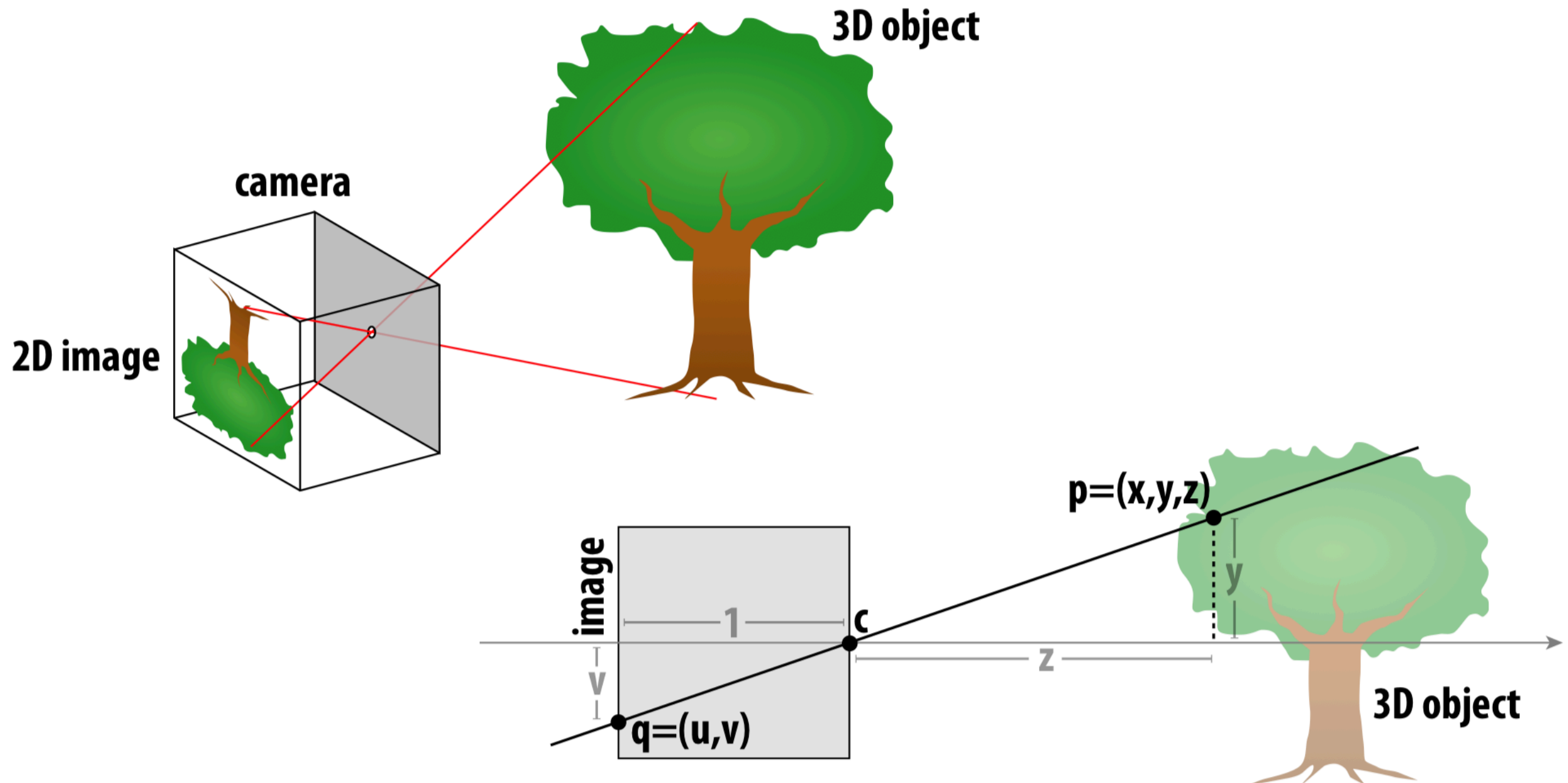


# Camera Rays



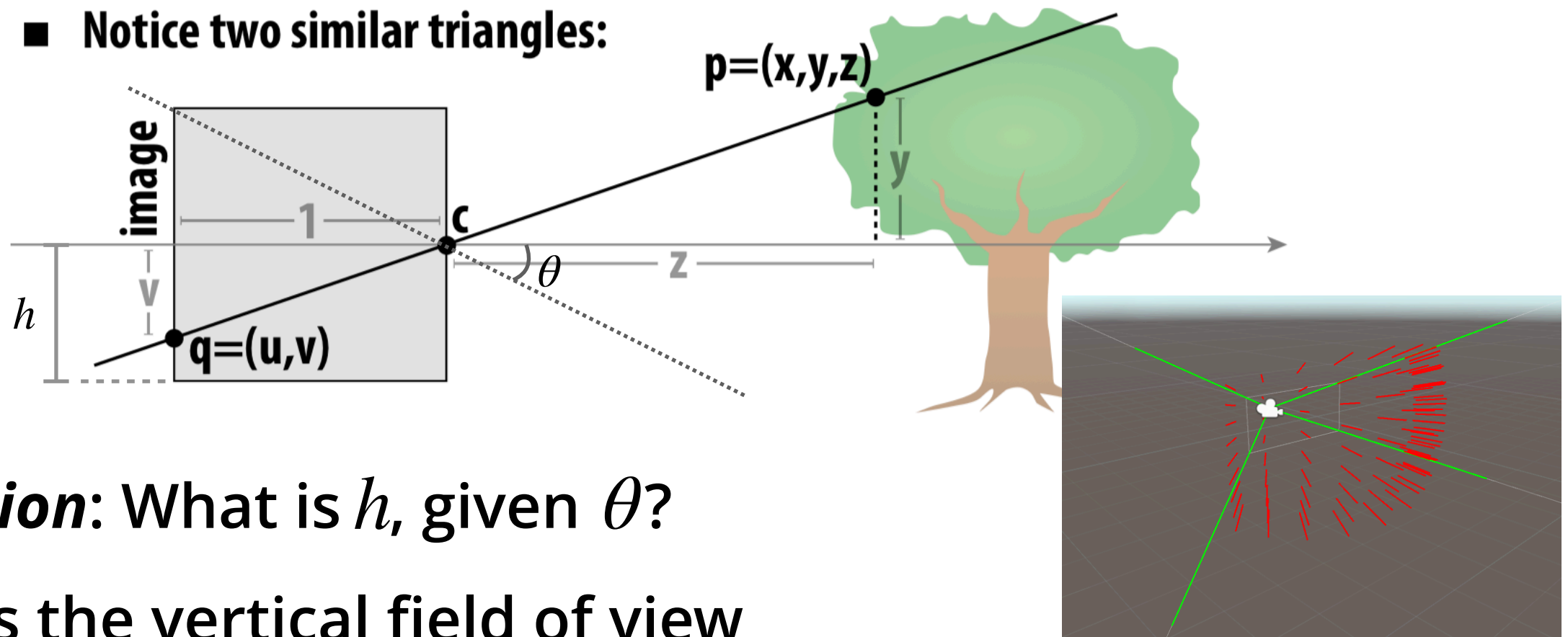
# Recall Lecture 1: Perspective Projection

- In lecture 1 we considered the *pinhole* model for cameras:



# Recall Lecture 1: Perspective Projection

■ Notice two similar triangles:



► **Question:** What is  $h$ , given  $\theta$ ?

- $2\theta$  is the vertical field of view

- $v$  is the projection of  $p_y$  on the image plane, with extents  $[-h, h]$

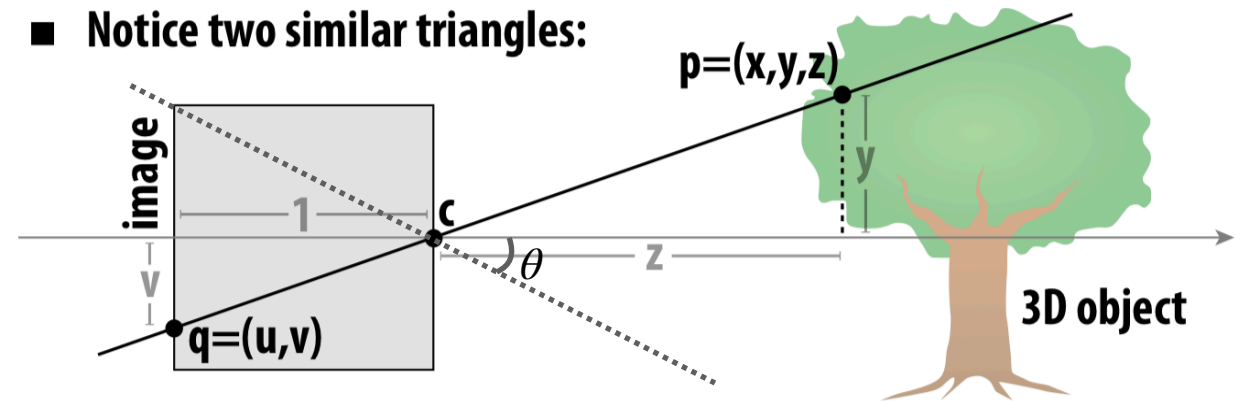
► **Goal of part 1:** generate the ray  $\overrightarrow{cp}$  in world space given the camera position, orientation, and  $(u, v)$  coordinate

- $u, v$  are given in  $[0, 1]$  range, not  $[0, h]$

# Implementation

- Where is each variable in the figure, in camera.h?
  - Do we need anything more than the excerpt on the bottom right?
- Suggestion: Calculate the camera ray in camera space first
  - Take advantage of similar triangles —  $y/v = ??$
  - How do we convert a camera space vector to world space?
  - Camera space: camera forward vector is  $(0,0,-1)$  and camera origin is at  $(0,0,0)$  - much easier to reason about camera rays!

■ Notice two similar triangles:



```
// camera.h
class Camera {
public:

    // ...
    double v_fov() const { return vFov; }           // !!
    double aspect_ratio() const { return ar; }       // !!
    // ...

    // worldspace -> cameraspace transformation matrix
    Matrix4x4 getTransformation();                   // !!

    // Task 1
    Ray generate_ray(double x, double y) const;

    // ...
};
```

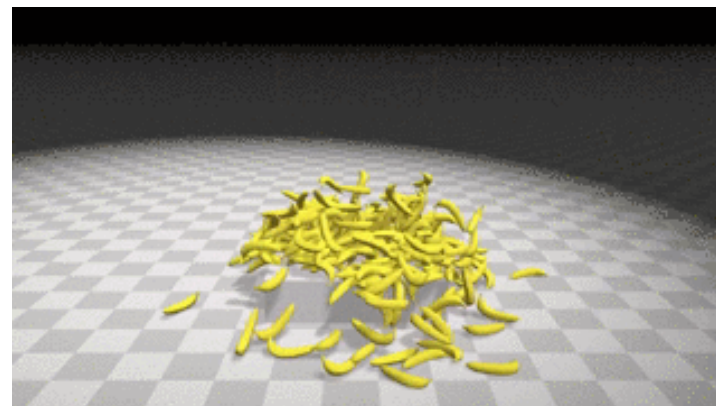
# **Bounding Volume Hierarchies (BVHs)**

# Recall: Spatial Data Structures

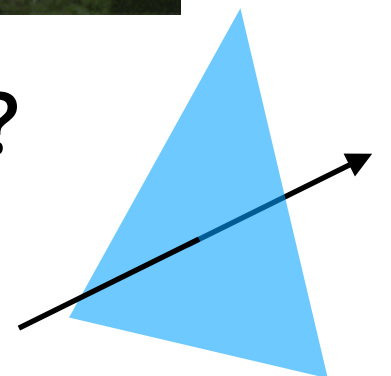
- ▶ Problem: *I want to efficiently perform a query on primitives that are ordered/arranged spatially. This query is only going to be relevant locally to some volume of space, and we would like to bail out of the computation early for “far-away” primitives that are outside that volume.*
  - Examples: Collision detection, frustum/occlusion culling

Left:  
<https://youtu.be/-S8wq0dz1H4>

Right:  
<https://youtu.be/VqH8kcmD-HI>

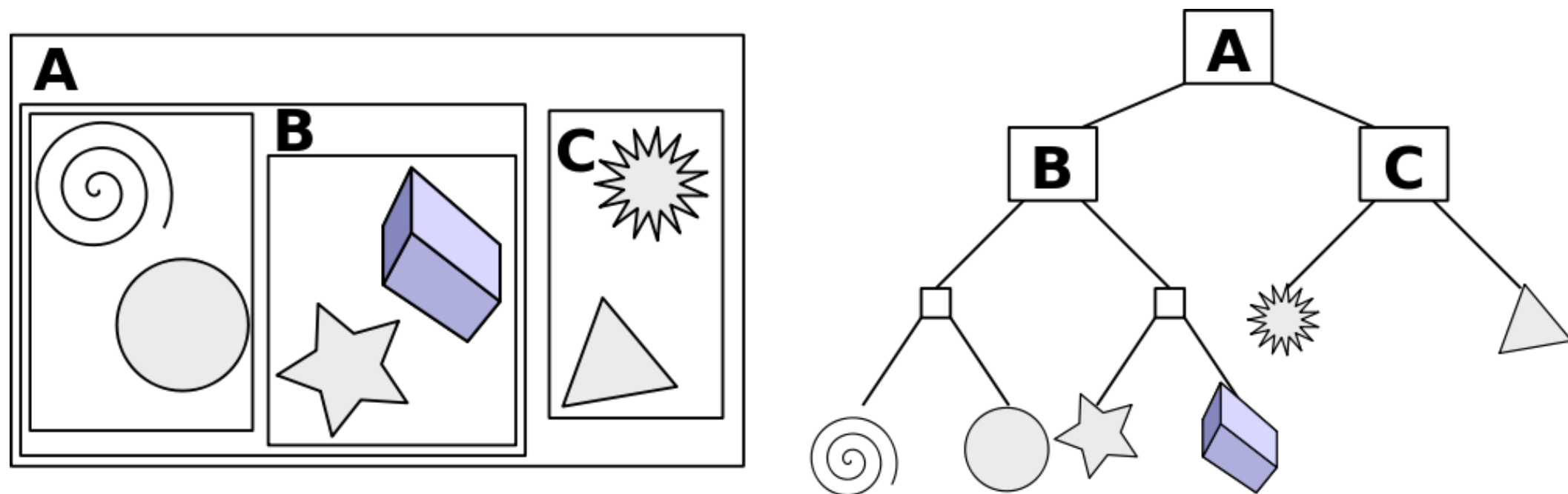


- How do we cheaply figure out when to bail out?
- ▶ What's the query in raytracing? The primitive?
  - In Scotty3D: Ray-Triangle / Ray-Sphere intersection



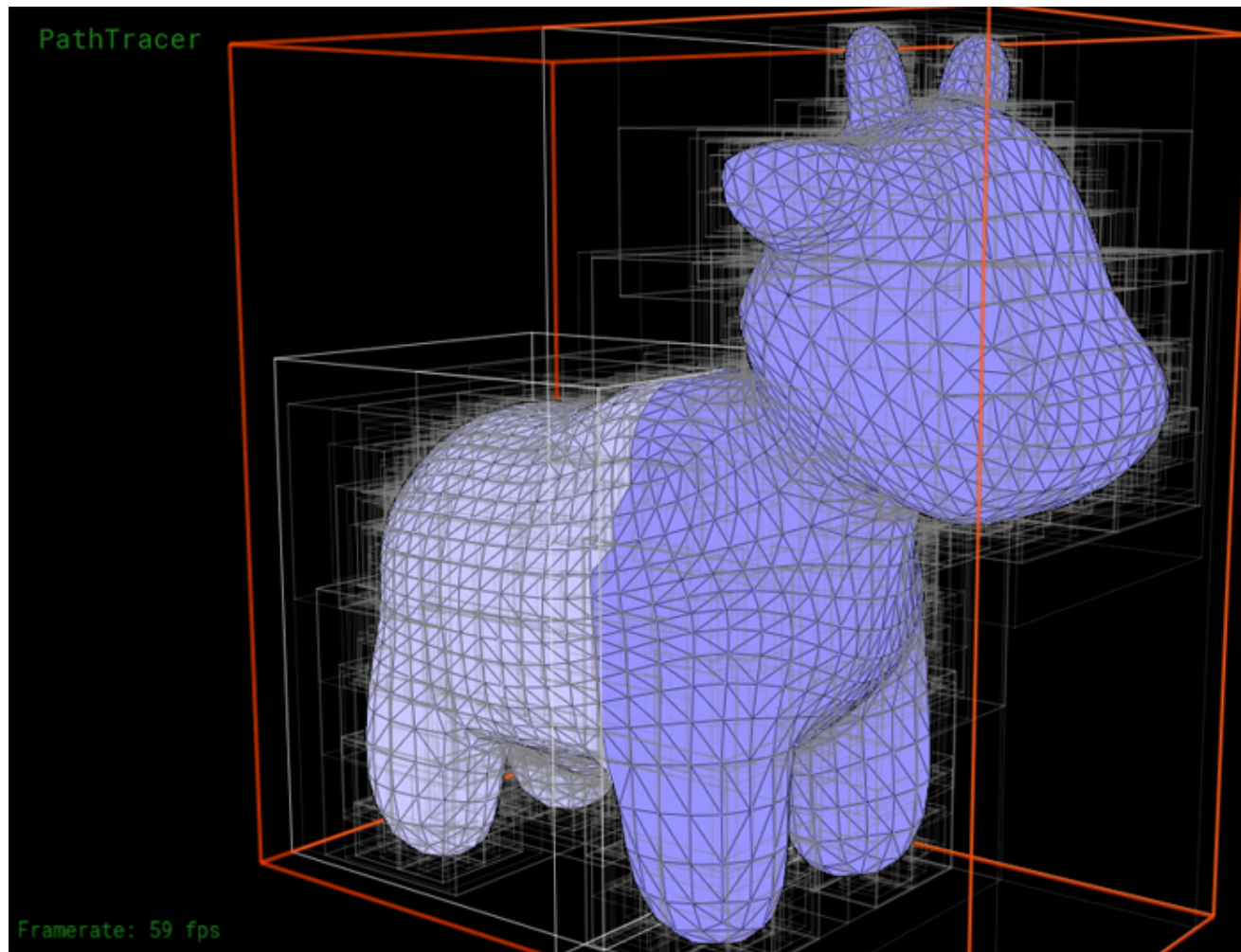
# Recall: Bounding Volume Hierarchy

- ▶ Divide all of your primitives into a hierarchy: a binary tree
  - The leaves are individual primitives
  - The nodes are *bounding volumes*



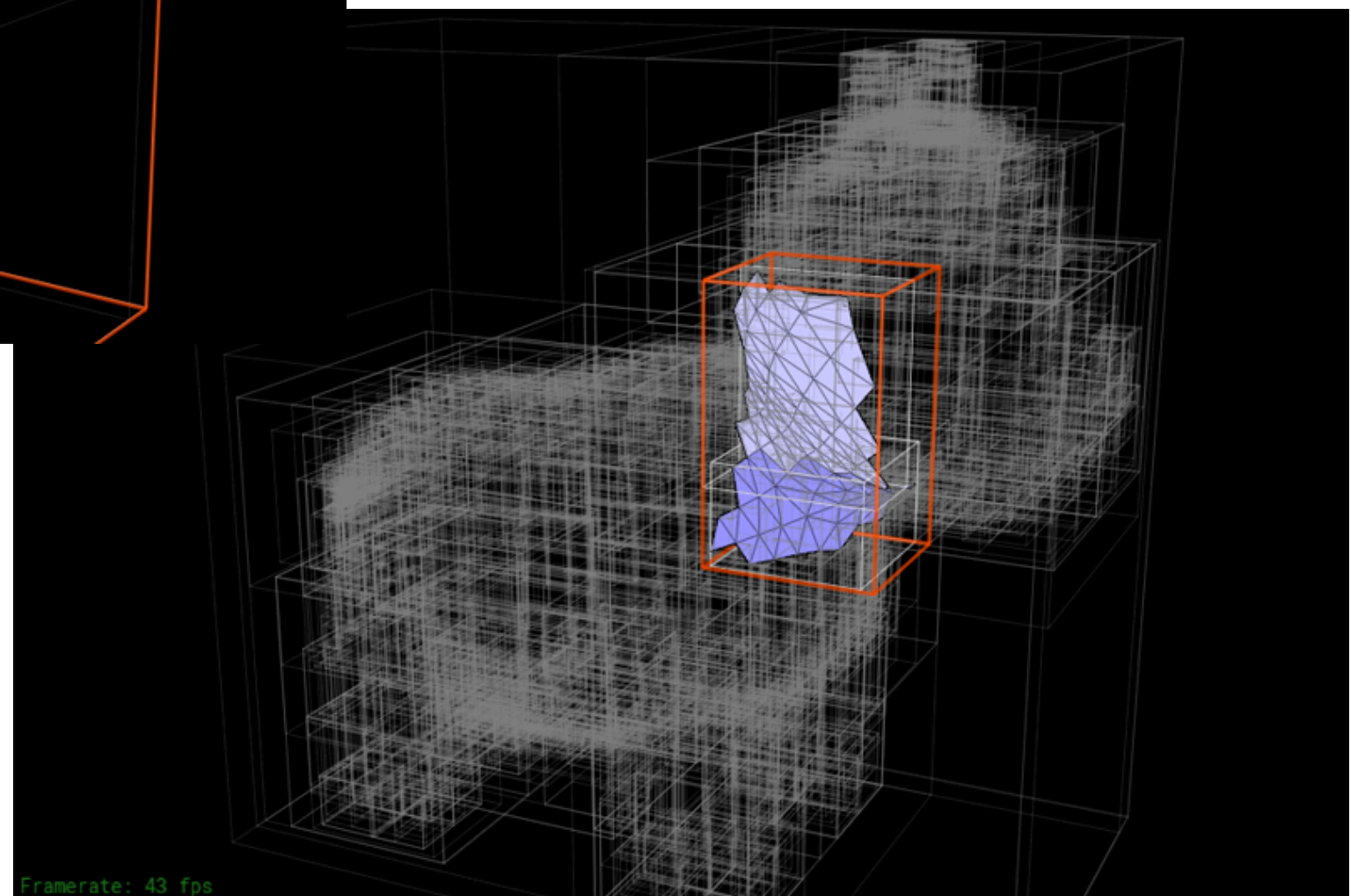


# BVH in Scotty3D



Red box: Currently selected node  
Dark blue triangles: "right" subtree  
Light blue triangles: "left" subtree

*Note: "right" / "left" does not have to do with the spatial positioning, only the topology of the BVH graph!*

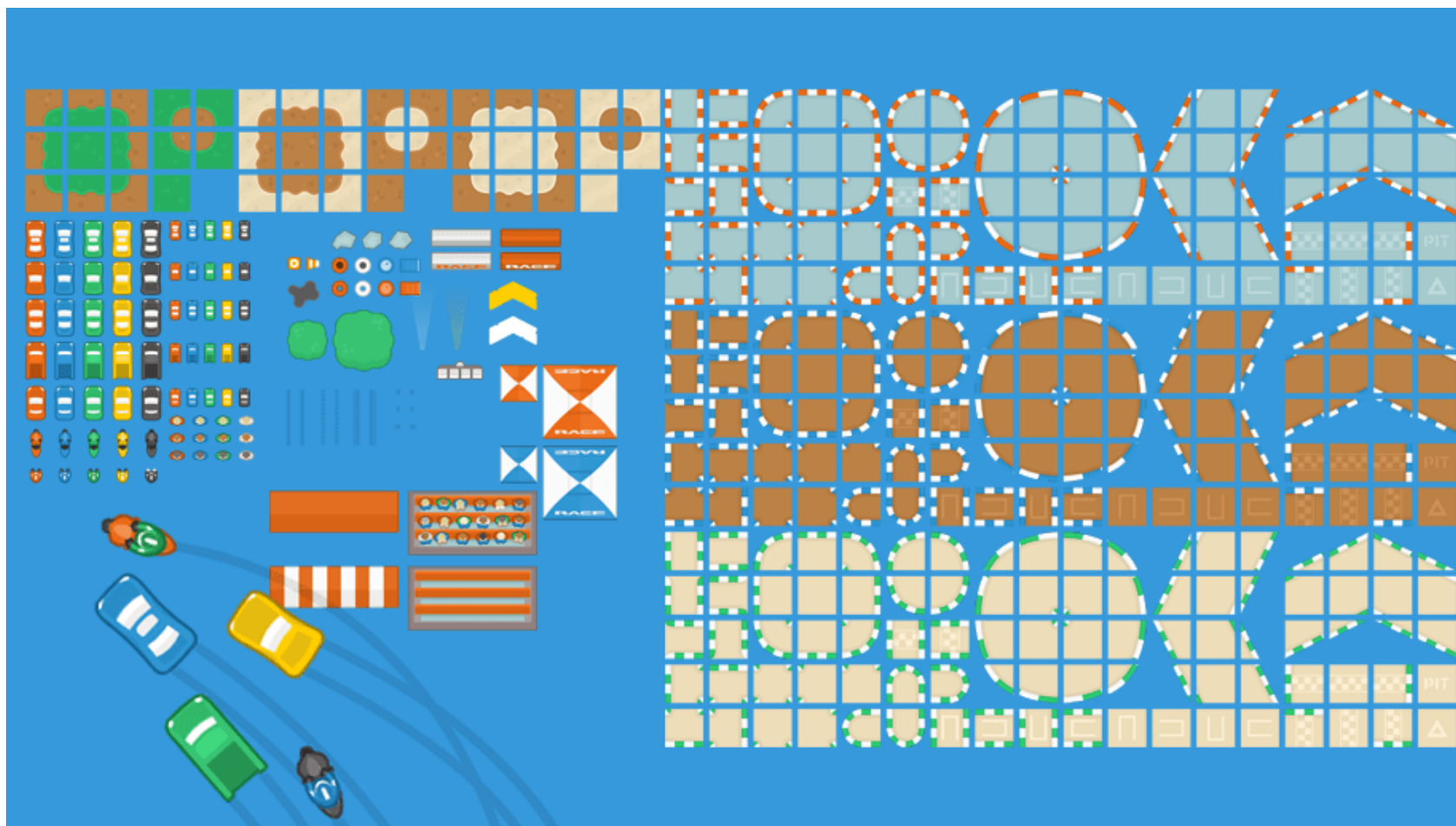


Once you've implemented BVH, you can look at this visualization via the V key after rendering

Press the < and > keys to descend the tree and ? to move to the parent

# Exercise: Bounding Circle Hierarchy

- ▶ Suppose you're a graphics engineer working on a 2D video game in which you need to draw many thousands of vector graphics (like DrawSVG) on screen at a time. To speed up rendering, you propose using vector objects (polygons, curves, etc) as the primitives and circles as the volumes



Vector Object  
Assets (the  
primitives)



# “BCH” Example Exercise

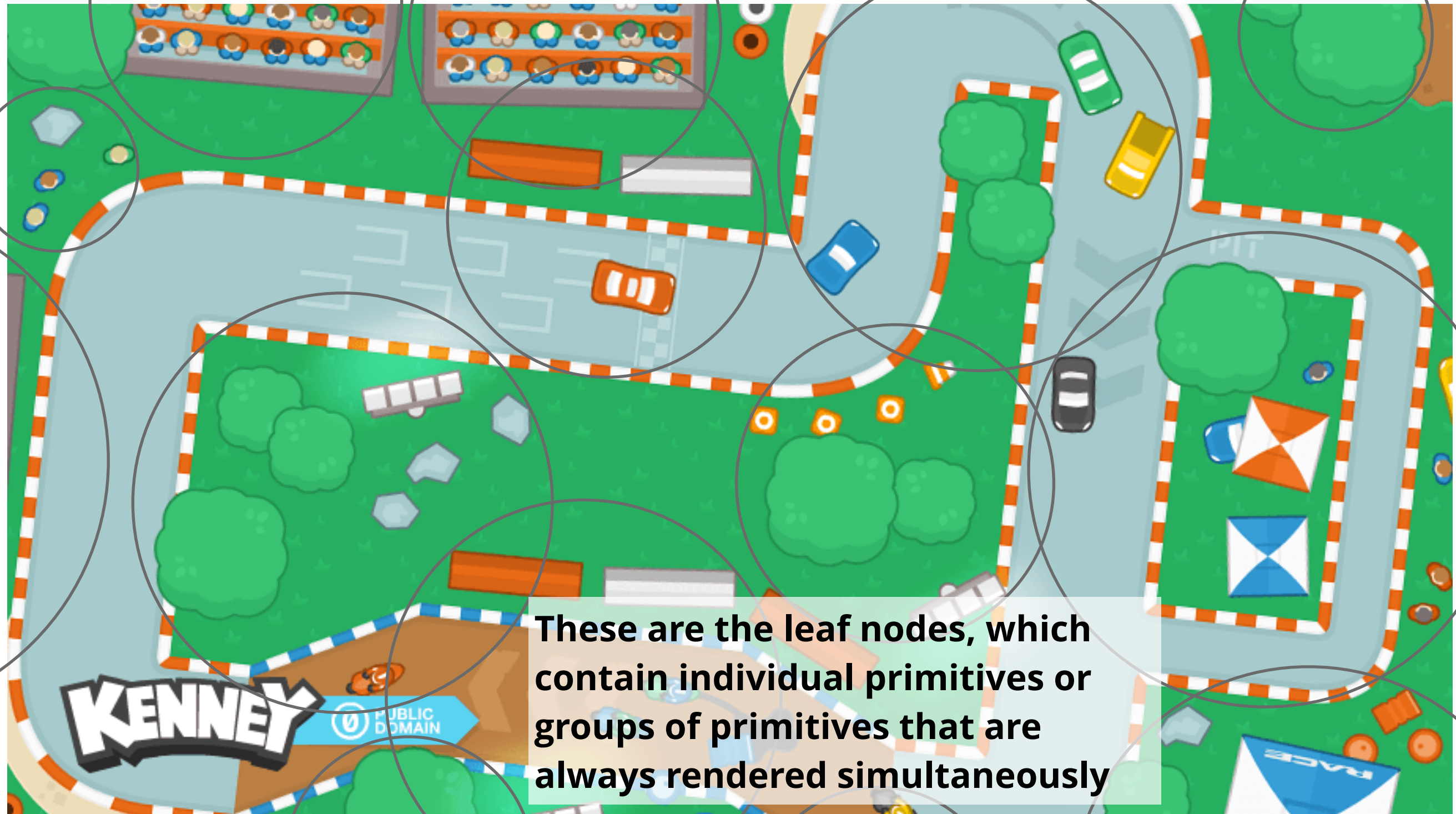
We are only considering foreground objects (not the racetrack/grass) here for clarity (we would have to draw way more circles otherwise)



Source: <https://kenney.nl/assets/racing-pack>

# "BCH" Example Exercise

We are only considering foreground objects (not the racetrack/grass) here for clarity (we would have to draw way more circles otherwise)

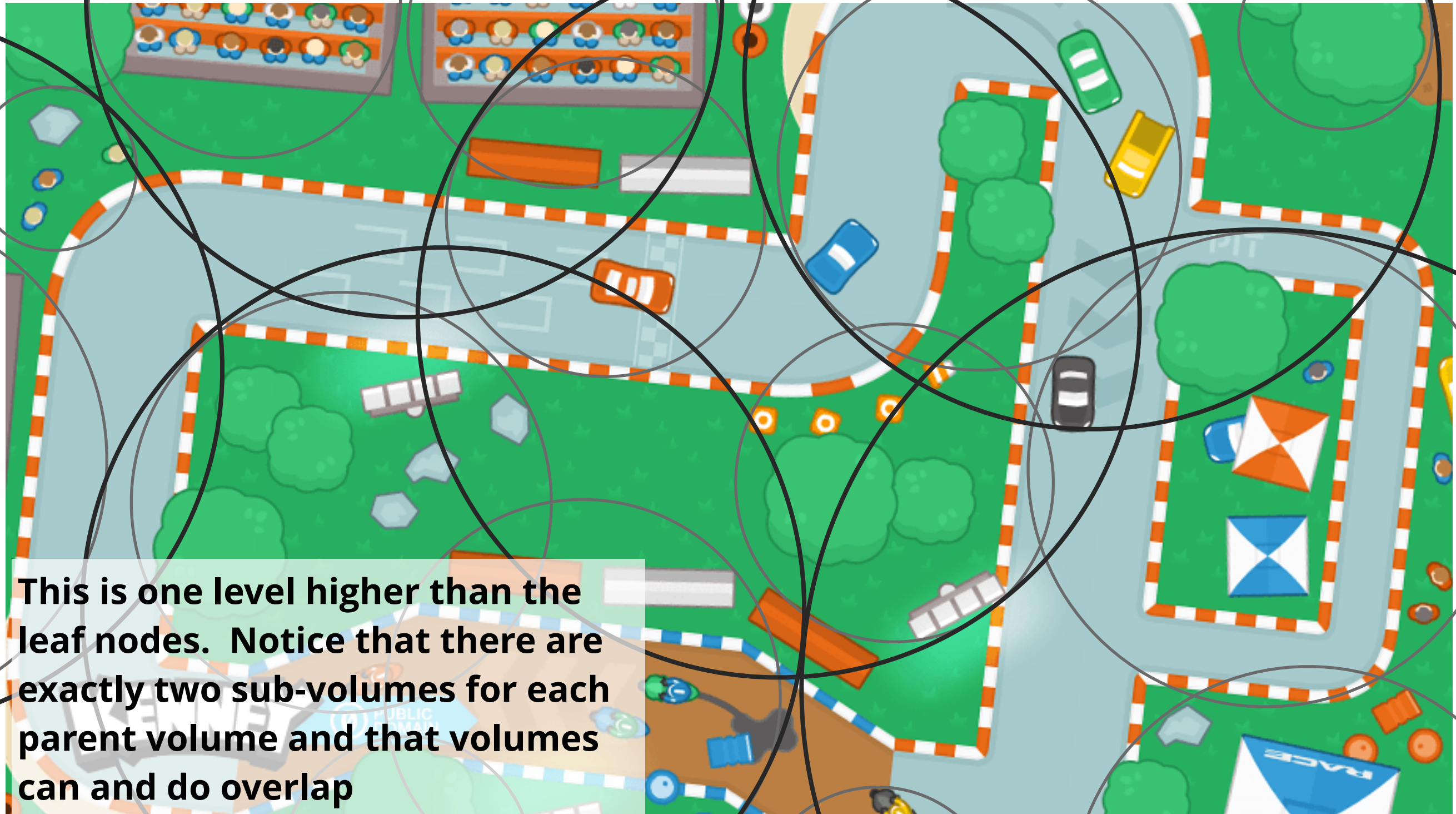


These are the leaf nodes, which contain individual primitives or groups of primitives that are always rendered simultaneously



# "BCH" Example Exercise

We are only considering foreground objects (not the racetrack/grass) here for clarity (we would have to draw way more circles otherwise)

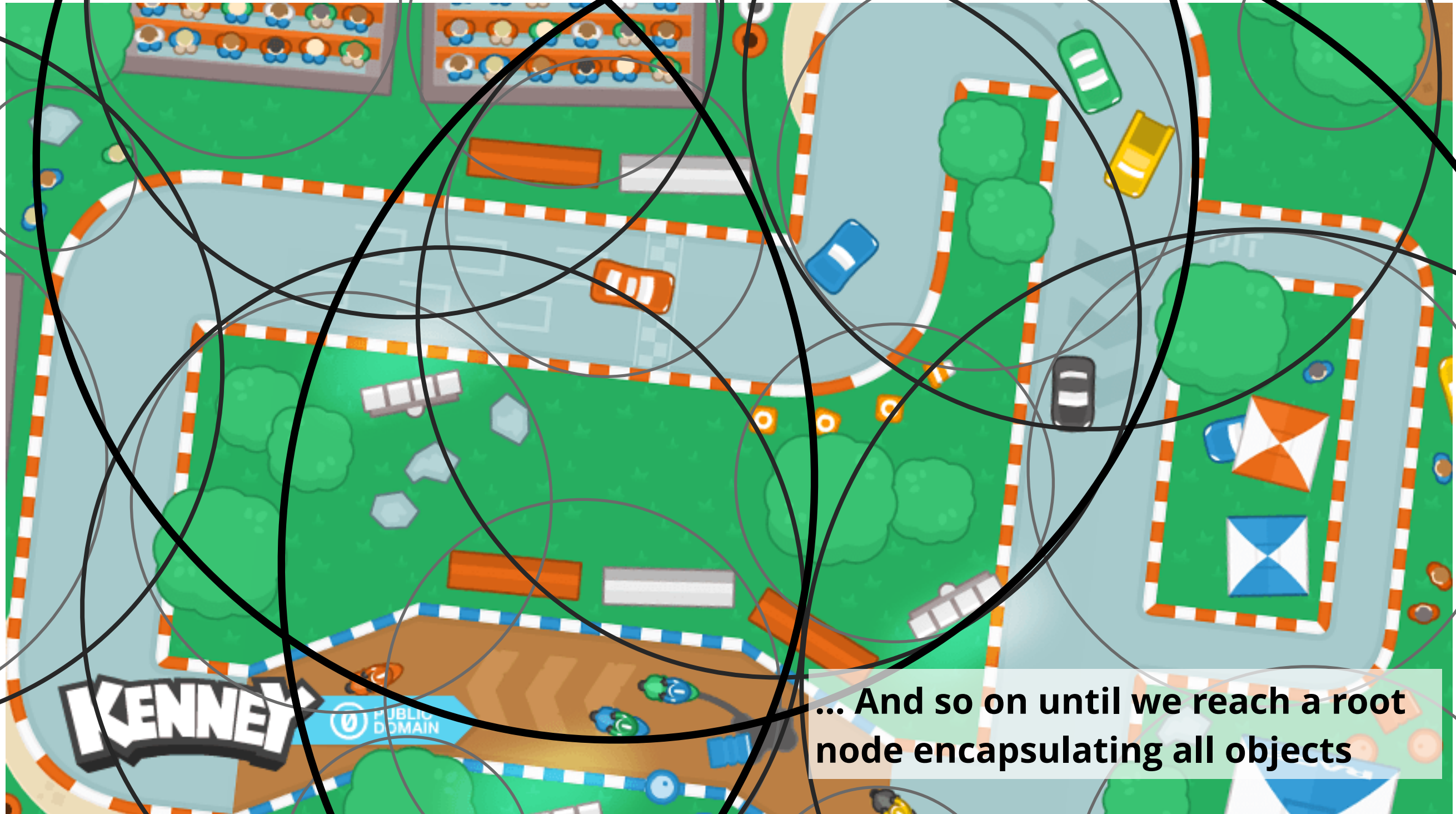


This is one level higher than the leaf nodes. Notice that there are exactly two sub-volumes for each parent volume and that volumes can and do overlap

Source: <https://kenney.nl/assets/racing-pack>

# "BCH" Example Exercise

We are only considering foreground objects (not the racetrack/grass) here for clarity (we would have to draw way more circles otherwise)



... And so on until we reach a root node encapsulating all objects



# “BCH” Implementation

- ▶ Here’s how we could write the header file for this rendering engine.
- ▶ “Flattened Tree” arrangement in the primitives vector
  - start and range are valid for parent nodes and leaf nodes (same in Scotty3D!)

```
/**
 * A node in the BVH accelerator aggregate.
 * The accelerator uses a "flat tree" structure where all the primitives are
 * stored in one vector. A node in the data structure stores only the starting
 * index and the number of primitives in the node and uses this information to
 * index into the primitive vector for actual data. In this implementation all
 * primitives (index + range) are stored on leaf nodes. A leaf node has no child
 * node and its range should be no greater than the maximum leaf size used when
 * constructing the BVH.
 */
struct BVHNode {
    BVHNode(BBox bb, size_t start, size_t range)
        : bb(bb) start(start) range(range) l(NULL) r(NULL) {}
};
```

↳ From bvh.h in Scotty3d

```
class Primitive {
    // ...
    Color color_at_pixel(float x, float y);
    // ...
};

struct Circle {
    float x;
    float y;
    float rad;

    inline bool isect(float x_, float y_) {
        return (x_-x)*(x_-x)+(y_-y)*(y_-y) < rad*rad;
    }
};

struct BCHNode {
    BCHNode *l;
    BCHNode *r;

    Circle bounds;
    size_t start; // start index in Scene::primitives
    size_t range; // number of elts in primitives
};

struct BCHAccel {
    // ...
    BCHNode *root;
};

class Scene {
    // ...
    std::vector<Primitive *> primitives;
    BCHAccel *accel;

    Color color_at_pixel(float x, float y);
};
```

# “BCH” Implementation

- ▶ Basic implementation of `color_at_pixel(...)` without acceleration structure.
- ▶ Query input: Pixel position  
Query output: Color at pixel

```
Color Scene::color_at_pixel(float x, float y) {
    Color cur = Color(0,0,0,0);
    for(int i = 0; i < primitives.size(); ++i) {
        Color top = primitives[i].color_at_pixel(x, y);
        // Alpha blend "over" operator (like DrawSVG)
        cur = Color::over(cur, top);
    }
    return cur;
}
```

- ▶ How would we traverse the BCH, given this header?

```
class Primitive {
    // ...
    Color color_at_pixel(float x, float y);
    // ...
};

struct Circle {
    float x;
    float y;
    float rad;

    inline bool isect(float x_, float y_) {
        return (x_-x)*(x_-x)+(y_-y)*(y_-y) < rad*rad;
    }
};

struct BCHNode {
    BCHNode *l;
    BCHNode *r;

    Circle bounds;
    size_t start; // start index in Scene::primitives
    size_t range; // number of elts in primitives
};

struct BCHAccel {
    // ...
    BCHNode *root;
};

class Scene {
    // ...
    std::vector<Primitive *> primitives;
    BCHAccel *accel;

    Color color_at_pixel(float x, float y);
};
```

# “BCH” Implementation

- Implementation of `color_at_node` that traverses a BCH

```
void Scene::color_at_node(float x, float y,
                          BCHNode *node, Color *cur) {
    if(!node->bounds.isect(x, y)) return;

    if(node->l == nullptr && node->r == nullptr) { // leaf
        for(int i = 0; i < node->range; ++i){
            int j = node->start + i;
            Color top = primitives[j].color_at_pixel(x, y);
            *cur = Color::over(*cur, top);
        }
        return;
    }

    color_at_node(x, y, node->l, cur);
    color_at_node(x, y, node->r, cur);
}

Color Scene::color_at_pixel(float x, float y) {
    Color ret(0,0,0,0);
    color_at_node(x, y, root, &ret);
    return ret;
}
```

- Sidebar: While faster, this method subtly changes (breaks) alpha blending behavior —  
*Why? How do you fix it?*

```
class Primitive {
    // ...
    Color color_at_pixel(float x, float y);
    // ...
};

struct Circle {
    float x;
    float y;
    float rad;

    inline bool isect(float x_, float y_) {
        return (x_-x)*(x_-x)+(y_-y)*(y_-y) < rad*rad;
    }
};

struct BCHNode {
    BCHNode *l;
    BCHNode *r;

    Circle bounds;
    size_t start; // start index in Scene::primitives
    size_t range; // number of elts in primitives
};

struct BCHAccel {
    // ...
    BCHNode *root;
};

class Scene {
    // ...
    std::vector<Primitive *> primitives;
    BCHAccel *accel;

    Color color_at_pixel(float x, float y);
    void color_at_node(float x, float y,
                       BCHNode *node, Color *cur);
};
```

# BCH Takeaways / Questions



- ▶ Our aim has been to minimize the number of primitives considered by our renderer at each pixel
- ▶ Is the BCH successful? How much wasted space (space in each bounding volume that is not covered by a primitive) is there? How much overlap between nodes is there?
  - Both of these issues lead to redundant BVH traversals
- ▶ Is a circle the best bounding shape for this scene?
- ▶ What types of scenes would a BCH be most effective for?

# Back to 3D: Building a BVH

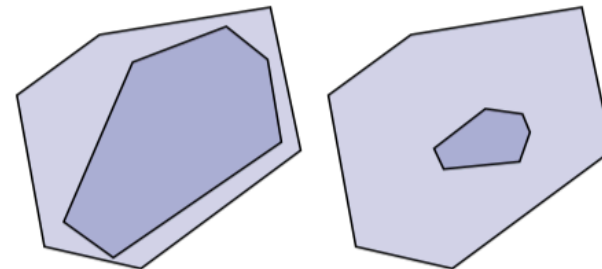
- ▶ With the BCH, we want to minimize “wasted space” in building the actual bounding partition.
  - We could theoretically find the best partitioning of a BCH by brute-forcing all possible partitions and maximizing the ratio of primitive areas to circle areas.
  - Try coming up with faster partitioning schemes...
- ▶ Now consider a pathtracer. What do we want to minimize in our acceleration structure to improve performance?
  - Recall, our query is ray / primitive intersections
  - Intersecting ray directions are totally unpredictable given the randomness of BRDFs
  - How do we “score” a particular partitioning?

# The Surface Area Heuristic

Recall From previous lecture:

- For convex object A inside convex object B, the probability that a random ray that hits B also hits A is given by the ratio of the surface areas  $S_A$  and  $S_B$  of these objects.

$$P(\text{hit } A | \text{hit } B) = \frac{S_A}{S_B}$$



Leads to surface area heuristic (SAH):

$$C = C_{\text{trav}} + \frac{S_A}{S_N} N_A C_{\text{isect}} + \frac{S_B}{S_N} N_B C_{\text{isect}}$$

Assumptions of the SAH (*which may not hold in practice!*):

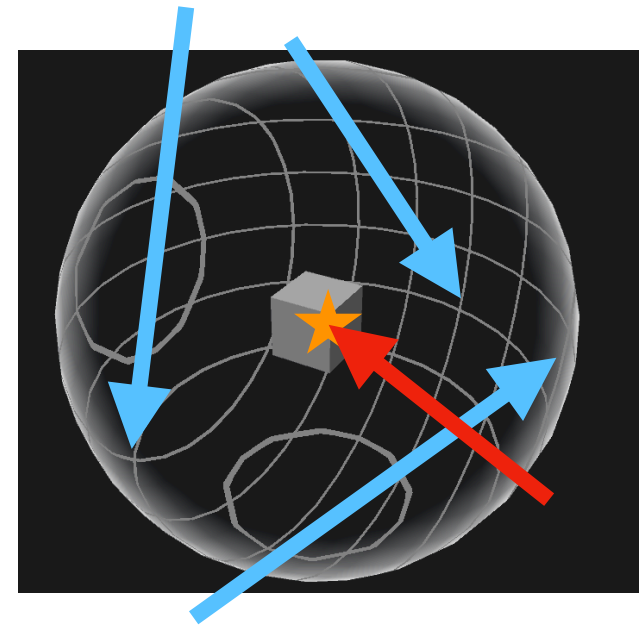
- Rays are randomly distributed
- Rays are not occluded

In short, the SAH is the way that we “score” a BVH, for the specific application of ray-primitive intersection. What kind of queries would the SAH be *bad* at? What alternatives to the SAH are there?

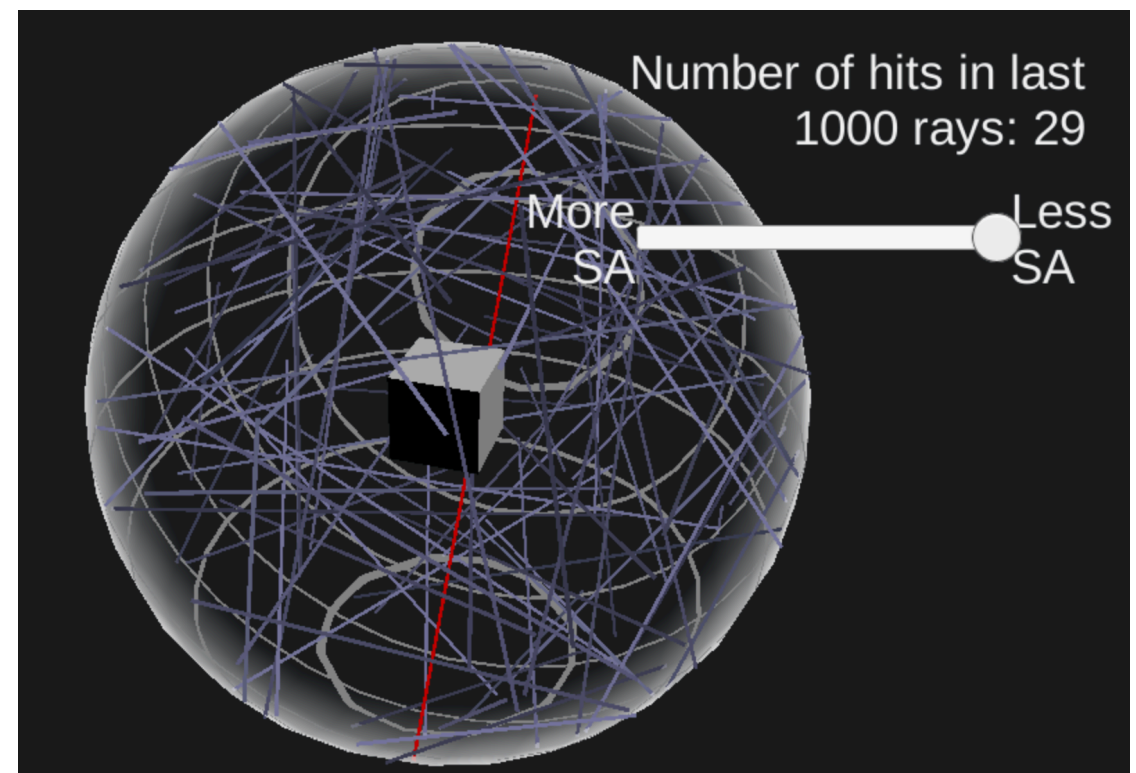
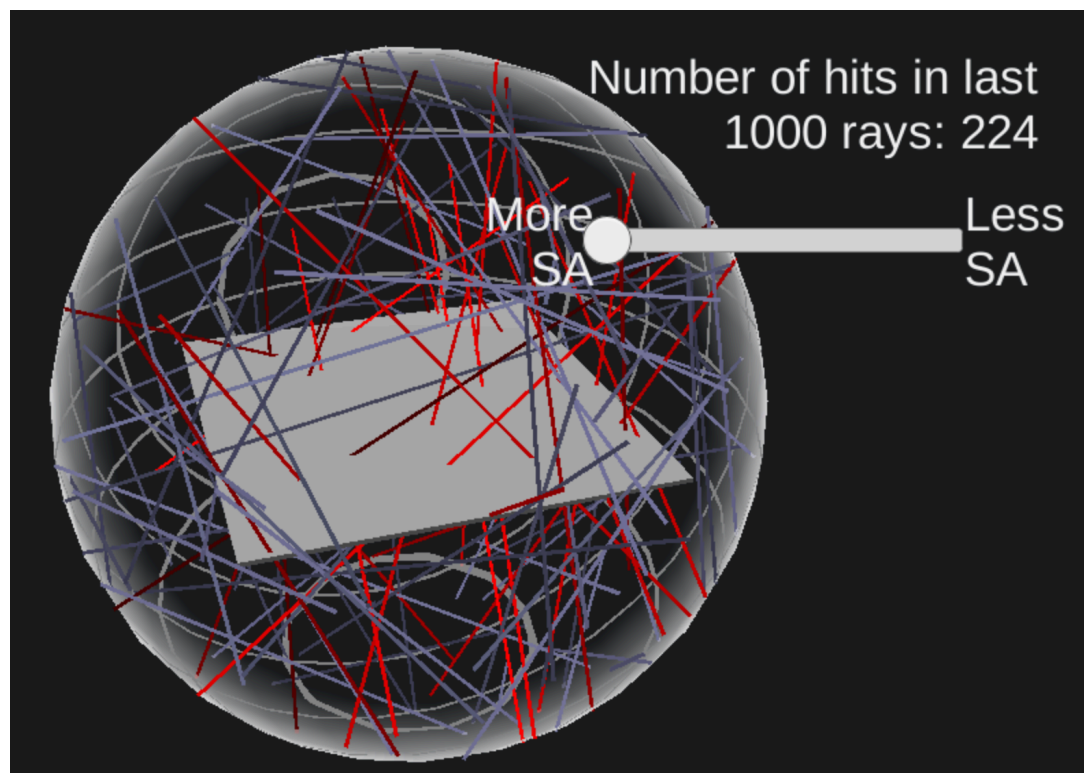


# The Surface Area Heuristic

- ▶ **Demo:** Squashing a cube while preserving its volume increases its Surface Area.
- ▶ How often should we expect to hit cubes with the same volume but different surface areas, with the sample rays randomly distributed about a sphere?



<http://flafla2.github.io/demos/sah-vis/index.html>



# SAH in an axis-aligned BVH

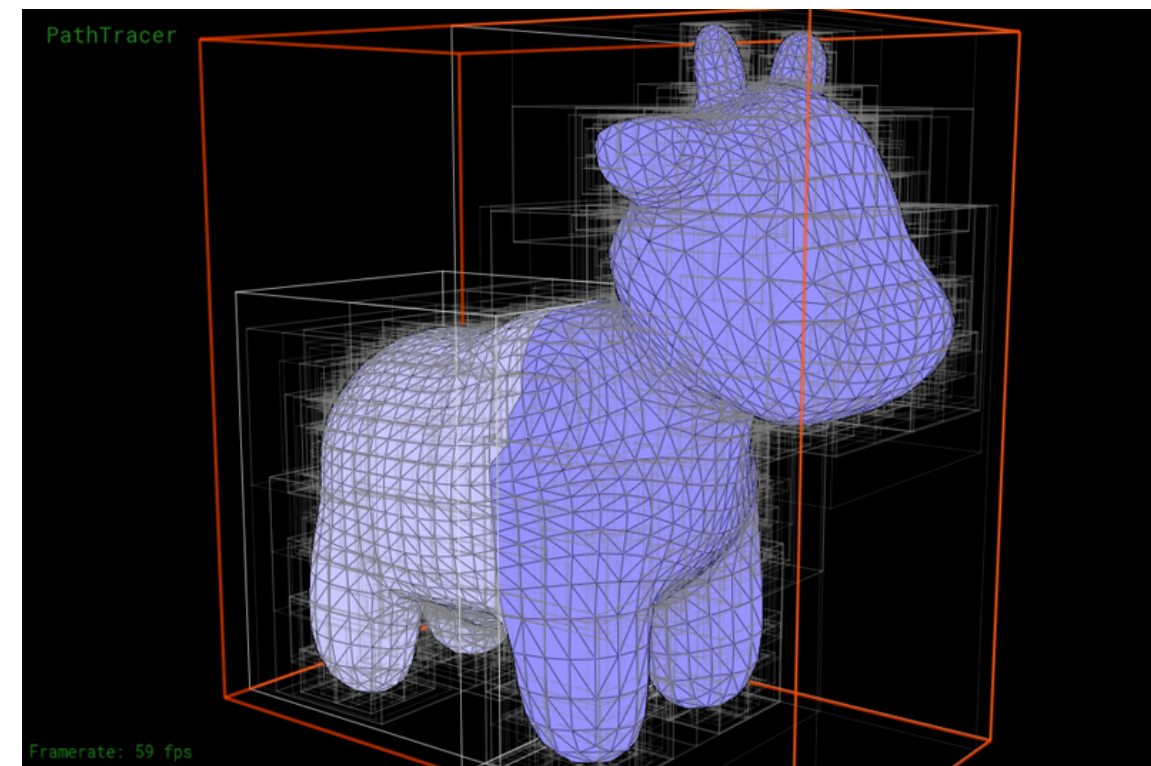
- ▶ When building a BVH, we need to figure out how to partition the primitives, starting at the root node (partitioning into initial left/right subtree) and then recursively partitioning each subtree.
  - We use the SAH to choose our partitioning (minimize  $C$ )
  - Why do we need the  $N_{\{A,B\}}$  term? The  $\frac{S_{\{A,B\}}}{S_N}$  term?

# primitives in subtree A      # primitives in subtree B

SA of bounding box of subtree A      SA of bounding box of subtree B

$$C = C_{\text{trav}} + \frac{S_A}{S_N} N_A C_{\text{isect}} + \frac{S_B}{S_N} N_B C_{\text{isect}}$$

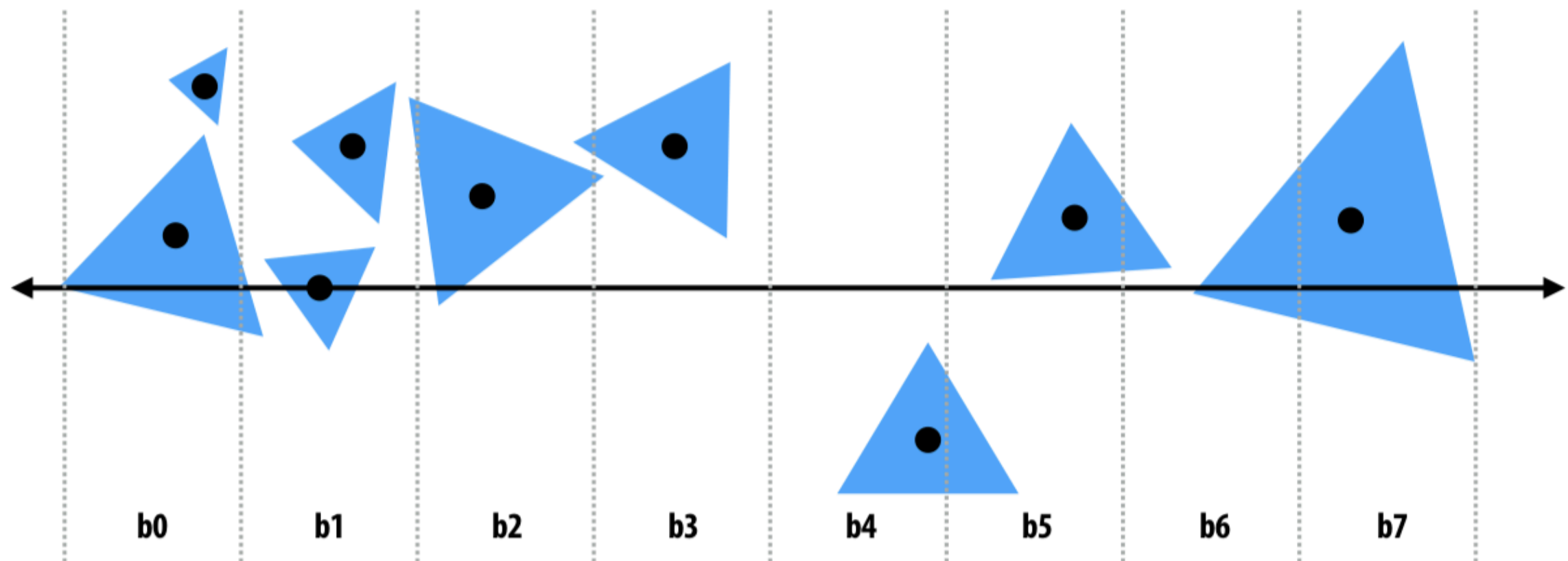
SA of bounding box of parent



- ▶ You can assume  $C_{\text{trav}}$  and  $C_{\text{isect}}$  are 1 as they are constants (irrelevant for comparisons)

# Efficiently implementing partitioning

- Efficient modern approximation: split spatial extent of primitives into  $B$  buckets ( $B$  is typically small:  $B < 32$ )



For each axis:  $x, y, z$ :

initialize buckets

For each primitive  $p$  in node:

$b = \text{compute\_bucket}(p.\text{centroid})$

$b.\text{bbox.union}(p.\text{bbox});$

$b.\text{prim\_count}++;$

For each of the  $B-1$  possible partitioning planes evaluate SAH

Recurse on lowest cost partition found (or make node a leaf)

*(from previous lecture)*

# min\_t, max\_t?

- ▶ Important: Each ray object in Scotty3D has variables min\_t and max\_t
  - min\_t is a *lower bound* on the distance along the ray of the ray / scene intersection
  - max\_t is an *upper bound* on the distance along the ray of the ray / scene intersection
- ▶ When a ray is found to intersect a primitive (e.g. in Triangle::intersect()), does min\_t change? max\_t?
- ▶ These variables are very important for the BVH to actually work! (Why?)
- ▶ Also will find in Task 5, it is helpful to set min\_t to some epsilon when a ray's origin is on the surface of an object.

# Questions?